Econ 301(001) **UBC** **Mid-Term Exam.**
Prof. H. Neary **Dept of Economics** **October 23, 2012**

Student Name ______________________ Student Number: ______________

Student Signature: ______________________________________

1. (10 points) The market demand function for a good is given by

$$Q_d = D(p, p_{og})$$

where p is the price of the good and p_{og} is the price of other goods in the market. Further, the supply function for the good is given by

$$Q_s = S(p, p_I)$$

where p is the price of the good and p_I is the price of a key input in the good's production.

(a) Write out an equation describing equilibrium in the market. What is the endogenous (equilibrating) variable that enables this equilibrium condition to be satisfied?

Solution:

$$D(p, p_{og}) = S(p, p_I).$$

The market price of the good, p adjusts according to the Law of Supply and demand until eventually a price is reached where quantity demanded equals quantity supplied. The equilibrium price solves the above equation and can be written as as a function of the endogenous variables.

$$p^* = p^*(p_{og}, p_I).$$

(b) Write a simplified algebraic expression to express how the equilibrium price in the market would change if the price of the input, p_I changed. What is the sign of this comparative static expression? What assumptions are you making in order to ensure this sign?

Solution: Totally differentiate the above equilibrium condition with respect to p and p_I to get

$$\frac{\partial D}{\partial p} dp = \frac{\partial S}{\partial p} dp + \frac{\partial S}{\partial p_I} dp_I$$

and solve for the ratio dp/dp_I to get

$$\frac{dp}{dp_I} = \frac{\partial S/\partial p_I}{(\partial D/\partial p - \partial S/\partial p)}.$$

 Question 1 continues on next page. . .

Solution: *continued* We wish to determine the sign of this expression. The denominator is *negative* because the demand curve slopes downward ($\partial D/\partial p < 0$) and the supply curve slopes upwards ($\partial S/\partial p > 0$). Therefore the derivative of $\partial p/\partial p_I$ will be opposite in sign to $\partial S/\partial p_I$. Quantity supplied of a good is decreasing in the price of an input. Hence we have

$$\frac{dp}{dp_I} > 0.$$

For completeness, note that the comparative static equation could be derived as follows. If we substitute the solution function $p^*(p_{og}, p_I)$ into the demand and supply functions we get an identity

$$D\big(p^*(p_{og}, p_I), p_{og}\big) \equiv S\big(p^*(p_{og}, p_I), p_I\big).$$

Differentiating across this identity with respect to p_I gives

$$\frac{\partial D}{\partial p}\frac{\partial p^*}{\partial p_I} = \frac{\partial S}{\partial p}\frac{\partial p^*}{\partial p_I} + \frac{\partial S}{\partial p_I}.$$

This can be solved immediately for

$$\frac{\partial p^*}{dp_I} = \frac{\partial S/\partial p_I}{(\partial D/\partial p - \partial S/\partial p)}.$$

(c) Write out an equation describing equilibrium in the market if the government imposes an ad valorem tax (% sales tax) at tax rate τ on the consumer.

Solution: An ad valorem tax on the consumer involves increasing the sales price by a certain percentage τ. The amount paid by the consumer is the marked-up price $p \times (1+\tau)$. The price received by the producer is p. The equilibrium condition determining the price p, given the tax, is

$$D\big(p \times (1+\tau), p_{og}\big) = S(p, p_I).$$

This gives an equilibrium price p^* as a function of the endogenous variables τ, p_{og}, p_I:

$$p^* = p^*(\tau, p_{og}, p_I).$$

(d) Write a simplified algebraic expression to express how the equilibrium price in the market would change if the tax rate τ changes? What is the sign of this comparative static expression?

Solution: Use either of the comparative static methods of part b to derive the comparative static expression

$$\frac{\partial p^*}{\partial \tau} = \frac{(\partial D/\partial p)p^*}{-\big((\partial D/\partial p)(1+\tau) - \partial S/\partial p\big)}.$$

Because $\partial D/\partial p < 0$ and $\partial S/\partial p > 0$ the sign of this comparative static expression is *negative*. An increase in the sales tax rate reduces the equilibrium pre-tax price p. (Note that the tax-inclusive price $p \times (1+\tau)$ rises with τ.)

