1. Find a scalar equation for the plane with vector parametric equation

$$v = (0, 2, -2) + s(1, -1, 2) + t(3, -5, 1); \; s, t \in \mathbf{R}.$$

A normal will be $n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & -5 & 1 \end{vmatrix} = (9, -(-5), -2) = (9, 5, -2)$

A. $4x - 9y + 36z = 18$

B. $9x - 2y + 2z = 0$

C. $9x + 5y - 2z = 14$

D. $7x - 8y + 5z = 6$

E. $9x - 11y + 18z = -40$

F. $3x + 2y - z = 0$

Hence, (B) is correct

2. The distance from the point (4, 0, 0) to the plane $2x - y + 8z = -3$ is:

A. $\frac{11}{\sqrt{69}}$

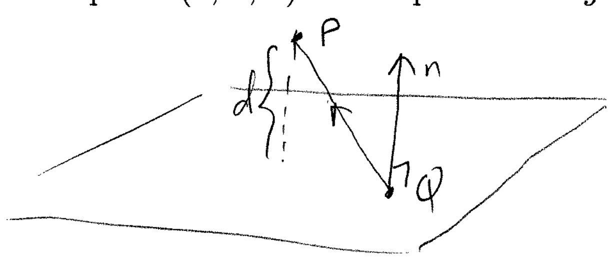
B. $\frac{13}{\sqrt{69}}$

C. $\frac{15}{\sqrt{69}}$

D. 0

E. $\frac{11}{69}$

F. $\frac{13}{69}$



$Q = (0, 3, 0)$ belongs to the plane

$$d = \| \text{proj}_n (P - Q) \| = \frac{|(P-Q) \cdot n|}{\|n\|}$$

$P - Q = (4, -3, 0)$ so $d = \frac{(4,-3,0) \cdot (2,-1,8)}{\sqrt{4+1+64}}$

$$= \frac{11}{\sqrt{69}}$$

3. If $u = (1,\ 5,\ -3)$, $v = (0,\ 2,\ -1)$, $w = (1,\ 1,\ 1)$ then the cosine of the angle between $(v \times w)$ and $(u \times v)$ is:

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A. $\frac{2}{21}$

B. $\frac{\sqrt{2}}{\sqrt{21}}$

C. $-\frac{1}{\sqrt{7}}$

D. $-\frac{1}{\sqrt{21}}$

E. $-\frac{1}{21}$

F. $\frac{2}{\sqrt{7}}$

$$v \times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = (3, -1, -2)$$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 5 & -3 \\ 0 & 2 & -1 \end{vmatrix} = (1, -(-1), 2) = (1, 1, 2)$$

Hence $$\cos\theta = \frac{(3,-1,-2) \cdot (1,1,2)}{\sqrt{9+1+4}\,\sqrt{1+1+4}} = \frac{-1}{\sqrt{21}}$$

4. If $\mathbf{u} = (2,\ 0,\ 2)$ and $\mathbf{v} = (3,\ -4,\ -10)$, the orthogonal projection of $\mathbf{v}$ along $\mathbf{u}$ is:

A. $(\frac{7}{2},\ 0,\ \frac{7}{2})$

B. $(\frac{11}{2},\ 0,\ \frac{11}{2})$

C. $(7, 0, 7)$

D. $(-7, 0, -7)$

E. $(-\frac{7}{2},\ 0,\ -\frac{7}{2})$

F. $(-\frac{11}{2},\ 0,\ -\frac{11}{2})$

$$\text{proj}_u v = \frac{v \cdot u}{\|u\|^2} \cdot u$$

$$= \frac{-14^{\,7}}{8^{\,4\,2}} \cdot (2, 0, 2)$$

$$= -\frac{7}{2} \cdot (1, 0, 1)$$

5. An equation of the plane parallel to the vector $(1, 1, -2)$ and which passes through the points $(0, -1, 5)$ and $(2, 0, -2)$ is:

A. $5x - 3y + z = 8$
B. $x + 5y + z = 0$
C. $9x - 6y + 5z = 8$
D. $5x - 11y + 2z = 11$
E. $7x - 9y + 2z = 13$
F. $5x - 7y - z = 2$

A normal is $n = u \times (P-Q)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ -2 & -1 & 7 \end{vmatrix}$$

$= (5, -3, 1)$

Hence (A) is correct.

(You can check it passes through P, e.g.)

6. Parametric equations of the line containing $(1,\ 0,\ 1)$ and which is parallel to the two planes with equations $x - 3y - 2z = 1$ and $x - y + 3z = 0$ are:

A. $x = 1 + 11t,\ y = 5t,\ z = 1 + 2t,\ t \in \mathbf{R}$
B. $x = -1 + 5t,\ y = -5t,\ z = 1 - 10t,\ t \in \mathbf{R}$
C. $x = -1 + 11t,\ y = -3t,\ z = 1 + 2t,\ t \in \mathbf{R}$
D. $x = t,\ y = 0,\ z = t,\ t \in \mathbf{R}$
E. $x = 1 + 11t,\ y = 5t,\ z = 1 - 2t,\ t \in \mathbf{R}$
F. $x = -t,\ y = 0,\ z = t,\ t \in \mathbf{R}$

A direction vector d for this line is

$$d = (1, -3, -2) \times (1, -1, 3) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -2 \\ 1 & -1 & 3 \end{vmatrix}$$

