SOLUTIONS

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2011

TEST 3

Any non-programmable calculator permitted, 1 blank sheets permitted for roughs (do not hand in)

Print Name : _____

Student Number: _____

Tutorial Section (A1, B2, A4, ...): _____

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Let $f(t) = 4^{\sqrt{t+1}}$. Then its derivative $f'(8)$ is equal to:

c (a) 64, (b) 16, (c) $\frac{64\ln 2}{3}$, (d) $\frac{9\ln 4}{2}$.

2. **[2 marks]** Let $f(x) = \log_2(\sqrt{x})$. Then its derivative $f'(3)$ is equal to:

d (a) $\frac{1}{\log_2 8}$, (b) 0 (c) $\frac{1}{2}$, (d) $\frac{1}{6\ln 2}$

3. **[2 marks]** Find all the critical points of the function f defined by $f(x) = \frac{2x}{4+x^2}$ where $-\infty < x < +\infty$.

a (a) $x = \pm 2$, (b) $x = 0$, (c) $x = \pm 1$, (d) f has no critical points.

4. **[2 marks]** Let $f(x) = e^{-x^2}$. Then f has a relative maximum at:

b (a) $x = 1$, (b) $x = 0$, (c) $x = -1$, (d) $x = 2$

5. **[2 marks]** Answer TRUE or FALSE:
Let f be defined by $f(t) = t^4 + t + 1$. The f has a point of inflection at $t = 0$.

b (a) TRUE (b) FALSE

PART II: Show all work here.
No additional pages will be accepted

6. **[5+5 marks]** : a) Evaluate the limit $\lim_{x\to 0^+} \frac{\log_4 \sqrt{x}}{\ln x}$.
b) A function f defined on $-\infty < x < +\infty$ has the property that its derivative is given by $f'(x) = x(x+3)(x^2-2)$. Find all the intervals where f is increasing and decreasing.

a) Use L'Hospital's Rule ← (1)

$$\lim_{x\to 0^+} \frac{D(\log_4 \sqrt{x})}{D(\ln x)} = \lim_{x\to 0^+} \frac{\frac{1}{\sqrt{x}} \cdot \frac{1}{\ln 4} \cdot \frac{1}{2\sqrt{x}}}{\frac{1}{x}}$$

(1) (1) (1)

$$= \lim_{x\to 0^+} \frac{1}{2\ln 4} \cdot \frac{x}{x}$$

$$= \frac{1}{2\ln 4} \leftarrow (1)$$

Use a sign decomposition table (SDT) or something similar.
Here f is increasing/decreasing when $f'(x)>0$ / $f'(x)<0$ respectively. Note $x^2-2=(x-\sqrt{2})(x+\sqrt{2})$.

2	x	$x+3$	$x-\sqrt{2}$	$x+\sqrt{2}$	Sign of $f'(x)$	INCR? DECR?
$(-\infty,-3)$	−	−	−	−	+	INCR
$(-3,-\sqrt{2})$	−	+	−	−	−	DECR
$(-\sqrt{2},0)$	−	+	−	+	+	INCR
$(0,\sqrt{2})$	+	+	−	+	−	DECR
$(\sqrt{2},\infty)$	+	+	+	+	+	INCR.

$\therefore$ f increases for $x<-3$, $-\sqrt{2}<x<0$, and $x>\sqrt{2}$ (①, ①, ①)

f decreases for $-3<x<-\sqrt{2}$, and $0<x<\sqrt{2}$ (①, ①)

7. [6+4 marks] a) Let $f(x)=x2^{-x}$, $-\infty<x<+\infty$. Find all its critical points [1 mark], points of increase/decrease [2 marks], where it is concave up and down [2 marks], and its horizontal asymptotes [1 mark].

b) $f(x)=e^{-x}\log_4(e^x)$. Calculate $f'(0)$.

a) $f'(x)=2^{-x}(1-x\ln 2)=0$, $x=\frac{1}{\ln 2}$ ← ① ONLY crit pt

INCR when $f'>0$, i.e. $1-x\ln 2>0$, i.e. $x<\frac{1}{\ln 2}$ ← ①

DECR ——— $f'<0$ ——— <0, i.e. $x>\frac{1}{\ln 2}$ ← ①

$f''(x)=-2^{-x}\ln 2\,(2-x\ln 2)$

$\therefore$ CONC UP if $2-x\ln 2<0$, i.e. $x>\frac{2}{\ln 2}$ ← ①

& CONC-DOWN if ——— >0, i.e. $x<\frac{2}{\ln 2}$ ← ①

HA: $\lim_{x\to\infty}\left(\frac{x}{2^x}\right)=\lim_{x\to\infty}\frac{1}{2^x\ln 2}=0$ ← ①

(by L'Hospital's Rule)

b)

$f'(x)=D(e^{-x})\log_4(e^x)+e^{-x}D(\log_4 e^x)$

$=-\underbrace{e^{-x}\log_4(e^x)}_{①}+\underbrace{e^{-x}\cdot\left(\frac{1}{e^x}\right)\frac{e^x}{\ln 4}}_{②}$

$\therefore$ at $x=0$

$f'(0)=-1\cdot 0+\frac{1}{\ln 4}$

$f'(0)=\frac{1}{\ln 4}$ ← ①