2. (10 points) The diagram below contains several consumption bundles labeled H through O. Below also is a number of statements about a single consumer's preferences over these bundles. In each case circle the word *consistent* if the preferences are consistent with the standard properties, or circle one of *transitivity*, *more is better*, *completeness*, or *convexity* as appropriate if the preferences described **violate** one of these properties.

   (a) A consumer reports that he prefers bundle M to bundle J but can not tell you about preferences for K relative to M.
   Circle 1 of: *consistent* *transitivity* *more is better* *completeness* *convexity*

   (b) A consumer reports that she prefers bundle J to bundle N.
   Circle 1 of: *consistent* *transitivity* *more is better* *completeness* *convexity*

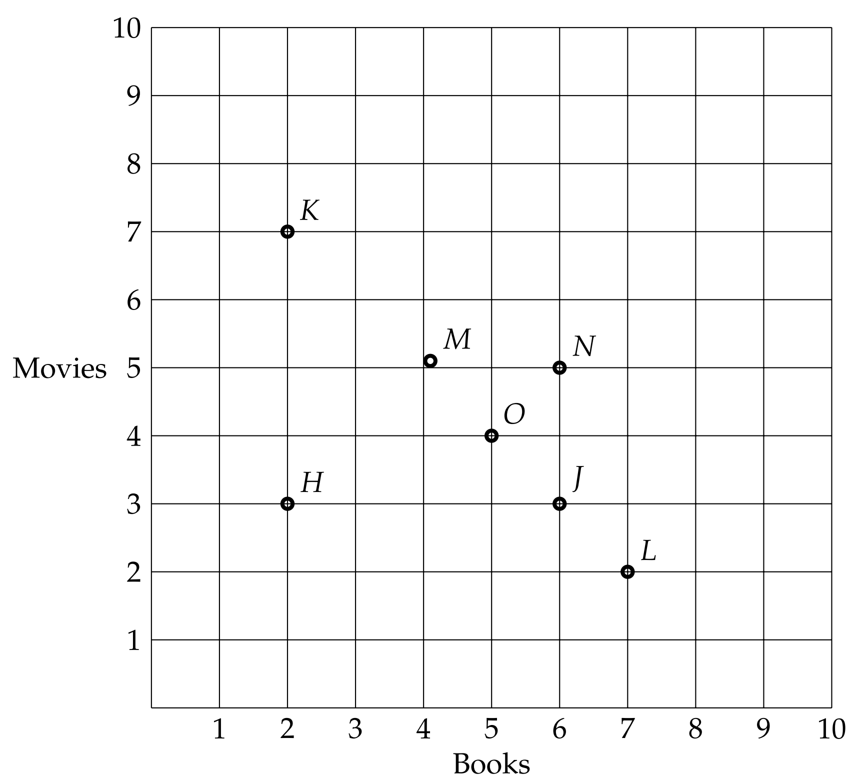
   (c) A consumer reports that she prefers bundle J to K, prefers bundle M to J, and prefers bundle K to M.
   Circle 1 of: *consistent* *transitivity* *more is better* *completeness* *convexity*

   (d) A consumer reports that she is indifferent between bundles K and J, and is indifferent between bundles N and L.
   Circle 1 of: *consistent* *transitivity* *more is better* *completeness* *convexity*

