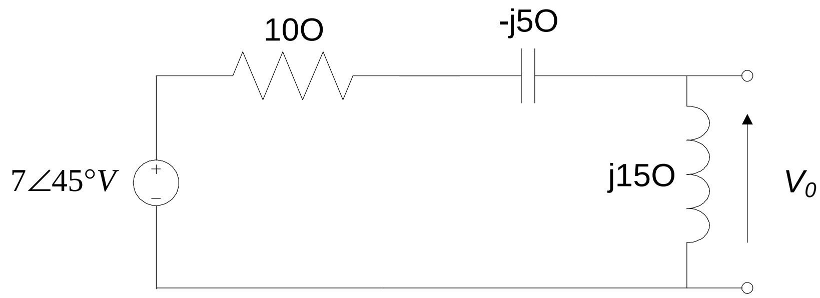
Problem Set 8 (Fall 2011)

8.1 Using voltage divider :



$$V_o = 7\angle 45^o \ . \ \frac{Z_L}{Z_R + Z_C + Z_L}$$

$$= 7\angle 45^o \ . \ \frac{j15}{10 - 5 + 15}$$

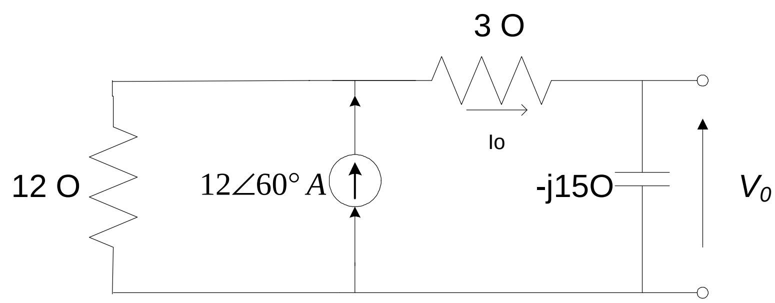
$$= 7\angle 45^o \ . \ \frac{15\angle 90^o}{10 + j10}$$

$$= 7\angle 45^o \ . \ \frac{15\angle 90^o}{10\sqrt{2}\angle 45^o}$$

$$= 5.25\sqrt{2}\ \angle 90^0 \ \text{v}$$

(* Note : According to the convention in this course, the arrow point to the higher potential , which might be the opposite to those physics background)

8.2 Using current divider:



$$I_o = 12\angle 60^o \ . \ \frac{12}{12 + 3 - j15}$$

$$= 12\angle 60^o \ . \ \frac{12}{15\sqrt{2}\angle -45^o}$$

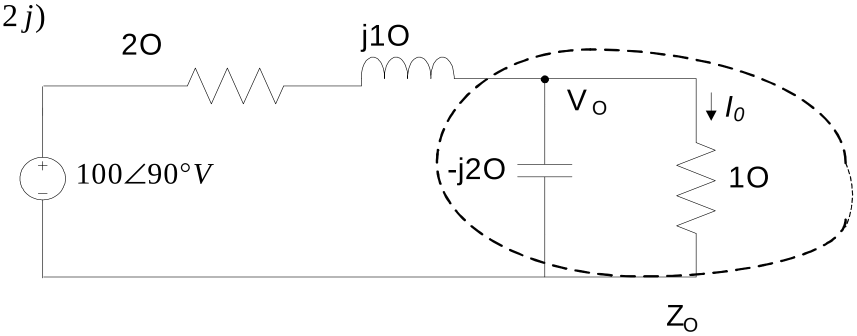
$$= \frac{24\sqrt{2}}{5}\angle 105^o \ \text{A}$$

$V_o = I_o$ (-j15Ω)

$$= \frac{24\sqrt{2}}{5}\angle 105^o \ . \ 15\angle -90^o$$

$$= 72\sqrt{2}\angle 15^o \ \text{V}$$

8. Z_o = -2j // 1 = $\dfrac{-2j}{1-2j}$ = $\dfrac{-2j(1+2j)}{1-2j(1+2j)}$



= $\dfrac{4-2j}{5}$ = 0.8 – 0.4j Ω

Using voltage divider:

