Solution Set 7 (Fall 2011)

7.1 Determine the phase angles by which $v_1(t)$ leads $i_1(t)$ and $v_1(t)$ leads $i_2(t)$, where :

$v_1(t) = 6\sin(377t + 25^\circ)V$

$i_1(t) = 0.04\cos(377t - 10^\circ)A$

$i_2(t) = -0.2\sin(377t - 75^\circ)A$

Solution:

$i_1(t) = 0.04\cos(377t - 10^\circ)A = 0.04\sin(377t - 10^\circ + 90^\circ)A = 0.04\sin(377 + 80^\circ)A$

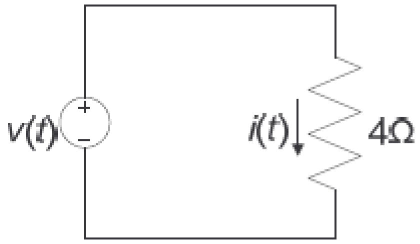
It means that $v_1(t)$ leads $i_1(t)$ by $25^\circ - 80^\circ = -55^\circ$.

$i_2(t) = -0.2\sin(377t - 75^\circ)A = 0.2\sin(377 - 75^\circ + 180^\circ)A = 0.2\sin(377 + 105^\circ)A$

It means that $25^\circ - 105^\circ = -80^\circ$

7.2 Calculate $i(t)$, the time-domain current in the resistor in the following circuit if the input voltage is:

1) $v_1(t) = 12\cos(377t + 180^\circ)V$
2) $v_2(t) = 16\sin(377t + 45^\circ)V$



Solution:

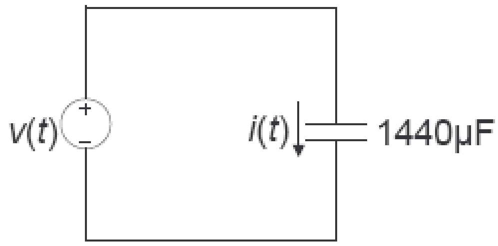
This is a purely resistive network: $i(t)$ and $v(t)$ are in phase

1) $i_1(t) = v_1(t)/R = \dfrac{12\cos(377t + 180^\circ)}{4}A = 3\cos(377t + 180^\circ)A$

2) $i_2(t) = v_2(t)/R = \dfrac{16}{4}\sin(377t + 45^\circ)A = 4\sin(377t + 45^\circ)A$

7.3 Calculate $i(t)$, the time-domain current in the capacitor in the following circuit if the input voltage is:

1) $v_1(t) = 8\cos(377t - 30^\circ)V$
2) $v_2(t) = 4\sin(377t + 60^\circ)V$



Solution:

This is purely Capacitive network: $i(t)$ leads $v(t)$ by 90°.

$i(t) = C\dfrac{dv(t)}{dt}$, However, this equation can be rewritten as $I = j\omega CV$ (eq.7.29 Irwin)

1) $I1 = j\omega C(8\angle -30^\circ)A,$ where $v_1(t) = 8\cos(377t - 30^\circ)V$

$I1 = 377*1440*10^{-6}\angle 90^\circ * 8\angle -30^\circ A = 4.34\angle 60^\circ A$

$i_1(t) = 4.34\cos(377 + 60^\circ)A$

2) $v_2(t) = 4\sin(377t + 60^\circ)V = 4\cos(377t + 60^\circ - 90^\circ)V = 4\cos(377 - 30^\circ)V$

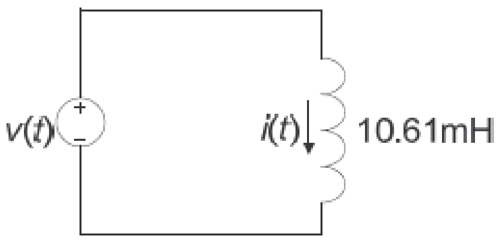
$I2 = j\omega C(4\angle -30^\circ)A = 377*1440*10^{-6}\angle 90^\circ * 4\angle -30^\circ A = 2.17\angle 60^\circ A$

$i_2(t) = 2.17\cos(377 + 60^\circ)A$

7.4 Calculate $i(t)$, the time-domain current in the inductor in the following circuit if the input voltage is:

1) $v_1(t) = 24\cos(377t + 12^\circ)V$

2) $v_2(t) = 18\sin(377t + 48^\circ)V$



Solution:

This purely Inductive network: $v(t)$ leads $i(t)$ by 90°

$v(t) = L\dfrac{di(t)}{dt}$ This can be simplified to $V = j\omega LI$

1) $v_1(t) = 24\cos(377t + 12^\circ)V$

$$I = \frac{V_1}{j\omega L} = \frac{24\angle 12^\circ}{377*10.61*10^{-3}\angle 90^\circ} = 6\angle(12^\circ - 90^\circ) = 6\angle -78^\circ$$