$= (-11, -5, 2)$ or $d' = (11, 5, -2)$. Hence E is correct

P Q

7. Let L be the line passing through $(1,\ 1,\ 0)$ and $(2,\ 3,\ 1)$. The point of intersection of L with the plane $x+y-z=1$ is:

A. $(\frac{1}{2},\ \frac{1}{2},\ 0)$

B. $(-1,\ 0,\ -1)$

(C.) $(\frac{1}{2},\ 0,\ -\frac{1}{2})$

D. $(1,\ 0,\ 0)$

E. $(0,\ \frac{1}{2},\ -\frac{1}{2})$

F. $(0,\ 1,\ 0)$

L has parametric equation

$r = P + t(Q-P) = (1,1,0) + t(1,2,1)$

or $x = 1+t$

$y = 1+2t$

$z = t.$

Hence L intersects this plane when

$(1+t) + (1+2t) - (t) = 1$ or

$2 + 2t = 1$ i.e. $t = -1/2$

$\therefore\ x = 1/2,\ y = 0,\ z = -1/2.$

8. Find the intersection of the lines $x = 2+2s,\ y = 2-s,\ z = 2-2s$ and $x = 6+5t,\ y = 2-t,\ z = 2-2t$.

A. $(8,\ 6,\ -6)$

B. $(-2,\ 7,\ 1)$

C. $\frac{1}{3}(20,\ -26,\ 7)$

(D.) $\frac{1}{3}(-2,\ 10,\ 14)$

E. $(2,\ 3,\ 2)$

F. $(4,\ -1,\ -4)$

We solve $2+2s = 6+5t$ (1)

& $2-s = 2-t$ (2)

$2-2s = 2-2t$ (3)

(1) + (3) yields $4 = 8+3t$ or $t = -4/3$.

Hence $x = 6 - 20/3 = -2/3$

$y = 2 + 4/3 = 10/3$

$z = 2 + 8/3 = 14/3$

9. The volume of the parallelepiped with edges given by the vectors $u = \sqrt{2}(1,\ 1,\ 1)$, $v = (1,\ 3,\ 2)$ and $w = (1,\ 1,\ 3)$ is:

A. 2

(B.) $4\sqrt{2}$

C. $\frac{\sqrt{2}}{2}$

D. $1/\sqrt{2}$

E. $1\sqrt{2}$

F. 4

The volume is $|u \times v \cdot w|$.

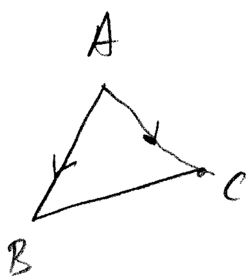
$$u \times v = \sqrt{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 3 & 2 \end{vmatrix} = \sqrt{2}(-1, -1, 2)$$

So $u \times v \cdot w = \sqrt{2}(-1,-1,2) \cdot (1,1,3)$

$= 4\sqrt{2} = |u \times v \cdot w|$

10. Find the area of the triangle with vertices $A = (0,\ 4,\ 1)$, $B = (2,\ -1,\ 5)$ and $C = (2,\ 3,\ 1)$.

(A.) 6
B. 5
C. 4
D. 3
E. 2
F. 1



The area is $\frac{1}{2}\|(B-A) \times (C-A)\|$; $(B-A)\times(C-A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 4 \\ 2 & -1 & 0 \end{vmatrix}$

$= (4, -8, 8) = 4(1, -2, 2)$. Hence the area of $\triangle ABC$

is $\frac{1}{2} \cdot 4 \cdot \|(1,-2,2)\| = 2 \cdot 3 = 6$

11. Find the polar form of:

$$\frac{\sqrt{3}+i}{1+i} = \frac{z}{w} \quad ; \quad z = |z|e^{i\theta} \quad ; \quad w = |w|e^{i\varphi}$$

A. $2\sqrt{2}(\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12}))$

B. $\sqrt{2}(\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12}))$

C. $2\sqrt{2}(\cos(-\frac{5\pi}{12}) + i\sin(-\frac{5\pi}{12}))$

D. $2\sqrt{2}(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12}))$

E. $\sqrt{2}(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12}))$

F. $\sqrt{2}(\cos(-\frac{5\pi}{12}) + i\sin(-\frac{5\pi}{12}))$

$|z| = \sqrt{\sqrt{3}^2 + 1^2} = 2$

$\cos\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$ $\therefore \theta = \pi/6$ $\therefore z = 2e^{i\pi/6}$

$|w| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\cos\varphi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\sin\varphi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\therefore \varphi = \pi/4$

$\therefore w = \sqrt{2}e^{i\pi/4}$

$\therefore \frac{z}{w} = \frac{2}{\sqrt{2}} \cdot e^{i\pi(1/6 - 1/4)} = \sqrt{2}e^{-i\pi/12}$

12. Evaluate Im(z) if

$$z = \frac{i}{(2+2i)(-1+i)}.$$

A. $-\frac{1}{5}$

B. $\frac{1}{4}$

(C.) $-\frac{1}{4}$

D. $\frac{1}{2}$

E. $-\frac{1}{2}$

F. 1

$z = i \cdot \frac{2-2i}{8} \cdot \frac{-1-i}{2} = -\frac{1}{16} \cdot (2+2i)(1+i)$

$= -\frac{1}{16}(0+4i)$

$= -\frac{i}{4} = \frac{-1}{4} \cdot i$