UNIVERSITÉ D'OTTAWA • UNIVERSITY OF OTTAWA

Sciences • Science

Mathématiques et statistique
Mathematics and Statistics

MAT 1341A Mini-test 1

September 26, 2003. Duration: 80 minutes.

Instructor: Barry Jessup.

1	
2	
3	
4	
5	
sub-total	
6	
7	
Total	

Family Name:_____

First Name:_____

Student number:_____

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are allowed. The use of calculators, cell phones, pagers or any text storage or communication device is not permitted.
3. Read each question carefully -you will save yourself time and unnecessary grief later on.
4. Questions 1 to 5 are multiple choice. These questions are worth 2 points each and no part marks will be given. Please record your answers in the space provided above.
5. Questions 6 and 7 require a complete solution, and are worth 6 points each, so spend your time accordingly. **The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
6. Where it is possible to check your work, do so.
7. Good luck! Bonne chance!

1. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid xy = xz = 0\}$. Then,

A. $(0, 0, 0) \in W$ but W is not closed under multiplication by scalars
B. W is closed under addition and W is closed under multiplication by scalars
C. W is closed under addition but W is not closed under multiplication by scalars
D. W is not closed under addition but W is closed under multiplication by scalars
E. $(0, 0, 0) \notin W$ but W is closed under addition
F. None of the other statements is true.

2. Which of these sets: $U = \{x, x + y, y) \mid x,\ y \in \mathbf{R}\}$, $V = \{(x + y,\ y,\ y) \mid x,\ y \in \mathbf{R}\}$ and $W = \{(x^2,\ y,\ x) \mid x,\ y \in \mathbf{R}\}$ are subspaces of $\mathbf{R}^3$?

A. U and V only
B. U and W only
C. V and W only
D. U only
E. V only
F. W only

3. Which of the following are subspaces of $\mathbf{M}_{2\,2}$?

(1) $\{\begin{bmatrix} x & y \\ y & z \end{bmatrix} \in \mathbf{M}_{2\,2} \mid x,\ y,\ z \in \mathbf{R}\}$

(2) $\{\begin{bmatrix} 0 & y \\ -y & 0 \end{bmatrix} \in \mathbf{M}_{2\,2} \mid y \in \mathbf{R}\}$

(3) $\{\begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathbf{M}_{2\,2} \mid xw - zy = 0 \in \mathbf{R}\}$

(4) $\{\begin{bmatrix} x & y \\ z & -x \end{bmatrix} \in \mathbf{M}_{2\,2} \mid x,\ y,\ z \in \mathbf{R}\}$

A. (1) and (2)
B. (1) and (3)
C. (1), (2) and (3)
D. (2), (3) and (4)
E. (3) and (4)
F. (1), (2) and (4)

4. Which two of the following are subspaces of $\mathbf{F}(\mathbf{R}) = \{f \mid f\colon \mathbf{R} \to \mathbf{R}\}$?

$S = \{f \in \mathbf{F}(\mathbf{R}) \mid f(x) = f(2x),\ \forall x \in \mathbf{R}\}$
$T = \{f \in \mathbf{F}(\mathbf{R}) \mid f(0) > 0\}$
$U = \{f \in \mathbf{F}(\mathbf{R}) \mid f(-x) = -f(x),\ \forall x \in \mathbf{R}\}$
$V = \{f \in \mathbf{F}(\mathbf{R}) \mid f(0) = 1\}$

A. V and T.
B. T and U.
C. S and T.
D. V and S.
E. S and U.
F. V and U.

5. Let $W = \{(x, y, z, w) \in \mathbf{R}^4 \mid x = 0 \text{ and } y + z + w = 0\}$. Then,

A. $W = \text{span}\{(1, 0, 0, 0),\ (0, 0, 1, 0)\}$
B. $W = \text{span}\{(0, 1, 0, 1),\ (0, 1, -1, 0)\}$
C. $W = \text{span}\{(0, 1, 0, 0)\}$
D. $W = \text{span}\{(0, -1, 1, 0),\ (0, -1, 0, 1)\}$
E. $W = \text{span}\{(0, 0, 1, 0)\}$
F. $W = \text{span}\{(0, 2, -1, -1),\ (0, 4, -2, -2)\}$

6. Let $u_0 = (1,1)$, $v_0 = (-1,-1)$and and

$$U = \{\, w \in \mathbf{R}^2 \mid ||w - u_0|| = ||w - v_0|| \,\}$$

a) Show that $||w - u_0||^2 = ||w - v_0||^2 \iff w \cdot (2,2) = 0$, and hence conclude that

$$U = \{(x,y) \in \mathbf{R}^2 \mid x + y = 0 \quad \}$$

(Hint: expand both sides using the dot product).

b) Using the conclusion of (a), give a complete geometric description of U.

c) Explain why U is a subspace of $\mathbf{R}^2$.

d) Find a spanning set for U.

7. Consider the vector space $\mathbf{F}([-1,1]) = \{f \mid f\colon [-1,1] \to \mathbf{R}\}$, with the standard operations. Define functions f, g, h, and k by the formulae (valid for all $x \in [-1,1]$):

$$f(x) = x, \quad g(x) = \frac{1}{x+2}, \quad h(x) = \frac{x^2+2x+3}{x+2}, \quad k(x) = 1,$$

and let $W = \operatorname{span}\{f, g\}$.

a) Explain why W is a subspace of $\mathbf{F}([-1,1])$.

b) Show that $h \in W$.

c) Explain why $k \notin W$. (Hint: Suppose $k \in W$ and evaluate both sides at 3 different points.)