School of Mathematics and Statistics
Carleton University
Math. 69.1004, Fall 2011

TEST 2

Any non-programmable calculator permitted, 1 blank sheets permitted for roughs (do not hand in)

Print Name : SOLUTIONS

Student Number:

Tutorial Section (A1, B2, A4, ...):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Let y be given implicitly as a differentiable function of x by $3x^2\cos y + 3y^2 - 3 = 0$. Then the slope of the tangent line of the curve $y = y(x)$ at the point (x, y) where $x = 0$, $y = 1$ is equal to:

d (a) 2, (b) $+\infty$, (c) $\frac{1}{2}$, (d) 0.

2. **[2 marks]** A differentiable function g has the property that $g(\pi) = 3$, $g'(3) = 2$ and $g'(\pi) = 2$. What is the value of the derivative of $g(g(x))$ at $x = \pi$?

a (a) 4, (b) 2, (c) 0, (d) -6

3. **[2 marks]** Let $f(x) = \text{Arctan}(\sqrt{x^2+1})$. Then $f'(1)$ is equal to:

b (a) 1/3, (b) $1/(3\sqrt{2})$, (c) $1/\sqrt{2}$, (d) $3/\sqrt{2}$

4. **[2 marks]** Find $L = \lim_{x\to 0} \frac{\text{Arcsin } 2x}{4x}$.

b (a) $L = 0$, (b) $L = 1/2$, (c) $L = -3/2$, (d) $L = 1$

5. **[2 marks]** Answer TRUE or FALSE:

a $\lim_{x\to\infty} \frac{x^{2010} - 2x + 7}{x^{2011}} = 0.$

(a) TRUE, (b) FALSE

PART II: Show all work here.
No additional pages will be accepted

6. **[5+5 marks]** : a) $f(x) = \text{Arctan}\,(\cos(\sqrt{x}))$. Find $f'(x)$ (there is no need to simplify your answer). b) Evaluate the following limit $\lim_{x\to\infty} 2\sqrt{x}\left(\sqrt{3x+1} - \sqrt{3x}\right)$. (Hint: Rationalize the denominator):

a)
(1) → $f(x) = \text{Arctan}\,\square$
(1) → $f'(x) = \frac{D\square}{1+\square^2}$
(1) → $\square = \cos\sqrt{x}$
(1) → $D\square = -\frac{\sin\sqrt{x}}{2\sqrt{x}}$

$$f'(x) = -\frac{\sin\sqrt{x}}{2\sqrt{x}\,(1+\cos^2\sqrt{x})}$$ (1)

b) $$2\sqrt{x}\left(\sqrt{3x+1} - \sqrt{3x}\right) = 2\sqrt{x}\,\frac{\left(\sqrt{3x+1} - \sqrt{3x}\right)\left(\sqrt{3x+1} + \sqrt{3x}\right)}{\sqrt{3x+1} + \sqrt{3x}}$$ (1)

$$= \frac{2\sqrt{x}\,(3x+1-3x)}{\sqrt{3x+1} + \sqrt{3x}} = \frac{2\sqrt{x}}{\sqrt{3x+1} + \sqrt{3x}} =$$ (1)

$$= \frac{2\sqrt{x}}{\sqrt{x}\left(\frac{\sqrt{3x+1}}{\sqrt{x}} + \frac{\sqrt{3x}}{\sqrt{x}}\right)} = \frac{2}{\sqrt{\frac{3x+1}{x}} + \sqrt{3}}$$

$$= \frac{2}{\sqrt{3+\frac{1}{x}} + \sqrt{3}} \xrightarrow[x\to\infty]{} \boxed{\frac{1}{\sqrt{3}}}$$

$$\therefore \lim_{x\to\infty} \frac{2}{\sqrt{3+\frac{1}{x}} + \sqrt{3}} = \boxed{\frac{1}{\sqrt{3}}}$$

7. **[5+5 marks]**

a) Evaluate the following limit using any method:

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}.$$

b) Consider the function f defined by $f(x) = \sqrt{x+1}$ with domain $\mathrm{Dom}(f) = \{x | -1 \le x \le 0\}$. Answer the following questions about its inverse function, $F(x)$.

(a) **[1 mark]** Does f have an inverse function, F? Explain.

(b) **[1 mark]** What is the domain of the inverse function, F?

(c) **[1 mark]** What is the range of the inverse function, F?

(d) **[2 marks]** What is the value of the inverse function at x, i.e., calculate $F(x)$, for $x \in \mathrm{Dom}(f)$?

a) Use L'Hopital's Rule, FORM $= \frac{0}{0}$

$$\therefore \lim_{x\to 0} \frac{D(1-\cos x)}{D(x^2)} = \lim_{x\to 0} \left(\frac{\sin x}{2x}\right) = \frac{1}{2}$$

it follows that $\lim_{x\to 0} \left(\frac{1-\cos x}{x^2}\right) = \frac{1}{2}$ also!

b) a) Yes, the graph satisfies the Horizontal line test!

b) $\mathrm{Dom}\, F = \mathrm{Ran}\, f = [0,1] = \{x : 0 \le x \le 1\}$.

c) $\mathrm{Ran}\, F = \mathrm{Dom}\, f = [-1,0] = \{x : -1 \le x \le 0\}$.

d) $y = \sqrt{x+1} \Rightarrow y^2 = x+1$

Interchange x & y $\Rightarrow$ $x^2 = y+1$, solve for y.

$\Rightarrow F(x) = x^2 - 1$.