

**STATISTICS for MANAGEMENT I
ADM2303**

Prof. Brand
Section
Week 4

Administrative Issues

- Assignment 1
 - Part II
- Assignment 2
- Office hour (rm 6149, Tues 4-5pm)
- Quiz
 - Mar 4th (Sunday morning)
 - Standard formulas provided (will upload on doc-depot).

Last time

- Conditional probability questions (tree diagram) [Ch6]
- Random variables, and probability models (Ch9)
- Expected value
 - Calculation (Ch9.1) and interpretation

Today

- Variance
- Examining “rules for operating on **independent random** variables” (Ch9.3) --- lectures go beyond textbook.
- Some standard discrete probability models (Ch9.4-6)
 - Geometric,
 - binomial

Review and Wrap-up

- Variance
- Operations of RV's review and wrap-up
- Example problem
- [SmallBiz.pdf](#)

Simple Mathematical Probability Models: Based on Bernoulli trials

- First, focus on class of simple discrete probability models. The basis for these probability models is the **Bernoulli trial**.
- We have Bernoulli trials if:
 - there are two possible outcomes (success and failure).
 - the probability of success, p , is constant.
 - the trials are independent.

The Geometric Model

- A single Bernoulli trial is less interesting.
- A **Geometric model** tells us the probability for a random variable that counts the number of Bernoulli trials until the first success (**event includes the first success**).
- Geometric models are completely specified by one parameter, p , the probability of success, and are denoted $\text{Geom}(p)$.

The Geometric Model (cont.)

Geometric probability model for Bernoulli trials:

$$\text{Geom}(p)$$

p = probability of success

$q = 1 - p$ = probability of failure

X = # of trials until the first success occurs

$$P(X = x) = q^{x-1}p$$

Ex: Let $p=0.10$. What is the Pr that it takes three trials until we land our first success?

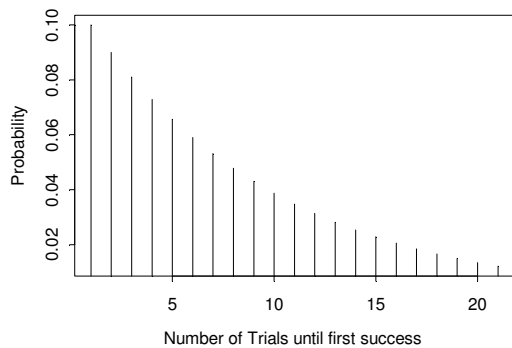
Applications of Geometric Distribution

- Producing units: $p=\text{Pr}[\text{defect}]$
- Sports (e.g., basketball): $p=\text{Pr}[\text{Basket}]$
- MP3 player: $p=\text{Pr}[\text{Song by XXX}]$
- Encountering specific customer

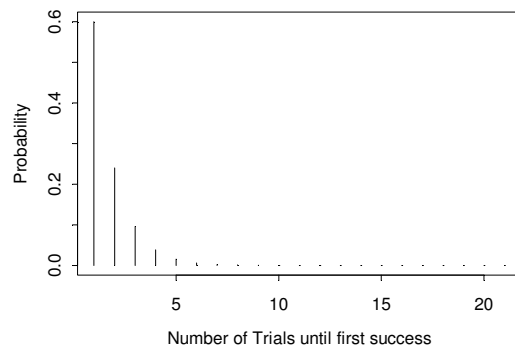
Enumerating Probability Model

X	Pr[X=x]
1	0.100
2	0.090
3	0.081
4	0.073
5	0.066
6	0.059
7	0.053
:	:
:	:

P[Sale] = 0.10



P[Sale] = 0.60



The Geometric Model (cont.)

- The expected value is

$$\mu = \frac{1}{p}$$

- The standard deviation is

$$\sigma = \sqrt{\frac{q}{p^2}}$$

The Binomial Model

- A **Binomial model** tells us the probability for a random variable that counts the number of successes in a fixed number of Bernoulli trials.
- Two parameters define the Binomial model: n , the number of trials; and, p , the probability of success. We denote this $\text{Binom}(n, p)$.

Example: Toss Coin 3 Times

- Let X represent the number of heads (successes)
- Note $p=0.5$
- What is probability of 2 heads?
 - Intuitively we might think that

$$\Pr[X=2] = p \cdot p \cdot (1-p) = p^2 \cdot (1-p)$$

Actually We've Seen Before

$X=x$	$P[X=x]$
$X=0$	$1/8$
$X=1$	$3/8$
$X=2$	$3/8$
$X=3$	$1/8$

H,H,T
H,T,H
T,H,H

3 outcomes with 2 heads

Counting Rule for Combinations

- A **combination** is an outcome of an experiment where x objects are selected from a group of n objects

$$C_x^n = \frac{n!}{x!(n-x)!}$$

where:

$$n! = n(n-1)(n-2) \dots (2)(1)$$

$$x! = x(x-1)(x-2) \dots (2)(1)$$

$$0! = 1 \text{ (by definition)}$$

Slide from: Prentice Hall 2005

Factorial

- $2! = 2 \times 1 = 2$
- $4! = 4 \times 3 \times 2 \times 1 = 24$
- $6! = 6 \times 5 \times (4!) = 6 \times 5 \times (24)$
- $6! / (2! \times (6-2)!) = 6 \times 5 \times (4!) / (2 \times (4!)) = (6 \times 5) / 2$

Binomial Distribution Formula

$$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$P(x)$ = probability of x successes in n trials,
with probability of success p on each trial

x = number of 'successes' in sample,
($x = 0, 1, 2, \dots, n$)

p = probability of "success" per trial

q = probability of "failure" = $(1 - p)$

n = number of trials (sample size)

