MATH 1007B **Tutorial #1 – solution** **13 September 2012**

Work in small groups (3-4 people). Do NOT divide up the problems and work on them separately. **Work together** on all problems by sharing insights and difficulties, but **each student must hand in a set of solutions.** Ask your TA if you need help.

1. **Simplify** as much as possible

a) $(-32)^{2/5} = (-2)^2 = 4$

b) $-32^{2/5} = -2^2 = -4$

c) $\left(\sqrt[7]{8x^3}\right)^{7/3} = \left(\sqrt[7]{(2x)^3}\right)^{7/3} = \left(\left((2x)^3\right)^{1/7}\right)^{7/3} = \left((2x)^{3/7}\right)^{7/3} = 2x$

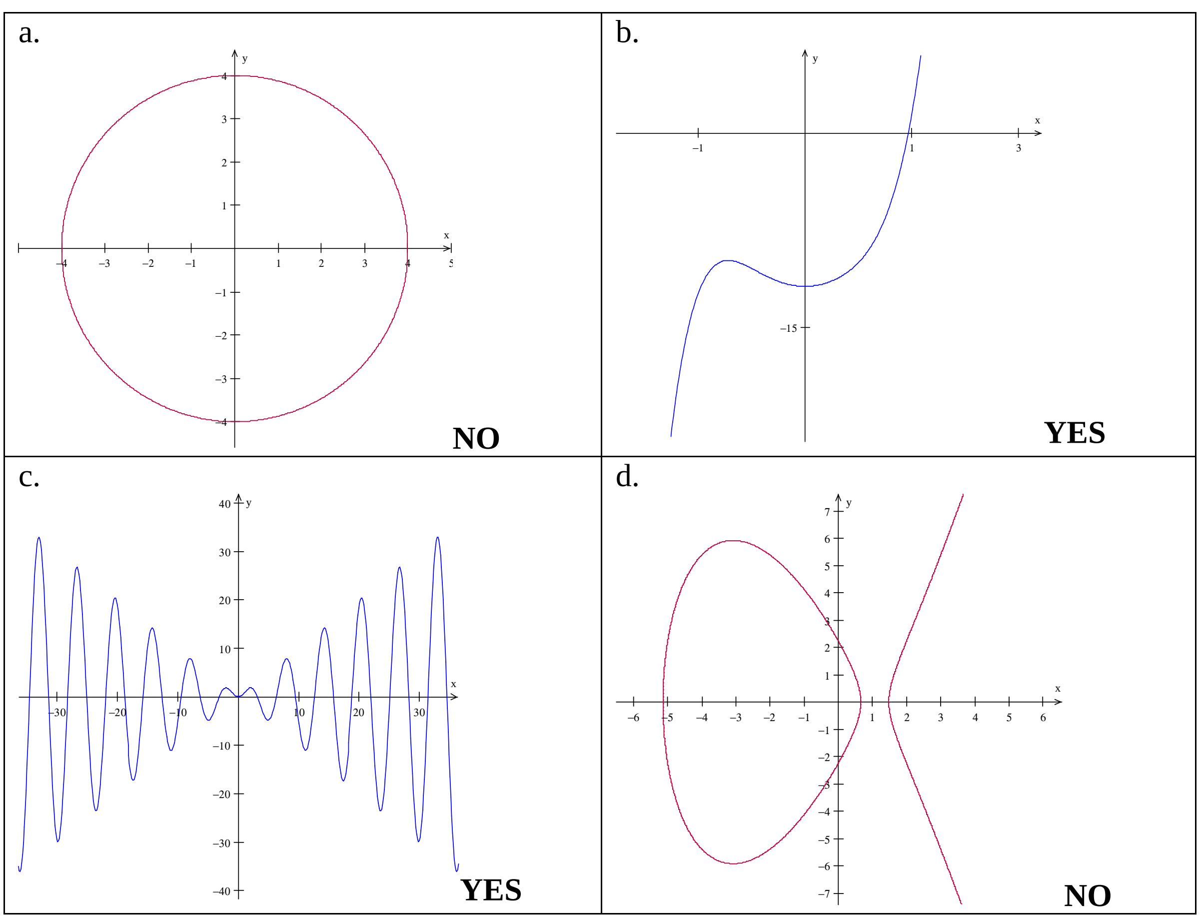
d)

$$\frac{\frac{a}{3b}-\frac{b}{a}}{b}+\frac{1}{a} = \frac{\frac{a^2-3b^2}{3ab}}{b}+\frac{1}{a} = \frac{a^2-3b^2}{3ab^2}+\frac{1}{a} = \frac{a^2-3b^2}{3ab^2}+\frac{3b^2}{3ab^2} =$$

$$\ldots = \frac{a^2-3b^2+3b^2}{3ab^2} = \frac{a}{3b^2}$$

e) $\left(\dfrac{2ab^{-1}}{6a^2b^{-3}}\right)^{-2} = \left(\dfrac{b^2}{3a}\right)^{-2} = \left(\dfrac{3a}{b^2}\right)^{2} = \dfrac{9a^2}{b^4}$

2. Which of the following is the graph of a function?



Think 'vertical line test'... a function crosses any vertical line once and only once at each value of the independent variable *x* in its domain.

