Be sure that this examination has 7 pages including this cover

The University of British Columbia

Midterm Examinations - February 7, 2012

Mathematics 101

Integral Calculus with Applications to Physical Sciences and Engineering

Closed book examination — Time: 1 hours

Last Name: _____ **First Name:** _____

Student Number: _____ **Instructor's Name:** _____

Signature: _____ **Section Number:** _____

Rules governing examinations

1. Each candidate must be prepared to produce, upon request, a UBC-card for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
   (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
   (b) Speaking or communicating with other candidates.
   (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material form the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		8
2		12
3		12
4		6
5		6
6		6
Total		50

Marks

Unless otherwise indicated, numerical answers must be simplified completely for full marks. Show all your work. Use backs of pages if necessary. Calculators are not allowed.

[8] **1.** (a) [4] Evaluate $\int_0^1 \sqrt{x}(x+\sqrt{x})\ dx$.

(b) [4] Evaluate $\int \frac{(\ln x)^2}{x}\ dx$

Continued on page 3

[12] **2.** (a) [4] Evaluate $\int (\ln x)^2 \, dx$.

(b) [4] Evaluate $\int_{-\pi}^{\pi} x^{101} \cos x \, dx$ *without* antidifferentiation. Explain your reasoning.

(c) [4] Evaluate $\int_0^2 (x+3)(x-1)^5 \, dx$.

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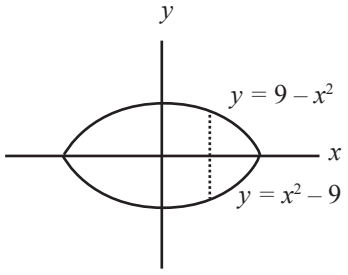
[12] **3.** (a) [4] Find the area of the region enclosed by the curves $y = x^2$ and $y = 2x$.

(b) [4] A solid is obtained by rotating the region enclosed by the curves $y = e^{-x}$, $y = 1$, and $x = 2$ about the line $y = -1$. Write down a definite integral giving the volume of the solid. **Do not evaluate this integral.**

(c) [4] The horizontal base of a solid is the region enclosed by the curves $y = 9 - x^2$ and $y = x^2 - 9$, as shown in the diagram below. Each vertical cross section perpendicular to the x-axis is a triangle, whose base is drawn as a dotted line in the diagram. The height of the triangle always equals the length of the base. Write down a definite integral giving the volume of the solid. **Do not evaluate this integral.**



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[6] **4.** Evaluate $\int_0^3 (x^2 + 4x^3)\, dx$ using a limit of Riemann sums. You may use the formulas

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

and

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

No credit will be given for a solution that uses antidifferentiation, but you may check your answer using antiderivatives.

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[6] **5.** A cable that has linear density 2 kg/m is attached to a bucket filled with coal that has mass 300 kg. The bucket is initially at the bottom of a 500-m-deep vertical mine shaft. The bucket is lifted to the midpoint of the shaft by winding up the top half of the cable. Find the amount of work done, in joules. You may use the value $g = 9.8$ m/s^2 for the acceleration due to gravity. *You do not need to simplify your answer in this problem, but you should evaluate any definite integral(s) that arise.*

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[6] **6.** (a) [3] Let $F(x) = \displaystyle\int_{x^2}^{7x} \frac{u}{1+u^2}\, du$. Find $F'(x)$.

(b) [3] A certain continuous function $f(x)$ defined on $[0, \infty)$ has the property that the average value of $f(x)$ on the interval $[0, x^2]$ equals x, for all $x > 0$. Determine $f(x)$, with explanation.

The End