STUDY!!

(A)

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1. A sandwich is taken out of the refrigerator, which has a constant temperature of 5 °C, at 10:00 and left on the counter. The temperature in the kitchen is constant at 21 °C. One hour later, the temperature of the sandwich is 12 °C. Let $T(t)$ be the temperature of the sandwich t hours after 10:00.

(i) If the temperature of the sandwich follows Newton's Law of Heating, what is the differential equation that $T(t)$ satisfies?

(ii) Solve the differential equation and give an explicit formula for $T(t)$.

(iii) What is the temperature of the sandwich at noon?

STUDY

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2. Parks Canada places a herd of 500 caribou on an island in Hudson Bay. They estimate that the carrying capacity of the island is 2000 caribou. Also, the relative growth rate in an unconstrained environment is estimated to be $k = 0.08$ per year. Assuming that the population follows the Logistic Model,
(i) write the differential equation that the population $P(t)$ will satisfy, where t is measured in years and
(ii) given that the solution of the Logistic equation is $P(t) = \dfrac{M}{1 + Ae^{-kt}}$, find the number of caribou on the island after 2 years.

3.

(i) Determine if the series $\sum_{n=1}^{\infty} \frac{3+\cos(n+1)}{n^3+2010n}$ converges or diverges. Explain your reasoning.

(ii) Give an example of a series that is convergent, but not absolutely.

(iii) Give an example of power series with an infinite radius of convergence.

4. Determine the radius and interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{9^n(n+3)}$.

5.

(i) Use the Maclaurin series of $\dfrac{1}{1-x}$ to find the Maclaurin series of $\dfrac{1}{1+x^3}$ and give its radius of convergence.

(ii) Then deduce the Maclaurin series of $\displaystyle\int \frac{1}{1+x^3}\,dx$ and give its radius of convergence.

(iii) Find a power series representation of $f(x) = \frac{-1}{(1+x)^2}$

(B)

1. A sandwich is taken out of the refrigerator, which has a constant temperature of 4 °C, at 10:00 and left on the counter. The temperature in the kitchen is constant at 22 °C. One hour later, the temperature of the sandwich is 11 °C. Let $T(t)$ be the temperature of the sandwich t hours after 10:00.

(i) If the temperature of the sandwich follows Newton's Law of Heating, what is the differential equation that $T(t)$ satisfies?

(ii) Solve the differential equation and give an explicit formula for $T(t)$.

(iii) What is the temperature of the sandwich at noon?

(B)

2. Parks Canada places a herd of 400 caribou on an island in Hudson Bay. They estimate that the carrying capacity of the island is 2000 caribou. Also, the relative growth rate in an unconstrained environment is estimated to be $k = 0.10$ per year. Assuming that the population follows the Logistic Model,
(i) write the differential equation that the population $P(t)$ will satisfy, where t is measured in years and
(ii) given that the solution of the Logistic equation is $P(t) = \dfrac{M}{1 + Ae^{-kt}}$, find the number of caribou on the island after 3 years.

3.

(i) Determine if the series $\sum_{n=1}^{\infty} \frac{2+\sin(n+2010)}{n^4+4n}$ converges or diverges. Explain your reasoning.

(ii) Give an example of power series with an infinite radius of convergence.

4. Determine the radius and interval of convergence of $\sum_{n=0}^{\infty} \frac{(x+3)^n}{8^n(n+2)}$.

5.

(i) Use the Maclaurin series of $\dfrac{1}{1-x}$ to find the Maclaurin series of $\dfrac{1}{1+x^4}$ and give its radius of convergence.

(ii) Then deduce the Maclaurin series of $\displaystyle\int \frac{1}{1+x^4}\,dx$ and give its radius of convergence.

(iii) Find a power series representation of $f(x) = \frac{3}{(1+x)^2}$.