From: Prentice Hall 2005

Example: Portfolio Question -- Modified

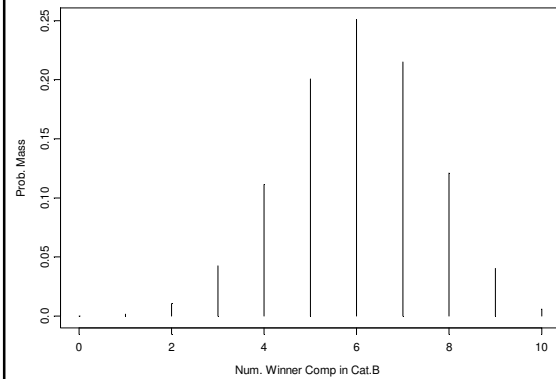
As the owner of a portfolio of companies you are examining your North American holdings. Using a certain profitability index you have categorized these companies into 4 categories (A, B, C, and D). ... Assume you have a total of 100 companies in your portfolio.

Category of Company	Fraction of Companies	Probability of Success per year
A	0.2	0.3
B	0.10	0.6
C	0.60	0.2
D	0.10	0.05

Enumerate the Probability Model

X	Pr[X=x]
0	0.0001
1	0.0016
2	0.0106
3	0.0425
4	0.1115
5	0.2007
6	0.2508
:	:
10	0.0060

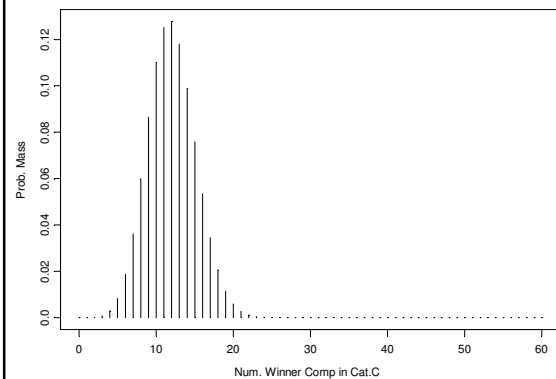
Comp. CatB : $P[W|B]=0.6$

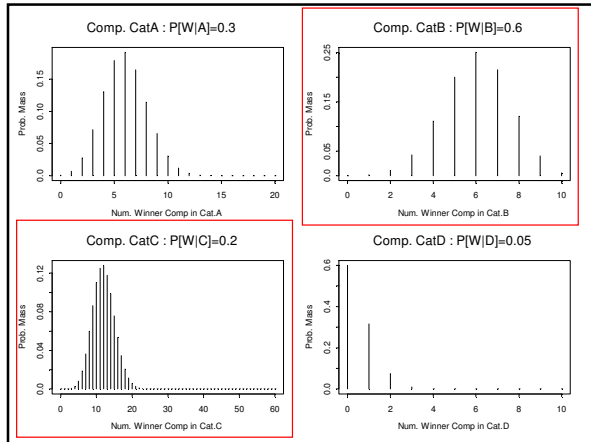


Category C

Category of Company	Fraction of Companies	Probability of Success per year
A	0.2	0.3
B	0.10	0.6
C	0.60 (60 companies)	0.2
D	0.10	0.05

Comp. CatC : $P[W|C]=0.2$





Binomial Distribution: EV and Var

- EV (mean)

$$\mu = E(x) = np$$

- Variance

$$\sigma^2 = npq$$

Where n = sample size
 p = probability of success
 $q = (1 - p)$ = probability of failure

From: Prentice Hall 2005

Sales

- Salesperson's is found to have a probability of success per customer that remains constant.
- When her shift starts, how many customers do you expect it will take before she gets her first sale?
 - $P[\text{Sale}] = p = 0.1$
 - What if $p=0.60$
- Bounds on valid outcomes
 - Integer
 - Lower bound of ?
 - Upper bound of ?

Sales Again

- Your salesman Josie had a consistent $P[\text{sale}] (=0.2)$. He returns from a "how to sell" workshop, which claims it 'will double your success.'
 - Out of 50 customers Josie gets 15 sales. How many was he expected to have?

Coin Flipping Again, and Again

- A coin will be tossed 10 times. Find the probability that there will be exactly 2 heads among the first five, and exactly 5 heads among the last five tosses.

Multiple Choice Exam Strategy

- Assume there are 5 possible answers offered on each question (a-e)
- What if you chose randomly for each question!
 - What is probability of correct choice?
- Consider 20 questions on exam
- How many correct answers will you get?

Multiple Choice Exam Strategy Con't

- What probability model to consider?
- Fully specify this model
- Find probability that student fails test
- Find probability that student gets a perfect score
- Find expected value and variance of X (X =incorrect answers)

What Can Go Wrong?

- Be sure you have Bernoulli trials—two outcomes per trial, a constant probability of success, and independence.
- Don't confuse Geometric and Binomial models.

Key Concepts

- We are particularly interested in **Bernoulli trials**.
- When we are looking for the probability for the number of Bernoulli trials until a success occurs, we have a **Geometric model**.
- When we are looking for the probability for the number of successes in a fixed number of Bernoulli trials, we have a **Binomial model**.
- The Normal model can be used to approximate the Binomial as long as certain conditions are met.

Checking if Independent versus Checking if Disjoint

Independent	Check whether $P(B A) = P(B)$ Or Check whether $P(A B) = P(A)$ Or Check whether $P(A \text{ and } B) = P(A) * P(B)$
Disjoint (mutually exclusive)	Check whether $P(A \text{ and } B) = 0$ Or Check whether the events A and B overlap in the sample space (Venn) diagram. Or Check whether both events can occur together