$V_o = 100\angle 90^o \cdot \dfrac{Z_O}{Z_O + 2 + j}$

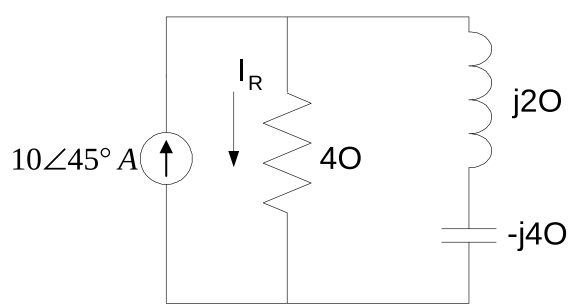
$= 100\angle 90^o \cdot \dfrac{0.8-0.4j}{0.8-0.4j+2+j} = 100\angle 90^o \cdot \dfrac{0.894\angle -26.6^o}{2.86\angle 12.1^o}$

$= 31.26\angle 51.3^o$ V

$I_o = \dfrac{v_O}{1\Omega} = 31.26\angle 51.3^o$ A

8.4 Power is absorbed only by the resistor. Inductors and capacitors only store energy. They do not consume.

Using current divider:



$I_R = 10\angle 45^o \cdot \dfrac{2j-4j}{2j-4j+4}$

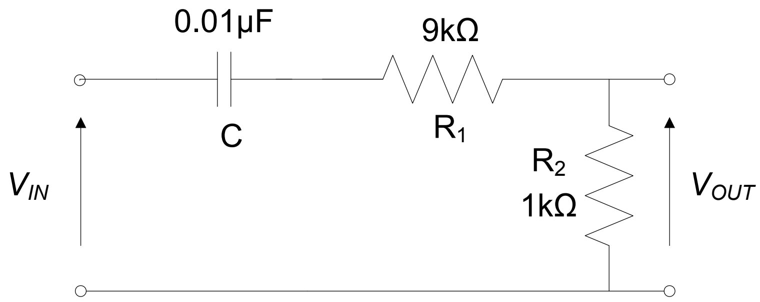
$= 10\angle 45^o \cdot \dfrac{-2j}{4-2j}$

$= 10\angle 45^o \cdot \dfrac{2\angle -90^o}{4.47\angle -26.6^o}$

$= 4.47\angle -18.4^o$

Average power $P_A = \dfrac{1}{2} \, |I_R|^2 \, R = \dfrac{1}{2} \, (4.47)^2 \cdot 4 \approx 40$ W

8.5



$$Z_c = \frac{1}{j\omega C}$$

Using Voltage Divider

$$V_{out} = \frac{R_2}{R_2 + R_1 + \frac{1}{j\omega C}} \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{j\omega C R_2}{1 + j\omega C(R_1 + R_2)} = \frac{j\omega \frac{R_2}{R_2 + R_1}}{j\omega + \frac{1}{C(R_1 + R_2)}}$$

Examine the denominator:

D(ω)= $j\omega + \frac{1}{C(R_1 + R_2)}$

At very low frequency $|j\omega| << \frac{1}{C(R_1 + R_2)}$, D(ω)= $\frac{1}{C(R_1 + R_2)}$

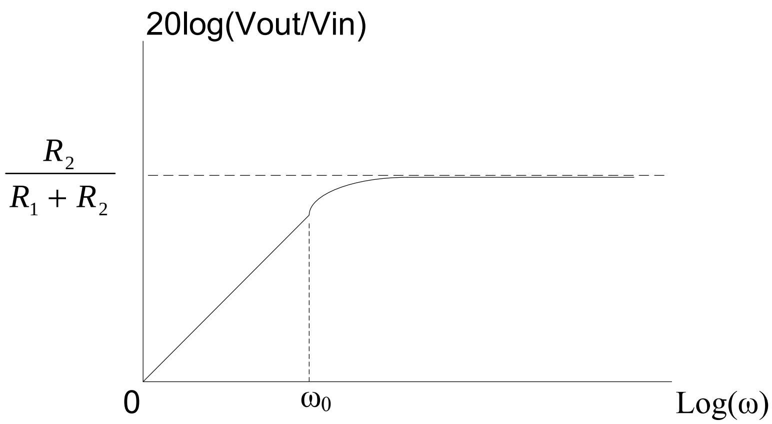
At very high frequency $|j\omega| >> \frac{1}{C(R_1 + R_2)}$, D(ω)= $j\omega$