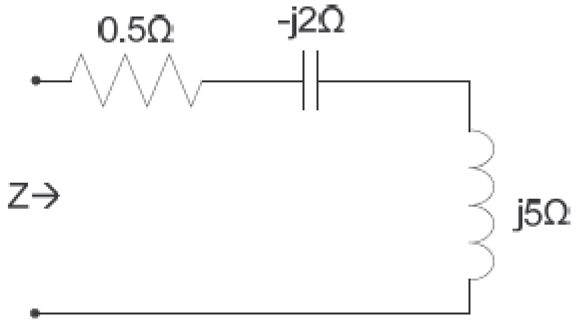
$i(t) = 6\cos(377 - 78^\circ)A$

2) $v_2(t) = 18\sin(377t + 48^\circ)V = 18\cos(377t + 48^\circ - 90^\circ)V = 18\cos(377t - 42^\circ)V$;

$$I_2 = \frac{18\angle -42^\circ}{377*10.61*10^{-3}\angle 90^\circ}A = 4.5\angle(-42^\circ - 90^\circ) = 4.5\angle -132^\circ$$

$i(t) = 4.5\cos(377t - 132^\circ)A$

7.5 Find the frequency-domain impedance Z of the following network:

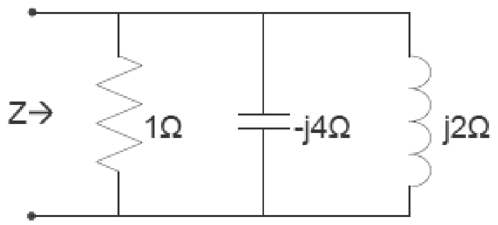


Solution:
This is a series RLC network:
Impedance $Z = Z_R + Z_C + Z_L$

$$Z = Z_R + Z_C + Z_L = 0.5\Omega - j2\Omega + j5\Omega = 0.5 + j3(\Omega) = \sqrt{0.5^2 + 3^2}\angle \tan^{-1}(\frac{3}{0.5}) = 3.04\angle 80.54^\circ$$

$$Z = 3.04\angle 80.54^\circ$$

7.6 Find the frequency-domain impedance Z of the following network:



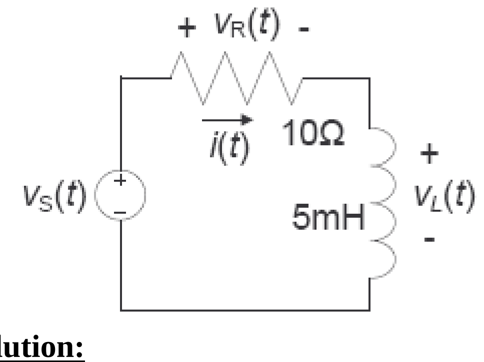
Solution:
This is a Parallel RLC network
Impedance $Z = (\frac{1}{Z_R} + \frac{1}{Z_C} + \frac{1}{Z_L})^{-1}$

$$\frac{1}{Z} = \frac{1}{1\Omega} + \frac{1}{-j4\Omega} + \frac{1}{j2\Omega} = 1 + j0.25 - j0.5 = 1 - j0.25$$

$$Z = \frac{1}{1 - j0.25}\Omega = \frac{1}{\sqrt{1 + (-0.25)^2}\angle \tan^{-1}(-0.25/1)}\Omega \approx \frac{1}{1.03\angle -14.04^\circ}\Omega = 0.97\angle 14.04^\circ\,\Omega$$

$$Z = 0.97\angle 14.04^\circ\,\Omega \quad \text{or } Z^{-1} = Y \approx 1.03\angle -14.04^\circ$$

7.7 The voltages $v_R(t)$ and $v_L(t)$ in the following circuit can be redrawn as phasors in a phasor diagram. Use a phasor diagram to show that $v_R(t) + v_L(t) = v_s(t)$ where
$v_s(t) = 120\cos(377t)V$

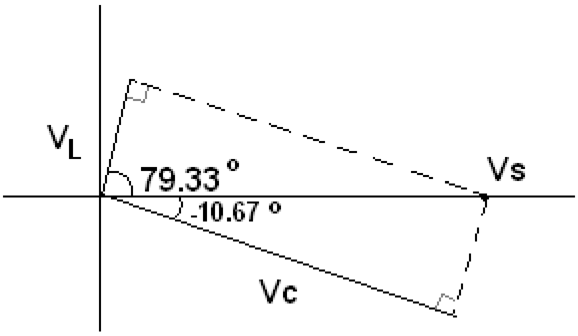


Solution:

$$I = \frac{V_s}{R + j\omega L} = \frac{120\angle 0°}{10 + j(377 \times 5m)} = 11.79\angle -10.67^\circ.$$