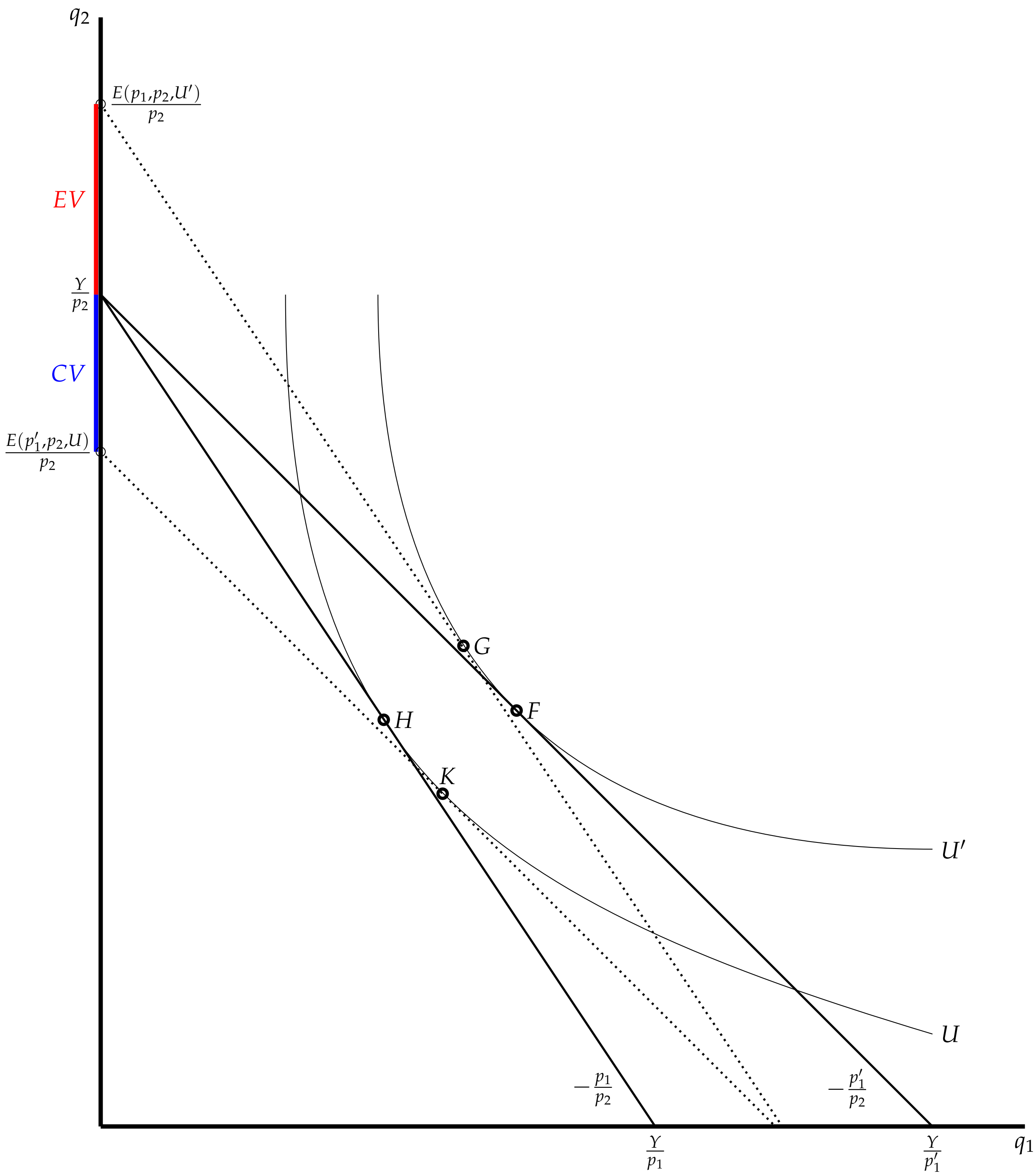
   (e) A consumer reports that she is indifferent between bundles J, M and K.
   Circle 1 of: *consistent* *transitivity* *more is better* *completeness* *convexity*

   (f) A consumer reports that he is indifferent between K and J, and prefers J to O.
   Circle 1 of: *consistent* *transitivity* *more is better* *completeness* *convexity*

   (g) A consumer reports that she prefers M to O.
   Circle 1 of: *consistent* *transitivity* *more is better* *completeness* *convexity*



3. (10 points) The figure below shows two indifference curves (solid black curves) and two budget lines (solid black) for a consumer facing goods q_1 and q_2. Initially the goods' prices are p_1 and p_2 respectively and the consumer's income is Y. Answer the following questions. Refer to the bundles by their labels, F, G, H, K respectively, in your answers.



 Question 3 continues on next page…

(a) Which bundle will the consumer choose in the initial equilibrium?

