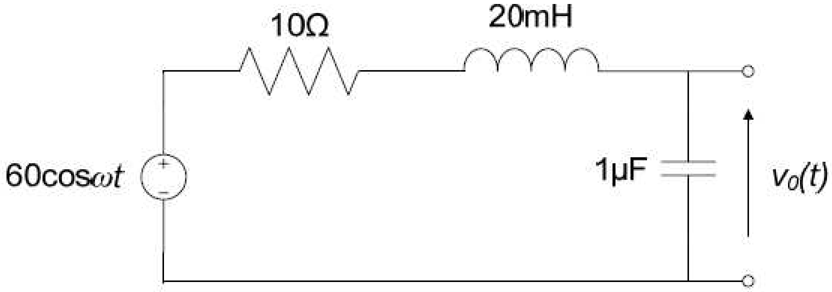
Solution Set 9 (Fall 2011)

9.1 Given the network below find ω_0, $|V_0\ (\omega_0)|$ and Q.



<u>Solution:</u>

Resonance occurs when $\omega L = \frac{1}{\omega C} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

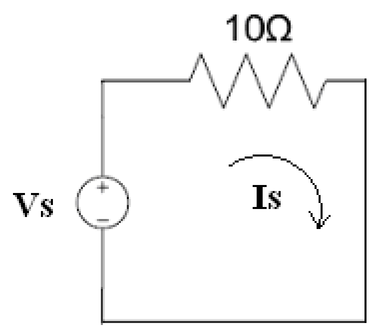
$$\omega_0 = \frac{1}{\sqrt{20mH * 1\mu F}} = 7071 rad/s$$

During the resonance:

$$I(\omega_0) = \frac{60\angle 0^\circ}{10} 6\angle 0^\circ$$

$$V(\omega_0) = I(\omega_0) * Z_C = 6\angle 0^\circ * (\frac{1}{j\omega C}) = 6\angle 0^\circ * 141.426\angle -90^\circ =$$

$$= 848.546\angle -90^\circ (V)$$



$$Q = \frac{|Vc|}{|Vs|} = \frac{|V_0(\omega_S)|}{|V_S|} = \frac{848.54}{60} = 14.14$$

Note: in this case $|V_C| = \frac{|V_S|}{R} * \frac{1}{\omega C} \rightarrow \frac{|V_C|}{|V_S|} = Q = \frac{1}{\omega CR}$;

9.2 Repeat the problem 9.1 if the value of R is changed to 1Ω.

<u>Solution:</u>

$\omega_0 = 7071 rad/s$ (as L and C are uncharged)

$$I(\omega_0) = \frac{60\angle 0^0}{1} = 60\angle 0^0$$

$$V_0(\omega_0) = I(\omega_0) * Z_C = 60\angle 0^0 * 141.42\angle -90^0 =$$

$=$

$$= 8485.36\angle -90^\circ (V)$$

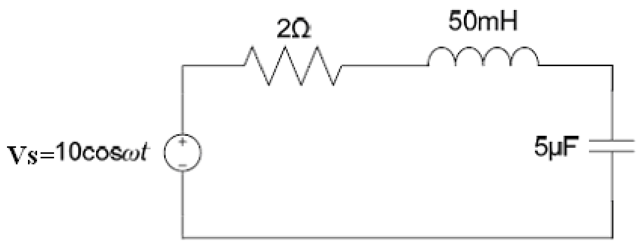
$$|V_0(\omega_0)| = 8485.36 (V)$$

$$Q=\frac{|V_0(\omega_0)|}{|Vs|}=\frac{8485.36}{60}=141.42$$

Note that since $Q \propto 1/R$ (in this case)

$$R\downarrow\times 10 \qquad Q\uparrow\times 10$$

9.3 Determine the resonant frequency, Q, BW and the average power dissipated by the network at resonance.



Solution:

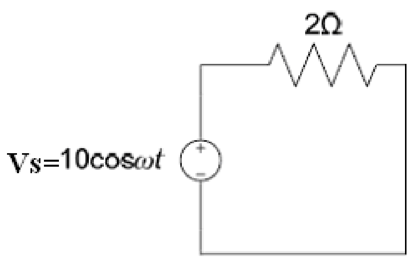
$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{50mH*5\mu F}}=2000rad/s$$

$$|V_L|=Is*\omega_0*L=\frac{|V_s|}{R}*\omega_0 L$$

$$Q=\frac{|V_L|}{|V_s|}=\frac{|V_s|}{R}*\omega_0 L/|V_s|=\omega_0 L/R=2000*50mH/2\Omega=50$$