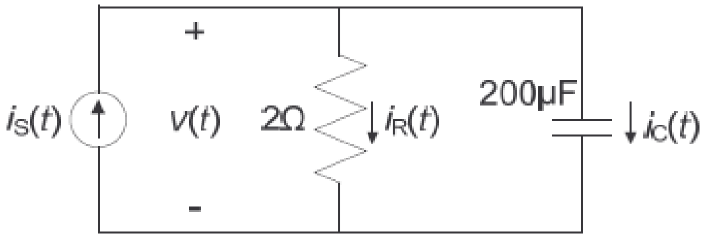
$$V_L = j\omega LI = 377 * 5 * 10^{-3}\angle 90^\circ \; * 11.79\angle -10.67^\circ = 22.22\angle 79.33^\circ$$

$$v_L(t) = 22.22\cos(377t + 79.33^\circ)V$$

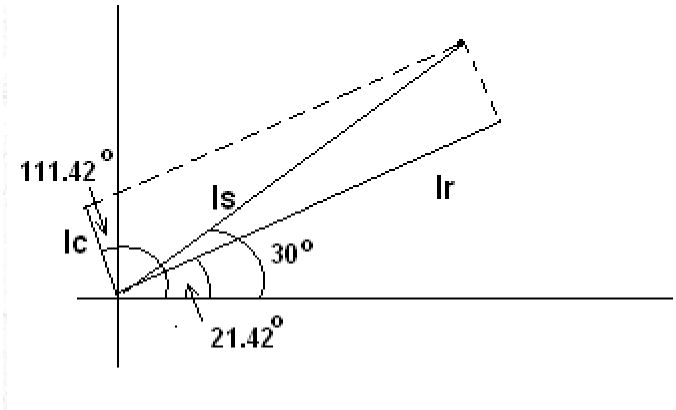


$$\Delta\angle = \tan^{-1}(\frac{V_L}{V_R}) = \tan^{-1}\frac{22.22}{117.9} = 10.67$$
$$V_S{}^2 = V_R{}^2 + V_L{}^2 = 117.9^2 + 22.22^2 = 14394.14$$
$$V_S \ = 120$$
$$\angle V_S \ = \angle V_R + \Delta\angle = -10.67 + 10.67 = 0$$

7.8 The currents $i_R(t)$ and $i_C(t)$ in the following circuit can be drawn as phasors in a phasor diagram. Use such a phasor diagram to show that $i_C(t) + i_R(t) = i_s(t)$ where:
$i_s(t) = 20\cos(377t + 30^\circ)A$



Solution:
This is a parallel network:
$$Z_c = \frac{1}{j\omega C} = \frac{1}{j*377*200*10^{-6}} = -j13.26 = 13.26\angle -90^\circ$$
Using a current divider,
$$I_R = I_S * \frac{Z_C}{Z_r + Z_C} = 20\angle 30^\circ \frac{13.26\angle -90^\circ}{2 - j13.26} =$$
$$= 20\angle 30^\circ \frac{13.26\angle -90^\circ}{13.41\angle -81.42^\circ} = 20\frac{13.26}{13.41}\angle 30^\circ - 90^\circ + 81.42^\circ =$$
$$= 19.78\angle 21.42^\circ$$
$$I_R = I_S * \frac{Z_R}{Z_R + Z_C} = 20\angle 30^\circ \frac{2}{13.41\angle -81.42^\circ} =$$
$$= 20\frac{2}{13.41}\angle 30^\circ + 81.42^\circ$$
$$Is = \sqrt{Ic^2 + I_R{}^2} = \sqrt{19.78^2 + 2.98^2} = 20;$$



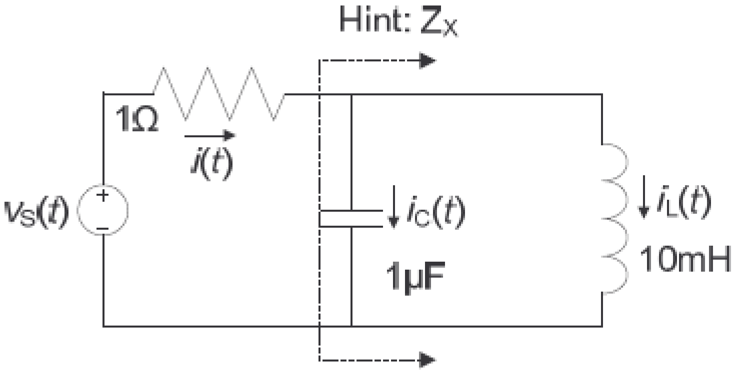
$$\Delta L = \tan^{-1}(\frac{I_C}{I_R}) = \tan^{-1}(\frac{2.98}{19.78}) = 8.57^\circ$$

$$\angle I_S = \angle I_R + \Delta L = 21.42^\circ + 8.57 \square\ 30^\circ$$

$$\angle I_S = \angle I_R + \Delta L = 21.42^\circ + 8.57^\circ \approx 29.9 \approx 30^\circ$$