Answer: Bundle H

(b) Which bundle will the consumer pick if the price of good 1 *falls* to p'_1?

Answer: Bundle F

(c) What is the substitution effect of this price change?

Answer: From bundle H to bundle K

(d) What is the income effect of this price change?

Answer: From bundle K to bundle F

(e) Is good 1 *normal* or *inferior*?

Answer: Good 1 is Normal

(f) Is good 2 *normal* or *inferior*?

Answer: Good 2 is Normal

(g) What is the Equivalent Variation (EV) for this price fall?

Answer: From bundle H to bundle G

(h) Write an algebraic expression for the EV, in terms of the expenditure function, and illustrate that expression in the figure.

Answer: The expression is $E(p_1, p_2, U') - E(p_1, p_2, U) = E(p_1, p_2, U') - Y.$

(i) What is the Compensating Variation (CV) for this price fall?

Answer: From bundle F to bundle K

(j) Write an algebraic expression for the CV, in terms of the expenditure function, and illustrate that expression in the figure.

Answer: The expression is $E(p'_1, p_2, U') - E(p'_1, p_2, U) = Y - E(p'_1, p_2, U).$

4. (10 points) Suppose that the compensated demand functions for goods q_1 and q_2 are given by

$$q_1^c = \frac{a\,p_2}{p_1} \qquad \text{and} \qquad q_2^c = U - a\,\mathrm{Log}(\frac{a\,p_2}{p_1})$$

where a is a utility parameter, p_1 and p_2 are given prices and U is a target utility level. (Hint for later use: $\partial \mathrm{Log}(x)/\partial x = 1/x$.)

(a) Derive the uncompensated (market) demand functions $q_1^*(p_1, p_2, Y)$ and $q_2^*(p_1, p_2, Y)$. Clearly indicate your logic.

Solution:

$$E = p_1 q_1^c + p_2 q_2^c = a p_2 + p_2 U - a p_2 \mathrm{Log}(\frac{a\,p_2}{p_1}) = p_2 U + a p_2 \Big(1 - \mathrm{Log}(\frac{a\,p_2}{p_1})\Big)$$

$$V = \frac{Y}{p_2} + a\Big(\mathrm{Log}(\frac{a\,p_2}{p_1}) - 1\Big); \qquad \frac{\partial V}{\partial p_1} = a\Big(\frac{p_1}{a p_2}\Big)\Big(\frac{-a p_2}{p_1^2}\Big) = \frac{-a}{p_1};$$

$$\frac{\partial V}{\partial p_2} = \frac{-Y}{p_2^2} + a\Big(\frac{p_1}{a p_2}\Big)\Big(\frac{a}{p_1}\Big) = \frac{-Y}{p_2^2} + \frac{a}{p_2}; \qquad \frac{\partial V}{\partial Y} = \frac{1}{p_2}.$$

$$q_1^* = \frac{-\partial V/\partial p_1}{\partial V/\partial Y} = \frac{a p_2}{p_1}; \qquad q_2^* = \frac{-\partial V/\partial p_2}{\partial V/\partial Y} = \frac{Y}{p_2} - a.$$

(b) Write a simplified algebraic expression for the Compensating Variation of a doubling of the price of good 1, from the initial p_1 to $p_1' = 2p_1$.

Solution:

$$\begin{aligned} CV &= E(p_1', p_2, U') - E(p_1', p_2, U) \\ &= p_2\Big[a + U' - a\mathrm{Log}(\frac{a\,p_2}{p_1'})\Big] - p_2\Big[a + U - a\mathrm{Log}(\frac{a\,p_2}{p_1'})\Big] = p_2\Big[U' - U\Big] \\ &= p_2\Big[\frac{Y}{p_2} + a\Big(\mathrm{Log}(\frac{a\,p_2}{p_1'}) - 1\Big) - \frac{Y}{p_2} - a\Big(\mathrm{Log}(\frac{a\,p_2}{p_1}) - 1\Big)\Big] \\ &= a p_2\Big[\mathrm{Log}(\frac{a\,p_2}{p_1'}) - \mathrm{Log}(\frac{a\,p_2}{p_1})\Big] = a p_2\Big[\mathrm{Log}(\frac{p_1}{p_1'})\Big] = a p_2 \mathrm{Log}(\frac{1}{2}). \end{aligned}$$

(c) Write a simplified algebraic expression for the Equivalent Variation of a doubling of the price of good 1, from the initial p_1 to $p_1' = 2p_1$.

