1341A.F05 Diag Solⁿ

P Q R

1. An equation of the plane containing (3, -1, 4), (-1, 5, 1) and (0, 2, -2) is:

A. $4x - 9y + 36z = 18$

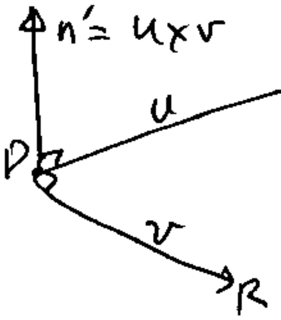
B. $9x + 5y - 2z = 14$

C. $7x - 8y + 5z = 6$

D. $8x - 11y + 18z = 24$

E. $3x - 2y + z = 0$

F. $3x + 2y - z = 0$



$$n' = u \times v = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 6 & -3 \\ -3 & 3 & -6 \end{vmatrix}$$

$$= (-27, -15, 6)$$

So a normal is $(9, 5, -2)$. (So B must be correct)

Use the point P:

$$0 = 9(x-3) + 5(y-(-1)) - 2(z-4) = 9x + 5y - 2z - 27 + 5 + 8$$

$$= 9x + 5y - 2z - 14$$

i.e. $9x + 5y - 2z = 14$. $\therefore$ B is correct

P

2. An equation of the plane containing the point (1, -1, 2) and the line with parametric equations $x = 4,\ y = -1 + 2t,\ z = 2 + t$ is:

A. $x + y - 2z + 5 = 0$

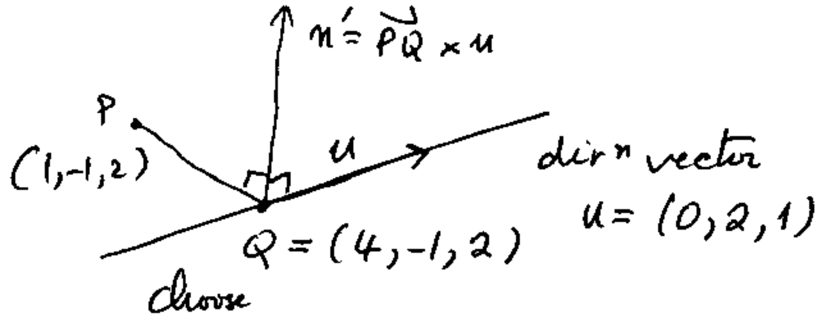
B. $y - 2z + 5 = 0$

C. $y + 2z + 5 = 0$

D. $y - 2z - 5 = 0$

E. $x + y + 2z - 5 = 0$

F. $y + 2z - 5 = 0$



$$n' = \vec{PQ} \times u = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 0 & 2 & 1 \end{vmatrix} = (0, -3, 6)$$; so a normal is

$n = (0, 1, -2)$. Using Q: $0 = 0(x-4) + 1(y-(-1)) - 2(z-2)$

i.e. $0 = y - 2z + 5$

$\therefore$ B is correct

3. Parametric equations of the line passing through $(1,\ \overset{P}{1},\ -1)$ and which is perpendicular to the plane with equation $2x - y + 3z = 4$ are:

A. $x = 1 - 2t,\ y = 1 + t,\ z = -1 + 3t,\ t \in \mathbf{R}$

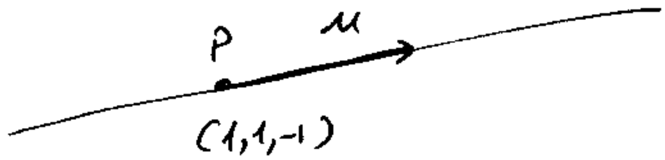
B. $x = 1 + 2t,\ y = 1 - t,\ z = -1 + 3t,\ t \in \mathbf{R}$

C. $x = 1 - t,\ y = 1 + t,\ z = -1 - 6t,\ t \in \mathbf{R}$

D. $x = 1 - 2t,\ y = 1 - t,\ z = -1 - 3t,\ t \in \mathbf{R}$

E. $x = 1 + t,\ y = 1 + t,\ z = -1 - 3t,\ t \in \mathbf{R}$

F. $x = 1 - 4t,\ y = 1 - t,\ z = -1 - 3t,\ t \in \mathbf{R}$



dir^n vector u = normal to plane above, so $u = (2,-1,3)$

$\therefore\ (x,y,z) = (1,1,-1) + t(2,-1,3)$

$\therefore$ B is correct

4. An equation for the plane which is equidistant from the points $\overset{P}{(2,\ -1,\ 4)}$ and $\overset{Q}{(1,\ 5,\ 2)}$ is:

A. $-2x - 5y + 4z = 4$

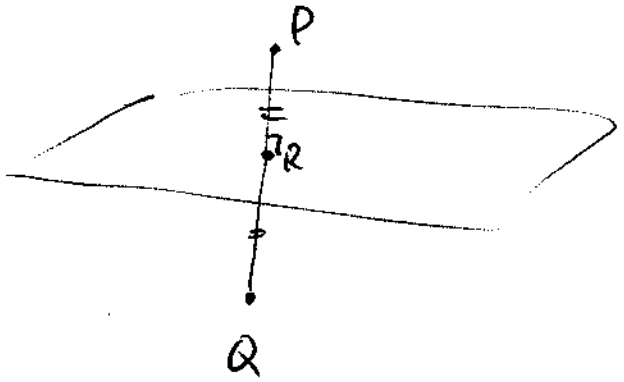
B. $2x - 5y - 4z = 3$

C. $-3x - 2y - z = 4/5$

D. $-x + 6y - 2z = 9/2$

E. $-x + 6y - 2z = -9/2$

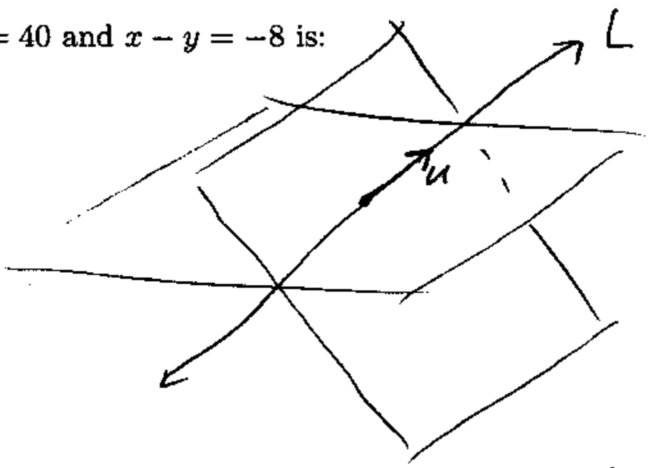
F. $3x - 2y + z = 4/5$



A normal is $\vec{PQ} = (-1,6,-2)$, and since $R = \frac{P+Q}{2} = (\frac{3}{2}, 2, 3)$ is on the plane, an equation is

$0 = -1(x - \frac{3}{2}) + 6(y-2) - 2(z-3)$ ie $-x + 6y - z = -\frac{3}{2} + 12 - 6 = \frac{9}{2}$

5. The intersection of the planes $x + 11y - 4z = 40$ and $x - y = -8$ is:



A. $(-4,\ 4,\ 0) + t(1,\ 3,\ 1),\ t \in \mathbf{R}$

B. $(4,\ 4,\ 0) + t(1,\ 1,\ 8),\ t \in \mathbf{R}$

C. $(-4,\ 4,\ 0) + t(1,\ 1,\ 3),\ t \in \mathbf{R}$

D. $(4,\ -4,\ 0) + t(-1,\ 1,\ -3),\ t \in \mathbf{R}$

E. $(0,\ 4,\ -4) + t(1,\ 3,\ -1),\ t \in \mathbf{R}$

F. $(4,\ 0,\ 4) + t(-1,\ 3,\ 1),\ t \in \mathbf{R}$

These planes are not parallel, so intersect in a line L, whose direction vector u is parallel to the cross product of the normals to the planes.

$$(1, 11, -4) \times (1, -1, 0) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 11 & -4 \\ 1 & -1 & 0 \end{vmatrix} = (-4, -4, -12); \text{ take } u = (1,1,3)$$

(So C must be correct : check that $(-4, 4, 0)$ belongs to both planes...)

6. Find the point of intersection of the plane with equation $2x + 2y - z = 5$ and the line with parametric equations $x = 4 - t,\ y = 13 - 6t,\ z = -7 + 4t$.

P **

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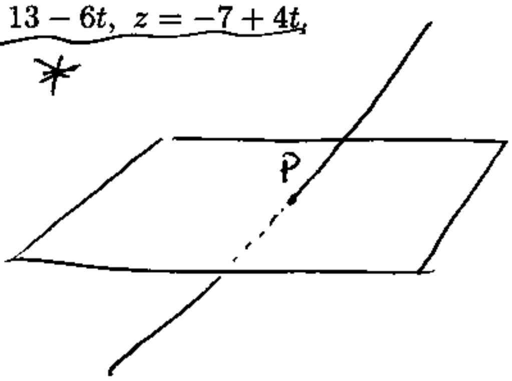


A. $(-1,\ 2,\ 2)$

B. $(2,\ 1,\ 0)$

C. $(0,\ 1,\ 1)$

D. $(-1,\ -1,\ 2)$

E. $(2,\ 1,\ 1)$

F. $(-1,\ 2,\ 0)$

We substitute * into ** and solve for t:

$$2(4-t) + 2(13-6t) - (-7+4t) = 5$$

i.e. $8 + 26 + 7 - 2t - 12t - 4t = 5$

or $-18t = -36$ i.e. $t = 2$

$$\therefore (x, y, z) = (2, 1, 1)$$

ℓ_1

7. Find the intersection of the line passing through the points (2, 1, -3) and (5, 4, -1) with the line passing through the points (0, 2, 3) and (-1, -2, -5).

