

ABW

CARLETON UNIVERSITY

FINAL
EXAMINATION
DECEMBER 2008

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, 69.1004 A, B, C, D.
Course Instructor(s): Dr. A.B. Mingarelli (A, B), Dr. M.R.R. Sadeghi (C), Dr. A. Masuda (D).

AUTHORIZED MEMORANDA

NON-PROGRAMMABLE CALCULATOR PERMITTED.

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1. Please count your pages now. **This examination has pages numbered from 1 - 15**, excluding this page. The pages numbered 8-15 are blank and are to be used for rough work only. If you feel there is a page missing please report this to your Proctor.
2. Next, please verify that you are in possession of a Scantron FORM
3. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
4. **The examination is out of a total of 100** and consists of 28 multiple choice of varying numerical values displayed next to the question number. **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question). Additional pages are added for your convenience and for rough work and are not to be submitted for grading purposes.

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Grading Instructions

- Please note that there are **28 questions** and most have **different values**. The maximum grade on this examination is 100% (which gets reduced to 50% of your final grade for the course by rounding up). Thus, for example, $53\% = 27/50$.
- In addition, the grade distribution (out of 100) is as follows:

1. 3 marks	11. 3 marks	21. 4 marks
2. 4 marks	12. 4 marks	22. 4 marks
3. 3 marks	13. 4 marks	23. 3 marks
4. 4 marks	14. 4 marks	24. 4 marks
5. 3 marks	15. 5 marks	25. 3 marks
6. 3 marks	16. 4 marks	26. 4 marks
7. 3 marks	17. 5 marks	27. 4 marks
8. 3 marks	18. 3 marks	28. 4 marks
9. 2 marks	19. 4 marks	
10. 3 marks	20. 3 marks	

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Multiple-Choice Questions

Please choose only one answer and insert in PENCIL in your Scantron sheet.

1. [3 marks] Evaluate $\lim_{x \rightarrow 0} \frac{\sin 8x}{16x}$.

(a) -1 ☒ (b) $1/2$ (c) $3/2$ (d) 0

2. [4 marks] Let $f(x) = \text{Arctan} \sqrt{x^2 + 1}$. Evaluate $f'(1)$. (Note that $\text{Arctan } x$ and $\tan^{-1} x$ represent the same function).

(a) $f'(1) = \frac{1}{\sqrt{3}}$ (b) $f'(1) = \frac{1}{2}$ ☒ (c) $f'(1) = \frac{1}{3\sqrt{2}}$ (d) $f'(1) = 1$

3. [3 marks] Let $f(x) = 2|x + 1| + 1$. Calculate

$$L = \lim_{h \rightarrow 0} \frac{f(-1 + h) - f(-1)}{h}.$$

(a) $L = 0$ (b) $L = 1$ (c) $L = -1$ ☒ (d) This limit does not exist

4. [4 marks] Find the derivative of the function f defined by $f(x) = x^{3x}$.

☒ (a) $3x^{3x}(1 + \ln x)$ (b) $3x^{3x-1}$ (c) $3x^{3x}$ (d) $x^{3x}(1 + 2 \ln x)$

5. [3 marks] A differentiable function f with a differentiable inverse, F , has the property that $f'(1) = 1/2$ and $f(1) = 1/2$. What is the value of the derivative of the inverse of f at $x = 1/2$? That is, calculate $F'(1/2)$.

Don't we need $F(x)$?

- (a) 1
☒ (b) 2
 (c) 0
 (d) $1/2$

$F'(x) = \frac{1}{f'(x)}$

6. [3 marks] Let $f(x) = 5^{\sqrt{x+1}}$. Evaluate $f'(8)$. In other words, find the derivative of f at $x = 8$.

- ☒ (a) $\frac{125 \ln 5}{6}$
 (b) $\frac{9 \ln 3}{2}$
 (c) 125
 (d) $\frac{9 \ln 5}{2}$

7. [3 marks] Find the derivative of the function f defined by $f(x) = \frac{x}{x^2 + 1}$.

- (a) $f'(x) = \frac{1}{2x}$
 (b) $f'(x) = \frac{1}{(x^2 + 1)^2}$
☒ (c) $f'(x) = \frac{1 - x^2}{(x^2 + 1)^2}$
 (d) $f'(x) = \frac{1}{x(x^2 + 1)}$

8. [3 marks] Evaluate the limit: $L = \lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{x+1} - \sqrt{x})$.

- (a) $L = 1/3$
☒ (b) $L = 1/2$
 (c) $L = 1$
 (d) $L = 0$

9. [2 marks] Let y be given implicitly as a differentiable function of x by $ye^{xy-1} = 1$. Calculate the value of the derivative $\frac{dy}{dx}$ at the point (x, y) where $x = 1$, $y = 1$:

- (a) 1,
 (b) $-1/3$,
 (c) 0,
☒ (d) $-1/2$

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10. [3 marks] Evaluate

$$I = \lim_{x \rightarrow 0^+} \frac{d}{dx} \int_1^{\sqrt{x}} \frac{\sin t^2}{2t} dt$$

- (a) $I = 0$
(b) $I = \frac{1}{4}$
(c) This limit does not exist
(d) $I = \frac{1}{2}$
11. [3 marks] Evaluate $L = \lim_{x \rightarrow 0^+} \frac{\log_2 \sqrt{x}}{\ln x}$ using any method.

- (a) $L = \frac{1}{2 \ln 2}$
(b) $L = 1$
(c) $L = 2$
(d) $L = \frac{1}{\log_2 2}$

12. [4 marks] The function
- f
- defined by
- $f(x) = 2x^3 - 6x^2 - 12x + 1$
- is concave up at each point of which of the following intervals?