Bandwidth BW= $\Delta\omega=\frac{\omega_0}{Q}=\frac{2000}{50}=40rad/s$

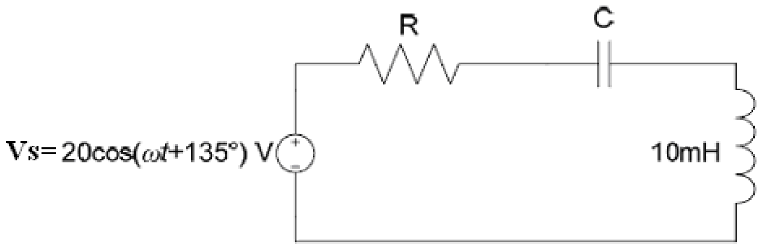
Power resonance:



$$V_{rms}=R.M.S(10\cos\omega t)=10/\sqrt{2}$$

$$P_{avg}=V_{rms}/R=\left(\frac{10}{\sqrt{2}}\right)^2\cdot\frac{1}{2}=25Watt$$

9.4 In the circuit below, if magnitude of the current at resonance is 10A, $\omega_0 = 1000 rad/s$, find ,Q and the bandwidth of the circuit.



Solution:

Given $\omega_0 = 1000 rad/s \qquad I_0 = 10A$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega_0^2} = \frac{1}{10mH*(1000)^2} = 100\mu F$$

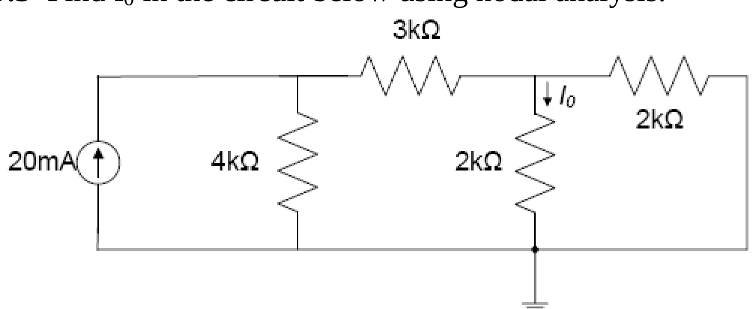
Resonance: $|I_0| = 10A \Rightarrow R = \frac{|V_S|}{|I_0|} = 20 \backslash 10 = 2\Omega$

$|V_L| = I_d \omega L = 10*1000*10mH = 100V$

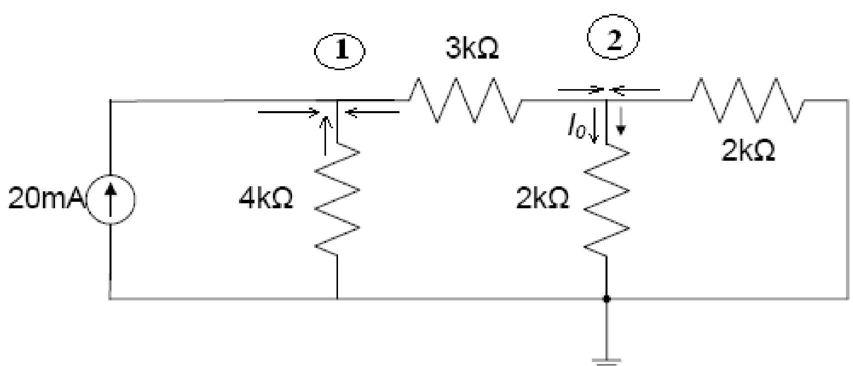
$$Q = \frac{|V_L|}{|V_S|} = 100/20 = 5$$

$$BW = \frac{\omega_0}{Q} = 1000/5 = 200 rad/s$$

9.5 Find I_0 in the circuit below using nodal analysis.



Solution:



At the node#1:

$$20m + \left(\frac{0 - V_1}{4k\Omega}\right) + \left(\frac{V_2 - V_1}{3k\Omega}\right) = 0$$

$$240 + (-3V_1) + 4V_2 + (-4V_1) = 0$$

$$4V_2 = 7V_1 - 240$$

$$V_2 = 7V_1 / 4 - 60 \quad ; [1]$$

At the node#2:

$$\left(\frac{V_1 - V_2}{3k\Omega}\right) - \left(\frac{V_2 - 0}{2k\Omega}\right) + \left(\frac{0 - V_2}{2k\Omega}\right) = 0$$