Solution:

$$\begin{aligned} EV &= E(p_1, p_2, U') - E(p_1, p_2, U) \\ &= p_2\Big[a + U' - a\mathrm{Log}(\frac{a\,p_2}{p_1})\Big] - p_2\Big[a + U - a\mathrm{Log}(\frac{a\,p_2}{p_1})\Big] = p_2\Big[U' - U\Big] \\ &= a p_2 \mathrm{Log}(\frac{1}{2}). \end{aligned}$$

5. (10 points) Suppose that the *inverse* market demand for an upcoming Bruce Springsteen concert at Philadelphia's 30,000-seat Wachovia Center is

$$p = 3400.00 - 0.1\,Q.$$

Mr. Springsteen and his promoters consider whether to auction the tickets to the concert. The auction works as follows: An auctioneer orders the bids from highest to lowest, and the price of each ticket equals the 30,000$^{\text{th}}$ highest bid. The tickets go to the highest bidders. In the auction, assume that each person bids his or her willingness to pay. In answering the following questions please enter a numeric response using a real number rounded to two decimal places.

(a) What is the price of the tickets?

Answer: The price of the tickets is $\boxed{p = 3400.00 - 0.1 \times 30,000 \Longrightarrow p = \$400.00.}$

(b) What is the market consumer surplus?
Answer: Market consumer surplus (CS) is

$$\boxed{CS = 0.5 \times (p_{max} - p) \times 30,000 = 0.5 \times (3400 - 400) \times 30,000 = \$45,000,000.}$$

(c) Instead, suppose that Mr. Springsteen, for the benefit of his fans, decides to sell each ticket for \$350.00. Based on the demand function, how many people are willing to pay \$350.00 or more to see the concert?

Answer: At \$350 a total of $\boxed{350 = 3400 - 0.1Q \Longrightarrow Q = 30,500}$ people are willing to pay.

(d) Suppose that 30,000 tickets at \$350 each are allocated to the people who bid the 30,000 highest willingnesses to pay. What is the consumer surplus?
Answer: In this case consumer surplus (CS) is

$$\boxed{45,000,000 + (400 - 350) \times 30,000 = \$46,500,000.}$$

(e) In deciding whether to auction the tickets or to set a price of \$350.00 suppose Bruce Springsteen's objective is to maximize consumer surplus. Which does he choose, an *auction* or a *\$350.00 price*?

Answer: He will choose $\boxed{\text{a \$350.00 price}}$

(f) How much money does Bruce S. lose as a result of this choice?

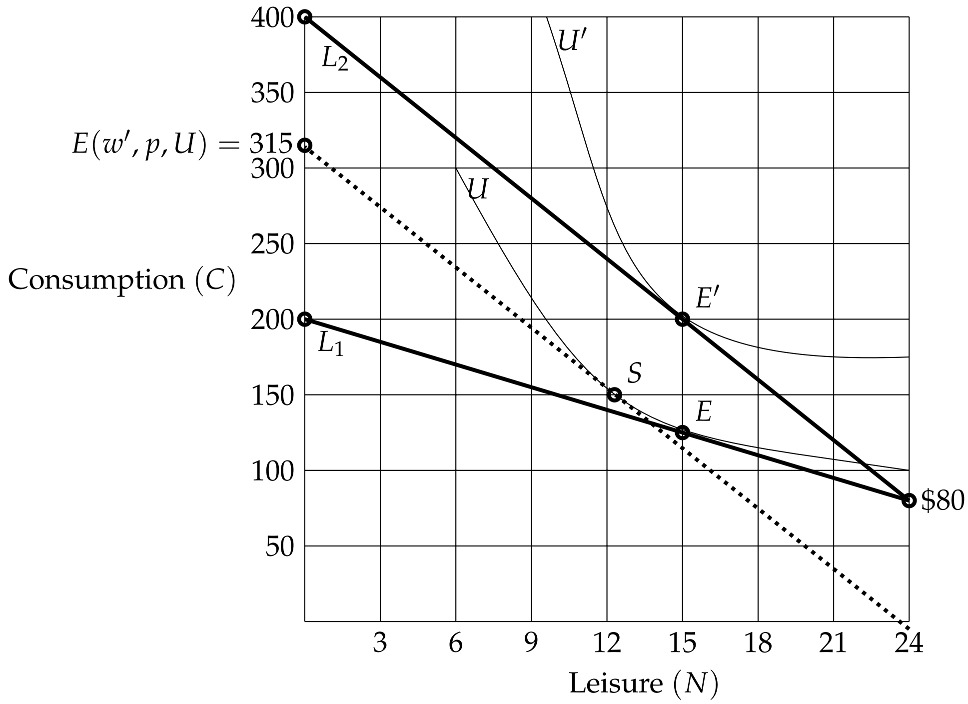
Answer: He loses $\boxed{(400 - 350) \times 30,000 = \$1,500,000.}$