Somewhere in between when $|j\omega| = \frac{1}{C(R_1 + R_2)}$ we call this cut off frequency ω_0. In this example $\omega_{0} = \frac{1}{C(R_1 + R_2)}$ =10k rad/s

Therefore

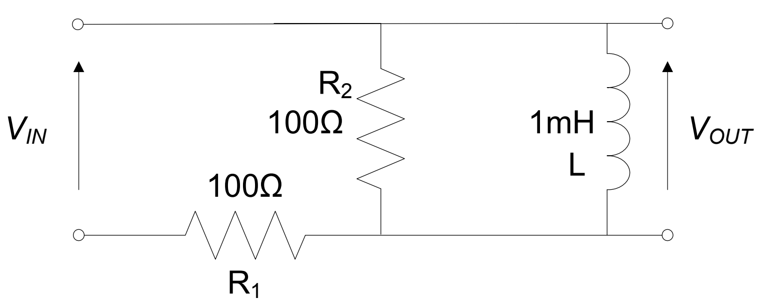
When $\omega_0 << \frac{1}{C(R_1+R_2)}$; $\frac{V_{out}}{V_{in}} = \frac{j\omega \frac{R_2}{R_1+R_2}}{\frac{1}{C(R_1+R_{2)}}} \approx j\omega R_2 C$

When $\omega_0 >> \frac{1}{C(R_1+R_2)}$; $\frac{V_{out}}{V_{in}} = \frac{j\omega \frac{R_2}{R_1+R_2}}{j\omega} \approx \frac{R_2}{R_1+R_2}$



High pass filter

8.6



$V_s=(1/2)V_{in}$
$R_s=100//100=50\Omega$

Using Voltage Divider

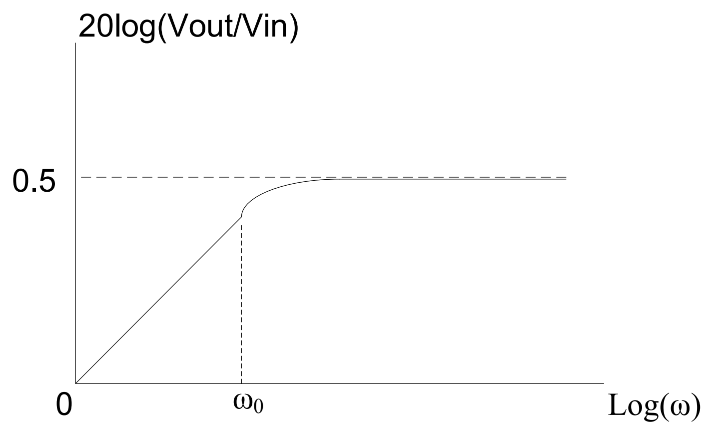
$V_{out} = \frac{j\omega(1mH)}{50\Omega + j\omega(1mH)} . \frac{V_{in}}{2}$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} \bullet \frac{j\omega}{\frac{50}{1m} + j\omega} = \frac{0.5j\omega}{j\omega + 50000}$$

$$\therefore \omega_0 = 50000 rad/s$$

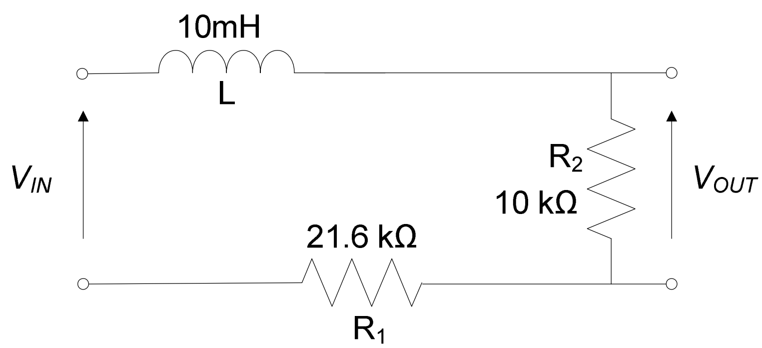
When ω<< $50000 rad/s$; $\frac{V_{out}}{V_{in}} \approx \frac{0.5j\omega}{50000} \propto \omega$