3. **Determine the rule of a linear function *f*,** given the following: the curve associated with this function passes through the points (–1, –4) and (2, 20)

The rule of the linear function takes on the following general form: $y = f(x) = mx + b$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, with $(x_1, y_1) = (-1, -4)$, $(x_2, y_2) = (2, 20)$ — the choice is arbitrary

$m = \frac{20 - (-4)}{2 - (-1)} = 8$

$\therefore$ The rule is $y = f(x) = 8x + b$

Now substitute (x_1, y_1) or (x_2, y_2) for (x, y):

(x_1, y_1): $y_1 = -4 = 8x_1 + b = 8(-1) + b \Rightarrow b = 4$

(x_2, y_2): $y_2 = 20 = 8x_2 + b = 8(2) + b \Rightarrow b = 4$

$\therefore y = f(x) = 8x + 4$

4. **Determine the domain** of the following functions, whose rule is given by:

a. $f(x) = 3x^3 - 5x + 12$ dom (f) = $\mathbb{R}$, because this is a polynomial	b. $f(x) = \ln(3x + 8)$ We require $3x + 8 > 0 \Rightarrow x > -8/3$ ... i.e. dom (f) = $(-8/3, +\infty)$
c. $g(x) = \frac{2x^2 + 5}{4 - 12x} - \sqrt{x+1}$ We require $4 - 12x \neq 0$ AND $x + 1 \geq 0$,so dom (g) = $x \neq 1/3$ AND $x \geq -1$ ... i.e. dom (g) = $[-1, 1/3) \cup (1/3, +\infty)$	d. $f(x) = \sqrt[4]{x^2 - 1} + \sqrt{2 - x}$ We require $x^2 - 1 \geq 0$ AND $2 - x \geq 0$, so dom (f) = ($x \leq -1$ OR $x \geq 1$) AND $x \leq 2$... i.e. dom (f) = $(-\infty, -1] \cup [1, 2]$

5. Let a function *f* be given by the following rule: $f(x) = \frac{1}{|x^2 - 3x + 2|}$

Rewrite the rule as a piece-wise defined function.

<u>Hint</u>: Recall that $f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

Note that the expression $x^2 - 3x + 2$ factors as $(x-1)(x-2)$ and is positive in the interval $(-\infty, 1) \cup (2, +\infty)$ and negative in the interval $(1, 2)$. (Do you see why that is? Think of a parabola having roots *x*=1 and *x*=2 and pointing upwards... for which values of *x* is the curve above/below the *x* axis?)

When the expression is positive (in the appropriate interval), its absolute value is equal to the expression itself, and so we may write:

$$f(x) = \frac{1}{|x^2 - 3x + 2|} = \frac{1}{x^2 - 3x + 2}, \quad x \in (-\infty, 1) \cup (2, +\infty)$$

When the expression is negative, its absolute value is equal to the negative of itself and so we may write:

$$f(x) = \frac{1}{|x^2 - 3x + 2|} = -\frac{1}{x^2 - 3x + 2}, \quad x \in (1, 2)$$

Putting it all together, it is possible to get rid of the absolute value sign and write the rule of the function as follows:

$$f(x)=\frac{1}{\left|x^2-3x+2\right|} = \begin{cases} \dfrac{1}{x^2-3x+2} & x<1 \\ -\dfrac{1}{x^2-3x+2} & 1<x<2 \\ \dfrac{1}{x^2-3x+2} & x>2 \end{cases}$$

6. If $f(x)=\dfrac{1}{x-1}$ and $g(x)=\sqrt{3x+5}$, **determine**:

$$f\circ f(x)=f(f(x))=\frac{1}{(f(x))-1}=\frac{1}{\left(\dfrac{1}{x-1}\right)-1}=\frac{1}{\left(\dfrac{1}{x-1}\right)-\dfrac{x-1}{x-1}}=\frac{x-1}{2-x}$$

$$g\circ f(x)=g(f(x))=\sqrt{3(f(x))+5}=\sqrt{3\left(\dfrac{1}{x-1}\right)+5}=\sqrt{\frac{3}{x-1}+\frac{5(x-1)}{x-1}}=\sqrt{\frac{5x-2}{x-1}}$$

$$f\circ g(x)=f(g(x))=\frac{1}{(g(x))-1}=\frac{1}{\left(\sqrt{3x+5}\right)-1}=\frac{\sqrt{3x+5}+1}{3x+4}$$

$$g\circ g(x)=g(g(x))=\sqrt{3(g(x))+5}=\sqrt{3\sqrt{3x+5}+5}$$

7. **BONUS:** We have not covered this yet :

Let f be a function and dom (f) = [–1, 27]. Also let $g(x)=x^3$. **Determine the domain of the composition** $f\circ g(x)$**.**

Hint: if the domain of *f* is specified, then the allowable values of any argument of *f* (including *g* (*x*)) are determined by that domain. The specific rule for the function *g* will then determine the allowable values for *x*.

SOLUTION: Please see the supplementary note on the domain and range of a composition of functions (posted in WebCT in the 'extra material' folder).

Note that ran (*g*) = ℝ, so that the intersection of dom (*f*) and ran (*g*) is $dom(f)=[-1,27]$. Thus,

dom ($f\circ g$) = $\{x\in\Re \mid u=g(x)=x^3\in[-1,27]\}\Rightarrow -1\le x^3\le 27$ = [–1, 3]

... and that is the domain we are looking for.