(g) Suppose that ticket scalpers, who do not appreciate the music at all, get 500 of the \$350 tickets. What is the minimum profit they can make selling them?

Answer: They can make a minimum profit of $\boxed{(400 - 350) \times 500 = \$25,000.}$

Exam. continues on next page...

6. (10 points) This figure shows an individual's space of Consumption and Leisure bundles. The consumer initially has an hourly unearned income of \$80, faces a wage $w =$ \$5 an hour, and faces a consumption price $p =$ \$1. The initial budget line is shown as L_1 and the initial equilibrium choice of leisure and consumption is shown as bundle E, where $N = 15$, $C =$ \$125.



(a) How many hours does the individual work in this equilibrium?

Answer: Works $H =$ $24 - N^* = 9$ hours.

(b) Suppose that the wage rate increases to $w' > w$, giving the new budget line L_2. What is the numerical value of the new wage w' that will give this new budget line?

Answer: The new wage w' : $400 = 80 + 24 \times w' \implies w' = 40/3 = \$13.33.$

(c) The new equilibrium is at E'. How many hours does the individual work in this new equilibrium?

Answer: Works $H =$ $24 - N^* = 9$ hours.

(d) On the figure draw a consistent pair of indifference curves to justify these two equilibrium bundles. (Zero marks if either of your indifference curve slopes upwards anywhere!)

Solution: See the diagram.

 Question 6 continues on next page…

(e) On the figure illustrate clearly the substitution effect and the income effect on leisure due to the change in the wage rate between w and w'.

> **Solution:** The dotted line has the slope of w', the new wage. U is the original utility level. Hence the movement from bundle E to bundle S represents the substitution effect, since S is the bundle chosen at the new prices and the old utility level: leisure (N) falls as its price (w) rises. To keep the individual on the initial utility level after the wage rises to w', the individual hypothetically must have a negative lump-sum compensation charged against them, to move the budget line from the actual L_2 down to the dotted budget line.
>
> The income effect is represented by the movement from bundle S to bundle E', as the hypothetical negative lump-sum compensation used to define the substitution effect is returned to the individual.

(f) From your diagram estimate (to the nearest $10) the amount of the Compensating Variation CV associated with the change in the wage rate.

> **Solution:** The CV is exactly the amount of compensation needed to get the individual from the new iso-utility level U' back to the original utility level U at the new wage rate w'. That is, it is the amount of compensation needed to move from the L_2 budget line to the dotted budget line. Reading from the C axis of the diagram the dotted line involves an expenditure of $E(w', p, U) = 315$; the L_2 budget line involves an expenditure of $E(w', p, U') = 400 = 80 + 24 \times w'$. The CV is therefore
>
> $$CV = E(w', p, U') - E(w', p, U) = 400 - 315 = 85.$$
>
> Note that the precise answer to this part depends on the way you have drawn the indifference curves, which determines the location of S and the dotted budget line. The answer given here is for the curves shown here.