$$2V_1 - 2V_2 - 3V_2 - 3V_2 = 0$$

$$2V_1 = 8V_2$$

$$V1 = 4V2 \,; [2]$$

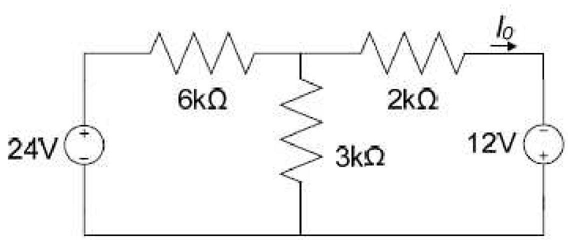
Let's substitute eq.[2] into eq.[1]:

$$V_2 = \frac{7}{4}(4V_2) - 60$$

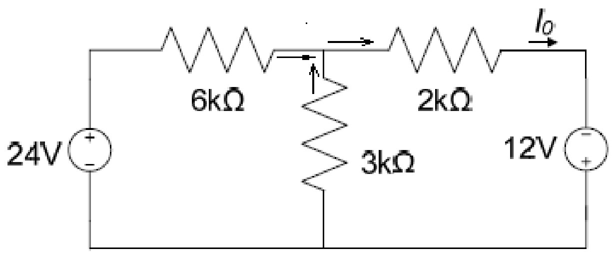
$$V_2 = 10V$$

$$I_0 = 10V / 2k\Omega = 5mA$$

9.6 Find I_0 in the circuit below using nodal analysis

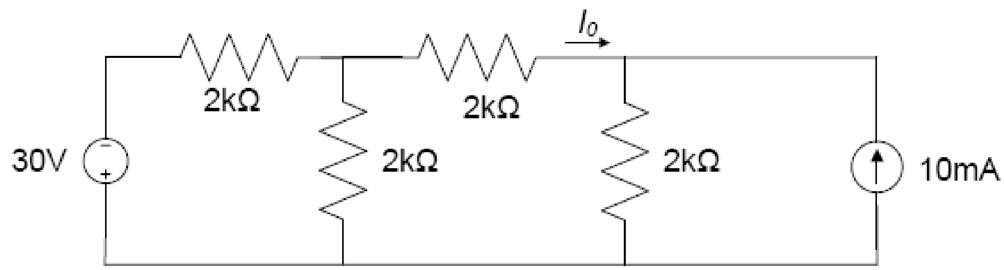


Solution:

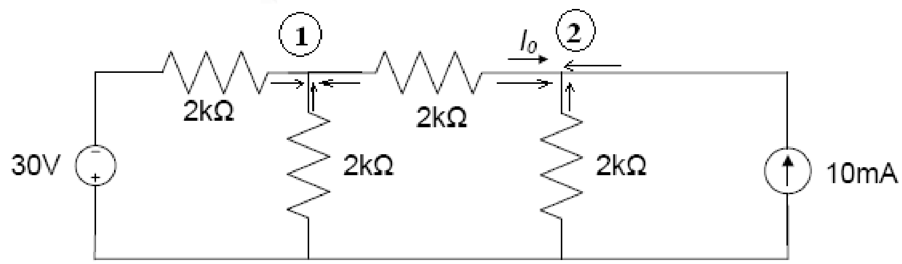


$$\left(\frac{24V - V_1}{6k\Omega}\right) + \left(\frac{0 - V_1}{3k\Omega}\right) - \left(\frac{V_1 - (-12V)}{2k\Omega}\right) = 0$$
$$24V - V_1 - 2V_1 - 3V_1 - 36 = 0$$
$$V_1 = -2V$$
$$I_0 = \frac{V_1 - (-12V)}{2k\Omega} = \frac{-2V + 12V}{2k\Omega} = 5mA$$

9.7 Find I_0 in the circuit below using nodal analysis



Solution:



At the node#1:
$$\left(\frac{-30V - V_1}{2k\Omega}\right) + \left(\frac{0 - V_1}{2k\Omega}\right) + \left(\frac{V_2 - V_1}{2k\Omega}\right) = 0$$
$$-30V - V_1 - V_1 + V_2 - V_1 = 0;$$
$$V_2 = 3V_1 + 30V \text{ ; [3]}$$

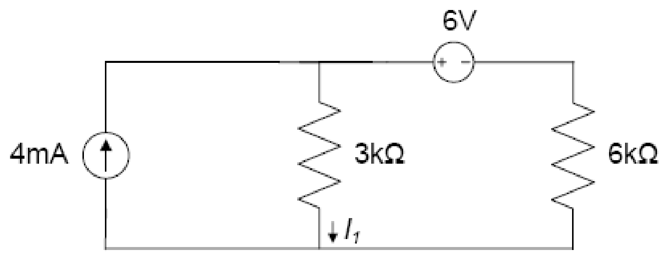
At the node#2:
$$\left(\frac{V_1 - V_2}{2k\Omega}\right) + \left(\frac{0 - V_2}{2k\Omega}\right) + 10mA = 0;$$
$$V_1 - V_2 - V_2 + 20V = 0;$$
$$V_1 = 2V_2 - 20V \text{ ; [4]}$$