ℓ_2

Dir^n vectors: ℓ_1: (3, 3, 2)
ℓ_2: (-1, -4, -8)

A. (2, -1, 5)

B. (2, -1, -5)

C. (2, 1, 5)

D. (-1, -2, -5)

E. (1, -2, -5)

F. (1, 2, -5)

ℓ_1: $x = 2 + 3t$, $y = 1 + 3t$, $z = -3 + 2t$

ℓ_2: $x = -s$, $y = 2 - 4s$, $z = 3 - 8s$

Equating x, y components yields
$$2 + 3t = -s$$
$$1 + 3t = 2 - 4s$$
$$\therefore 1 = -2 + 3s$$

This gives $s = 1$; and $t = -1$

Check z components:

ℓ_1: $z = -3 - 2 = -5$ ✓

ℓ_2: $z = 3 - 8 = -5$

$\therefore$ pt of int is $(-1, -2, -5)$

u $\quad$ v

8. The angle between the vectors (0, 3, -3) and (-2, 2, -1) is:

A. $\pi/6$

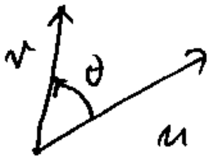
B. $\pi/2$

C. $\pi/4$

D. $\pi/3$

E. $\pi/5$

F. $\pi/7$



$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$

$$= \frac{9}{(3\sqrt{2})(3)} = \frac{\sqrt{2}}{2}$$

$$\therefore \theta = \pi/4$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad v \quad\quad\quad\quad\quad u$

9. The orthogonal projection of $(-5, 1, 8)$ along $(3, 0, 3)$ is:

A. (1, -1/5, -8/5)

B. (-1, 1/5, 8/5)

C. (-3/2, 0, -3/2)

D. (3/2, 0, 3/2)

E. (5, -1, -8)

F. (-5, 1, -8)

$$\text{proj}_u v = \frac{(u \cdot v)\, u}{\|u\|^2}$$

$$= \frac{9}{18} \cdot (3, 0, 3)$$

$$= \left(\frac{3}{2}, 0, \frac{3}{2}\right)$$

$u \quad v$

10. Find the volume of the parallelepiped determined by the vectors $(1, 1, -1)$, $(2, 0, 1)$ and $(1, -1, 3)$.

w

A. -2

B. 4

C. 6

D. 8

E. 16

F. 2

$$vol = |u \times v \cdot w|$$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix} = (1, -3, -2)$$

$$u \times v \cdot w = (1, -3, -2) \cdot (1, -1, 3)$$

$$= -2$$

$$\therefore vol = |-2| = 2$$

11. Find

$$\left|\frac{3-i}{2-4i}\right| = \frac{|3-i|}{|2-4i|} = \frac{\sqrt{9+1}}{\sqrt{4+16}}$$

$$= \frac{\sqrt{10}}{\sqrt{20}} = \frac{\sqrt{10}}{\sqrt{2}\sqrt{10}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

A. 1/2
B. $\sqrt{2}/2$
C. $\sqrt{8/29}$
D. 8/29
E. 3/2
F. $\sqrt{14/29}$

12. Find Im(z) if

$$z = \frac{i}{(1+i)(2+3i)}. = \frac{i(1-i)(2-3i)}{2 \cdot 13}$$

$$= \frac{1}{26}(i+1)(2-3i)$$

$$= \frac{1}{26}(5-i) = \frac{5}{26} - \frac{i}{26}$$

$$\therefore \operatorname{Im}(z) = -\frac{1}{26}$$

A. 1/2
B. -1/2
C. -1/5
D. -1/26
E. -5/26
F. 5/13

13 (bonus). For what values of p will the line with parametric equations $x = 4 + 2t,\ y = 5t,\ z = 1 + pt$, and the plane $x + 2y + 3z = 8$ have no point in common?

A. 3

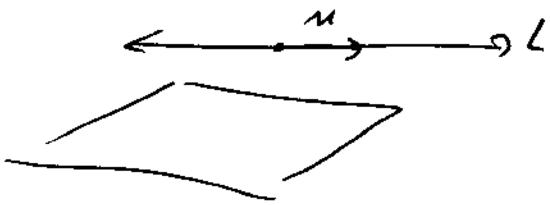
B. -12

C. -4

D. 6

E. 4

F. -8



We need any dirn vector for L to be $\perp$ to a normal for the plane, and for the line not to lie in the plane.

First: $u = (2, 5, p)$ so $u \cdot n = 2 + 10 + 3p = 0$

$n = (1, 2, 3)$

$\Leftrightarrow p = -4$

(so C must be correct)

(Check: In this case,

$(4+2t) + 2(5t) + 3(1-4t)$

$= 7 \neq 8$, so L does not lie in the plane)