School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2011
TEST 5

Any non-programmable calculator permitted, 1 blank ~~sheet~~ permitted for roughs

Print Name : SOLUTIONS

Student Number:

Tutorial Section (A1, A4, ...):

PART I: Multiple Choice Questions
(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Evaluate $\int_0^1 xe^x\,dx$.
   (a) 0, (b) 1, (c) $e-1$, (d) e. b

2. **[2 marks]** Evaluate $\int_1^2 \ln y\,dy$.
   (a) 1, (b) 1/2, (c) $\ln 2$, (d) $-1+2\ln 2$. d

3. **[2 marks]** Evaluate $\int_0^{\pi/2} x\sin x\,dx$.
   (a) π, (b) 1/4, (c) 1, (d) 0. c

4. **[2 marks]** Evaluate $\int (t-1)e^{2t}\,dt$.
   (a) $\frac{e^{2t}}{4}(2t-3)+C$, (b) $\frac{te^{2t}}{2}+\frac{e^{2t}}{4}+C$, (c) $te^{2t}-e^{2t}+C$, (d) $\frac{e^{2t}}{2}(t-1)+C$. a

5. **[2 marks]** Answer TRUE or FALSE:
   $\int (\ln x)^2\,dx = x(\ln x)^2 - 2\int \ln x\,dx$
   (a) TRUE, (b) FALSE a.

PART II: Show all work here.
No additional pages will be accepted

6. **[5+ 5 marks]** :
   a) Find an antiderivative of the function f defined by $f(t)=t\cos 2t$.
   b) Evaluate $\int (x^2-3)e^x\,dx$.

a)
$$\begin{array}{ccc} t & + & \cos 2t \\ \searrow & & \\ 1 & - & \frac{\sin 2t}{2} \quad \leftarrow (1) \\ \searrow & & \\ 0 & & -\frac{\cos 2t}{4} \quad \leftarrow (1) \end{array}$$

(1) (1) ← (1)

$\therefore \frac{t\sin 2t}{2} + \frac{\cos 2t}{4}$ is an antiderivative. (+C need not be added).

b)

x^2-3	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

(2)

(1) (1) (1)

$$\therefore \int (x^2-3)e^x\,dx = (x^2-3)e^x - 2xe^x + 2e^x + C.$$

7. **[5+5 marks]**

a) Evaluate $\int e^{2x}\sin x\,dx.$ $= I.$

b) Evaluate $\int x^2(\ln x)^3\,dx.$ $= J.$

a)

e^{2x}	$+$	$\sin x$
$2e^{2x}$	$-$	$-\cos x$
$4e^{2x}$		$-\sin x$

(1) (1) (1) (1) (1)

$$\therefore I = \boxed{\frac{1}{5}}\left\{-e^{2x}\cos x + 2e^{2x}\sin x\right\}.$$

My	-4
C	$+4$
A	$+5$
R	$1/5$

$$\therefore I = \frac{1}{5}\left(2e^{2x}\sin x - e^{2x}\cos x\right)$$ (1)

b). $\ln x = t$, $x = e^t$, $dx = e^t dt$ (1)

Then $J = \int e^{2t}t^3 e^t\,dt = \int t^3 e^{3t}\,dt$

t^3	$+$	e^{3t}
$3t^2$	$-$	$e^{3t}/3$
$6t$	$+$	$e^{3t}/3^2$
6	$-$	$e^{3t}/3^3$
0		$e^{3t}/3^4$

$$\therefore J = t^3\frac{e^{3t}}{3} - 3t^2\frac{e^{3t}}{3^2} + 6t\frac{e^{3t}}{3^3} - 6\frac{e^{3t}}{3^4} + C$$ (1)

$$= \frac{1}{3}x^3(\ln x)^3 - \frac{3x^3(\ln x)^2}{9} + \frac{6x^3\ln x}{27} - \frac{6x^3}{81} + C$$

either one is OK. (OR)

$$= \frac{1}{3}x^3(\ln x)^3 - \frac{x^3(\ln x)^2}{3} + \frac{2x^3\ln x}{9} - \frac{2x^3}{27} + C$$

(1) (1)

otal: /30