Let's substitute [4] into [3]:
$$V_2 = 3(2V_2 - 20V) + 30V$$
$$V_2 = 6V$$
Let's substitute V2 into eq. [4]:

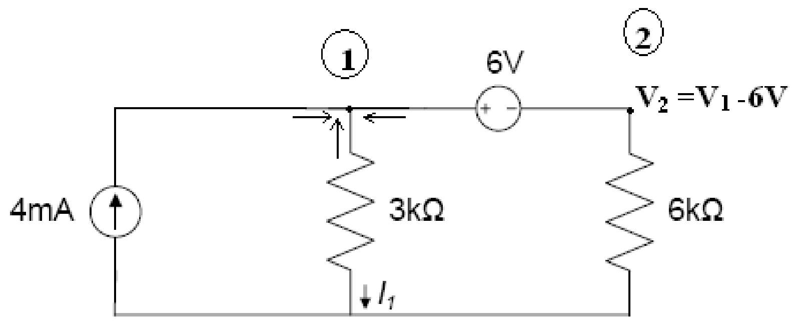
$$V_1 = 2(6V) - 20V = -8V$$

$$I_0 = \frac{V_1 - V_2}{2k\Omega} = -7mA$$

9.8 Find I_1 in the circuit below using nodal analysis



<u>Solution:</u>



It can be noted that node#2 is just (V_1-6V)
Node#1:

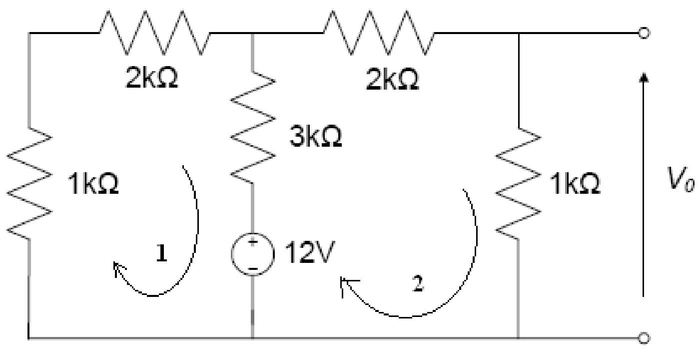
$$4mA + \left(\frac{0 - V_1}{3k\Omega}\right) - \left(\frac{0 - (V_1 - 6V)}{6k\Omega}\right) = 0;$$

$$24V - 2V_1 - V_1 + 6V = 0;$$

$$V_1 = 10V$$

$$I_1 = \frac{V_1 - 0}{3k\Omega} = 10V / 3k\Omega = 3.33mA$$

9.9 Use loop analysis to find V_0 in the circuit below:



Solution:

Loop 1:

$$1k\Omega I_1 + 2k\Omega I_1 + 3k\Omega(I_1 - I_2) + 12V = 0;$$

$$6k\Omega I_1 - 3k\Omega I_2 + 12V = 0;$$

$$I_1 - 0.5I_2 + 2mA = 0;\ [5]$$

Loop 2:

$$-12V + 3k\Omega(I_2 - I_1) + 2k\Omega I_2 + 1k\Omega I_1 = 0;$$

$$6k\Omega I_2 - 3k\Omega I_1 - 12V = 0;$$

$$I_2 = 0.5I_1 + 2mA;\ [6]$$

Let's substitute eq.[6] into eq.[5]:

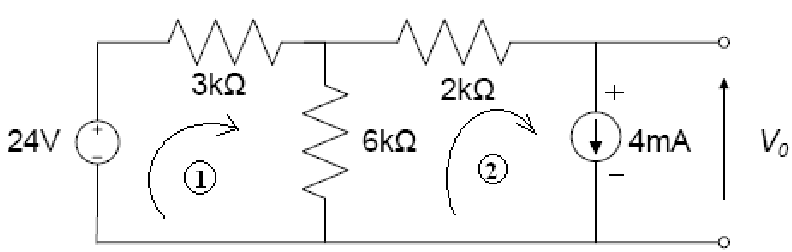
$$I_1 - 0.5(0.5I_1 + 2mA) + 2mA = 0;$$

$$I_1 = -1.33mA$$

$$I_2 = 0.5(4/3)mA + 2mA = 1.333mA$$

$$V_0 = 1.333mA * 1k\Omega = 1.333V$$

9.10 Use loop analysis to find V_0 in the circuit below:



Solution:

Loop 1:

$$-24V + 3k\Omega I_1 + 6k\Omega(I_1 - I_2) = 0;$$

$$9k\Omega I_1 - 6k\Omega I_2 - 24V = 0;$$

$I_1 = \frac{2}{3} I_2 + \frac{8}{3} mA$

But I_2 is known to be 4mA:

$I_1 = \frac{2}{3} 4mA + \frac{8}{3} mA = \frac{16}{3} mA$

Loop 2:

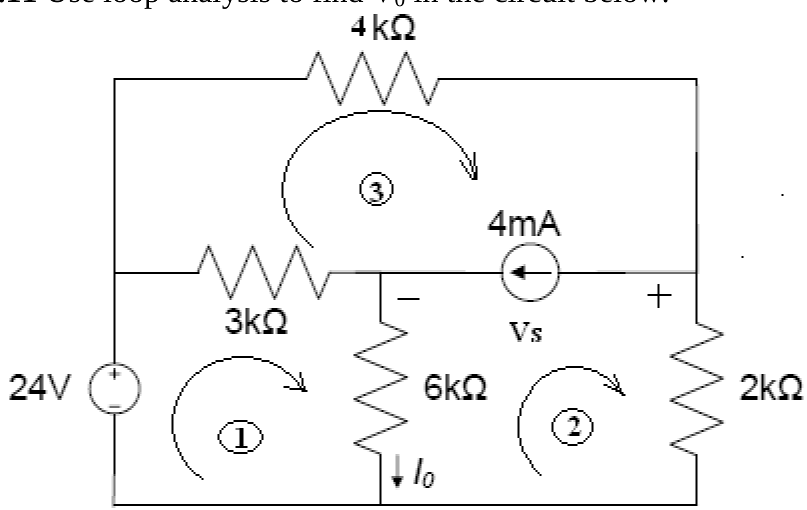
$6k\Omega(I_2 - I_1) + 2k\Omega I_2 + V_0 = 0$

$6k\Omega(4mA - \frac{16}{3} mA) + 2k\Omega 4mA + V_0 = 0$

$6k\Omega(-1.333mA) + 2k\Omega 4mA + V_0 = 0$

$V_0 = 0V$

9.11 Use loop analysis to find V_0 in the circuit below:



Solution:

Loop 1: $-24V + 3k\Omega(I_1 - I_3) + 6k\Omega(I_1 - I_2) = 0$; [1]

Loop 2: $6k\Omega(I_2 - I_1) - V_S + 2k\Omega I_2 = 0$; [2]

Loop 3: $3k\Omega(I_3 - I_1) + 4k\Omega I_3 + V_S$; [3]

$I_3 - I_2 = 4mA \rightarrow I_3 = I_2 + 4mA$ [4]

Let's insert eq. [4] into eq. [3]:

$3k\Omega((I_2 + 4mA) - I_1) + 4k\Omega(I_2 + 4mA) + V_s = 0$

$3k\Omega I_2 + 12V - 3k\Omega I_1 + 4k\Omega I_2 + 16V + V_s = 0$

$7k\Omega I_2 - 3k\Omega I_1 + 28V + V_s = 0$; [5]

Let's substitute [4] into [1]:

$$-24V + 3k\Omega(I_1 - (I_2 + 4mA)) + 6k\Omega(I_1 - I_2) = 0$$
$$-24V + 3k\Omega I_1 - 3k\Omega I_2 - 12V + 6k\Omega I_1 - 6k\Omega I_2 = 0$$
$$9k\Omega I_1 - 9k\Omega I_2 - 36V = 0\,; \quad [6]$$

When eq. [2]:
$$6k\Omega(I_2 - I_1) - V_S + 2k\Omega I_2 = 0;$$
$$8k\Omega I_2 - 6k\Omega I_1 - V_S = 0$$

Eq. [5] and [2] together:
$$15k\Omega I_2 - 9k\Omega I_1 + 28V = 0; \quad [7]$$

Eq. [6] and [7] together:
$$6k\Omega I_2 - 8V = 0$$
$$I_2 = 8/6mA = 1.333mA\,; \quad [8]$$
Let's substitute eq. [8] into eq. [6]:
$$9k\Omega I_1 - 9k\Omega(1.333mA) - 36V = 0;$$
$$9k\Omega I_1 = 12V + 36V$$
$$I_1 = 5.333mA$$
$$I_0 = I_1 - I_2 = 5.333mA - 1.333mA = 4mA$$