When ω>> $50000 rad/s$; $\frac{V_{out}}{V_{in}} = \frac{0.5j\omega}{j\omega} = 0.5$



High pass filter

8.7



Using Voltage Divider

$$V_{out} = \frac{10k}{10k + 21.6k + 10m \bullet j\omega} . V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{10k}{31.6k + 10m \bullet j\omega}$$

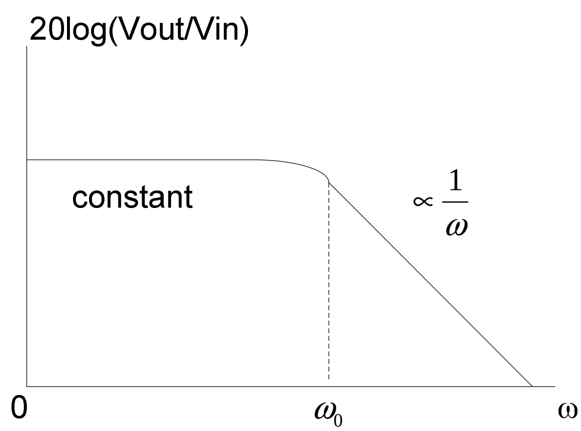
$$= \frac{1M}{3.16M + j\omega}$$

$$\therefore \omega_0 = 3.16\times10^6\, rad/s$$

When ω<< $3.16\times10^6\, rad/s$; $\frac{V_{out}}{V_{in}} = \frac{1\times10^6}{3.16\times10^6} \approx \frac{1}{3.16}$

When ω>> $3.16\times10^6\, rad/s$; $\frac{V_{out}}{V_{in}} = \frac{1\times10^6}{j\omega} \propto \frac{1}{\omega}$

It's a low pass filter



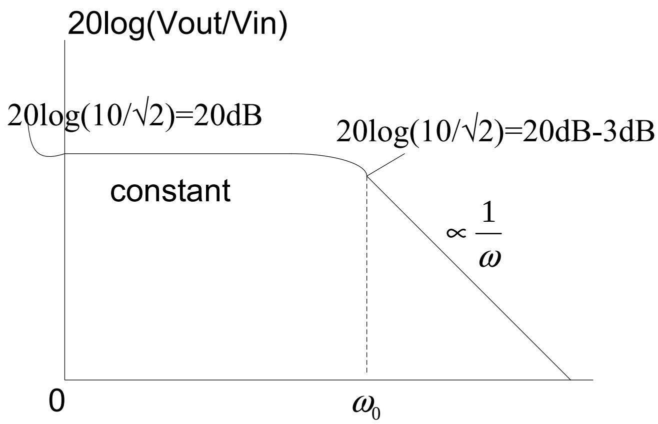
8.8 $\frac{V_{out}}{V_{in}} = \frac{10}{1+10\bullet j\omega} = \frac{1}{\frac{1}{10} + j\omega}$ $\therefore \omega_0 = \frac{1}{10} = 0.1 rad/s$

When ω>> ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{1}{j\omega} = \frac{1}{\omega}\angle -90$

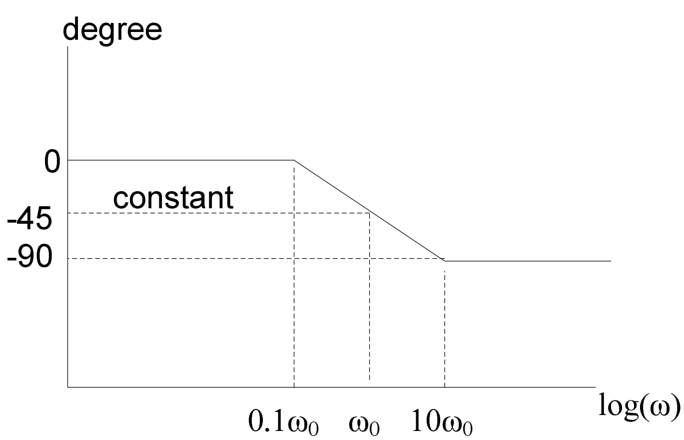
When ω= ω_0 ; $\frac{V_{out}}{V_{in}} = \frac{1}{\frac{1}{10} + j\frac{1}{10}} = \frac{10}{1+j} = \frac{10}{\sqrt{2}}\angle -45$