- (a) $-12 < x < 0$
(b) $0 < x < 1$
(c) $1 < x < \infty$
(d) $-\infty < x < 1$

13. [4 marks] Determine ALL the horizontal asymptotes of the function
- f
- defined by

$$f(x) = \frac{x|x|}{x^2 + 1}.$$

- (a) $y = 0$ is the only asymptote
(b) $y = -1, y = 1$
(c) $y = 1/2, y = 1/3$
(d) $y = 1, y = -1/2$

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14. [4 marks] For what value of a does the function defined by

$$f(x) = x^4 + ax^3 - 6$$

have a point of inflection at $x = 1$?

- ☒ (a) $a = -2$
 - (b) $a = 1$
 - (c) $a = -4$
 - (d) $a = 0$
15. [5 marks] Find all the critical points of the function f defined by $f(x) = x^2 e^{-x}$.
- (a) $x = 0$ only
 - (b) $x = 0$ and $x = 1$ only
 - (c) $x = e$ and $x = -1$ only
 - ☒ (d) $x = 0$ and $x = 2$ only
16. [4 marks] Determine the largest interval I on which the function f , whose domain is the set of all real numbers, and which is defined by $f(x) = e^{-x^2}$ is increasing on I .
- (a) $0 < x < \infty$
 - (b) This function is never increasing.
 - (c) $-\infty < x < \sqrt{2}/2$
 - ☒ (d) $-\infty < x < 0$
17. [5 marks] Find the most general antiderivative of the function f defined by

$$f(x) = \frac{1 - 2 \sin 2x}{x + \cos 2x}.$$

- (a) $\ln |1 + 2 \cos 2x| + C$, where C is a constant
- (b) $\ln |1 - \sin 2x| + C$, where C is a constant
- ☒ (c) $\ln |x + \cos 2x| + C$, where C is a constant
- (d) $\ln |x - \sin 2x| + C$, where C is a constant

18. [3 marks] Evaluate $f(x) = \int_0^{\pi/8} \tan 2x \, dx$.

(a) $\frac{1}{2} \ln 2 - 1$

(b) $\frac{1}{2} \ln 8$

(c) $\frac{1}{4} \ln 2$

(d) $\frac{1}{2} \ln |\sec^2 2x|$

19. [4 marks] Evaluate $\int_0^1 x e^x \, dx$

(a) $1 - \frac{2}{e}$

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{e-1}{2}$

20. [3 marks] The improper integral $\int_0^\infty x 3^{-x^2} \, dx$ has the value

(a) $-\frac{1}{3 \ln 3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{\ln 3}$

(d) $\frac{1}{2 \ln 3}$

21. [4 marks] Evaluate $\int x^2 \ln x \, dx$.

(a) $\frac{1}{3} x^3 (\ln x - \frac{1}{3}) + C$

(b) $\frac{1}{3} x^3 \ln x - \frac{x}{9} + C$

(c) $3(\ln x)^2 + \frac{1}{9} + C$

(d) $2x^2(\ln x) + \frac{x}{9} + C$

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22. [4 marks] Evaluate and simplify the indefinite integral: $\int \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$.

(a) $\frac{3^{\sqrt{x}}}{\ln x} + C$

(b) $\frac{3^{\sqrt{x}+1}}{4} + C$

• (c) $\frac{2 \cdot 3^{\sqrt{x}}}{\ln 3} + C$

(d) $\frac{4 \cdot 3^{\sqrt{x}}}{\ln 2} + C$

23. [3 marks] Evaluate

$$\int \frac{3x+1}{x(x-1)} dx$$

using the method of partial fractions.

(a) $2 \ln|x-1| + 3 \ln|x| + C$

(b) $\ln|x-1| - 2 \ln|x| + C$

• (c) $4 \ln|x-1| - \ln|x| + C$

(d) $2 \ln|x-1| + 4 \ln|x| + C$

24. [4 marks] Evaluate the indefinite trigonometric integral

$$\int \sec^6 x \tan^3 x dx.$$

(a) $\frac{\sec^3 x \tan^3 x}{3} + C$

• (b) $\frac{\sec^8 x}{8} - \frac{\sec^6 x}{6} + C$

(c) $\frac{\sec^8 x}{7} + \frac{\sec^6 x}{5} + C$

(d) $\frac{\sec^3 x}{5} - \frac{\tan^4 x}{4} + C$

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25. [3 marks] Evaluate the improper integral $I = \int_0^1 x \ln x \, dx$

(a) $I = -\frac{1}{4}$

(b) $I = 0$

(c) $I = \frac{1}{3}$

(d) $I = -\frac{5}{12}$

26. [4 marks] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \sin x$ between $x = 0$ and $x = \pi$ about the y -axis.

(a) $2\pi \int_0^\pi \sin^2 x \, dx$

(b) $2\pi \int_0^\pi x^2 \sin^2 x \, dx$

(c) $2\pi \int_0^\pi x \sin x \, dx$

(d) $2\pi \int_0^\pi \sin x \, dx$

27. [4 marks] Find the area A of the region bounded by the curves defined by $y = x^2$, $y = x^3$ and the lines $x = 0$ and $x = 1$.

(a) $A = \frac{3}{2}$

(b) $A = \frac{3}{4}$

(c) $A = \frac{1}{12}$

(d) $A = \frac{1}{3}$

28. [4 marks] Evaluate $I = \int_0^1 \sqrt{1-x^2} \, dx$. (Hint: There is an easy way to do this.)

(a) $I = \pi$

(b) $I = \frac{\pi}{4}$

(c) $I = \frac{\pi}{2}$

(d) $I = \frac{\pi}{3}$

END

Total: [100 marks]