When ω<< ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{1}{\frac{1}{10}} = 10\angle 0$

Magnitude plot:



Phase plot:

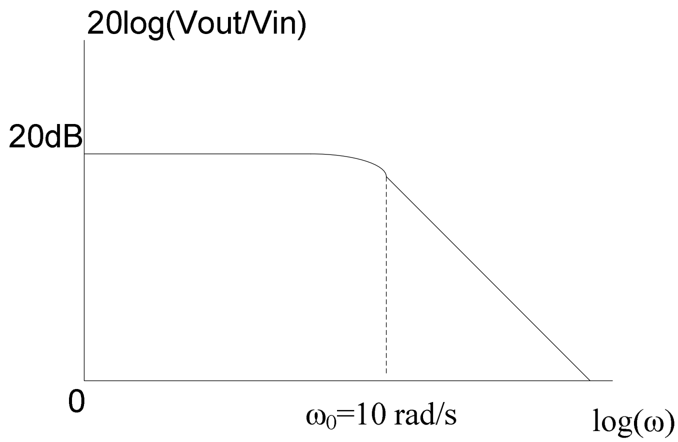


8.9 $\frac{V_{OUT}}{V_{IN}} = \frac{100}{j\omega - 10}$ $\quad \therefore \omega_0 = 10 rad/s$

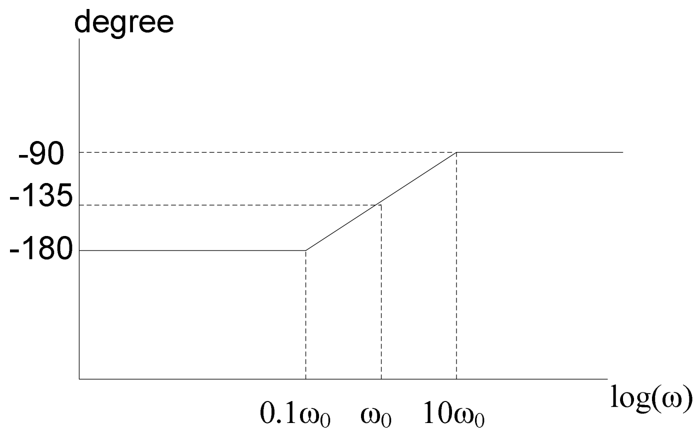
When ω>> ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{100}{j\omega} = \frac{100}{\omega}\angle -90$

When ω<< ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{100}{-10} = 10\angle -180$

Magnitude plot:



Phase plot:



8.10 $\frac{V_{OUT}}{V_{IN}} = \frac{1}{1+\frac{1}{j10\omega}} = \frac{j\omega}{j\omega+\frac{1}{10}} = \frac{j\omega}{j\omega+0.1}$

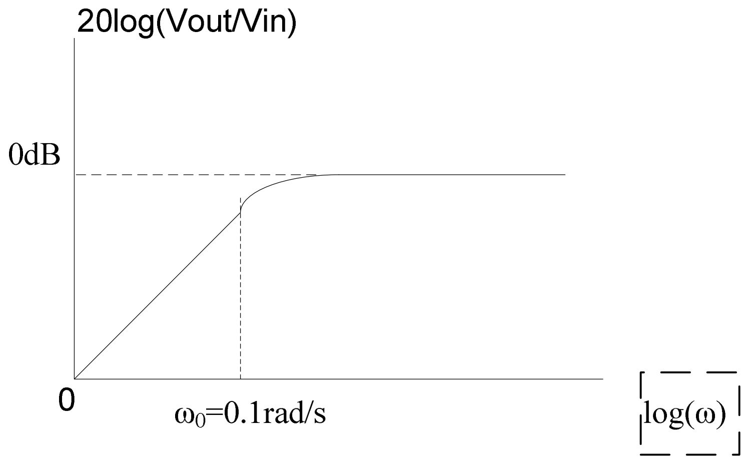
$\therefore \omega_0 = 0.1 rad/s$

When ω>> ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{j\omega}{j\omega} = 1$

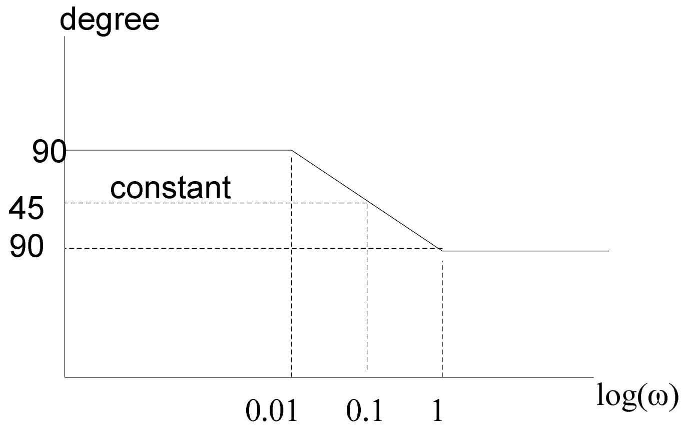
When ω= ω_0 ; $\frac{V_{out}}{V_{in}} = \frac{0.1j}{0.1(j+1)} = \frac{1\angle 90}{\sqrt{2}\angle 45} = \frac{1}{\sqrt{2}}\angle 45$

When ω<< ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{j\omega}{0.1} = 10.j\omega = 10\omega\angle 90$

Magnitude plot:



Phase plot:



8.11 $\frac{V_{OUT}}{V_{IN}} = \frac{200j\omega}{1+20j\omega} = \frac{10j\omega}{0.05+j\omega}$

$\therefore \omega_0 = 0.05rad/s$

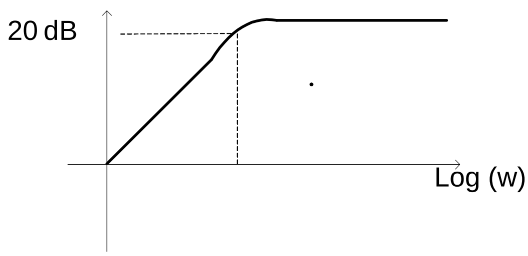
When ω>> ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{10j\omega}{j\omega} = 10\angle 0$

When ω= ω_0 ; $\frac{V_{out}}{V_{in}} = \frac{10j(0.05)}{0.05 + j(0.05)} = \frac{10j}{1+j} = \frac{10}{\sqrt{2}}\angle 45$

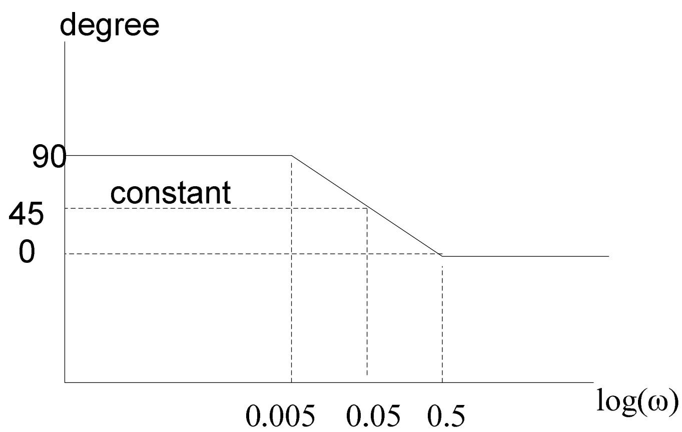
When ω<< ω_0 ; $\frac{V_{out}}{V_{in}} \approx \frac{10j\omega}{0.05} = 200j\omega = 200\omega\angle 90$

Magnitude plot:

20 Log | $\frac{V_{out}}{V_{in}}$ |



Phase plot:



8.12

8.8 $V_{out} = 10\angle 135.\dfrac{10}{1+j10} = \dfrac{100\angle 135}{10.05\angle 84} = 9.95\angle 51$

8.9 $V_{out} = 10\angle 135.\dfrac{100}{j-10} = \dfrac{100\angle 135}{10.05\angle 174.3} = 9.95\angle -39.3$

8.10 $V_{out} = 10\angle 135.\dfrac{j}{j+0.10} = 10\angle 135 \bullet \dfrac{1\angle 90}{1.004\angle 84} = 9.95\angle 141$

8.11 $V_{out} = 10\angle 135.\dfrac{10j}{0.05+j} = 10\angle 135 \bullet \dfrac{10\angle 90}{1.0012\angle 87} = 99.9\angle 138$