7.9 The currents $i_L(t)$ and $i_C(t)$ of the inductor and capacitor, respectively, in the following circuit can be drawn as phasors in a phasor diagram. Show in the phasor diagram that $i_L(t) + i_C(t) = i_S(t)$ where:

$v_s(t) = 10\cos(10^3 t + 30^\circ)V$



Solution:

Zx is a parallel network $Z_x = Z_L \| Z_C$

$$Z_C = 1/j\omega C = \frac{1}{j10^3 * 10^{-6}} = -j10^3$$

$$Z_L = j\omega L = j10^3(10*10^{-3}) = j10$$

$$Z_x = \frac{Z_C * Z_L}{Z_L + Z_C} = \frac{-j10^3 * j10}{-j10^3 + j10} = j10.1$$

Voltage divider:

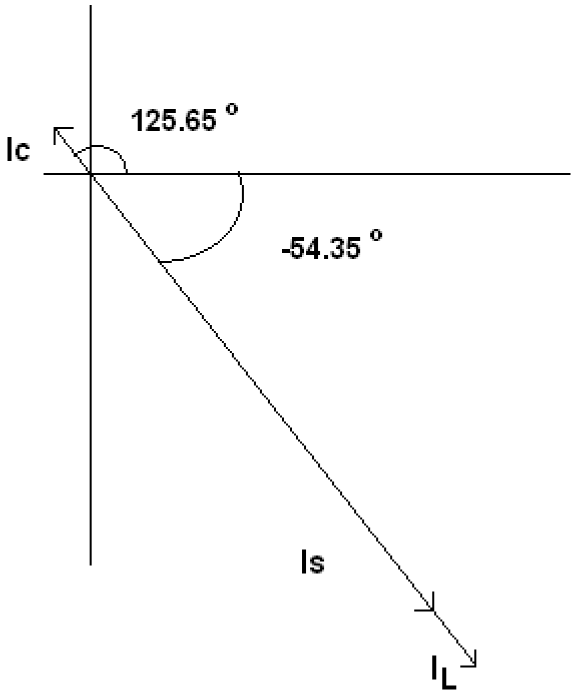
$$Va = Vs\frac{Z_X}{Z_X + Z_R} = 10\angle 30^\circ \frac{j10.1}{1 + j10.1} =$$

$$= 10\angle 30^\circ \frac{10.1\angle 90^\circ}{10.15\angle 84.35^\circ} = 9.95\angle 35.65^\circ$$

$$I_L = \frac{V}{Z_L} = \frac{9.95\angle 35.65^\circ}{j10} = \frac{9.95\angle 35.65^\circ}{10\angle 90^\circ} = 0.995\angle -54.35^\circ (A)$$

$$Ic = V / Z_C = 9.95\angle 35.65^\circ (\frac{1}{-j10^3}) = 9.95\angle 35.65^\circ (\frac{1}{10^3\angle -90^\circ}) =$$

$$= 9.95*10^{-3}\angle 125.65^\circ$$



$$Is = 0.995\angle -54.35^\circ + 9.95*10^{-3}\angle 125.65^\circ =$$

$$= 0.995\angle -54.35^\circ - 9.95*10^{-3}\angle -54.35^\circ =$$

$$= 0.985\angle -54.35^\circ$$

Verification:

$$Vs = 10\angle 30^\circ = 10\cos 30 + j10\sin 30 = 8.66 + j5.00$$

$$Va = 9.95\angle 35.65^\circ = 8.09 + j5.80$$

$$Is = \frac{Vs - Va}{R} = \frac{8.66 + j5.0 \_(8.09 + j5.8)}{1} = 0.57 - 0.8j =$$

$$= 0.982\angle -54.53^\circ$$

$Is_{phasor} = Is_{calculated}$

7.10. Repeat 7.9 for $v_s(t) = 10\cos(10^4 t + 30^\circ)V$ (surprising result called 'resonance').

Solution:

$$Z_C = 1/ j\omega C = \frac{1}{j10^4 * 10^{-6}} = -j100$$

$$Z_L = j\omega L = j10^4 (10*10^{-3}) = j100$$

$$Z_X = \frac{Z_c * Z_L}{Z_C + Z_L} = \frac{-j100 * j100}{-j100 + j100} = \frac{10^4}{0} j = +\infty$$

Voltage divider: $Va = Vs \frac{Z_x}{Z_x + Z_R} = Vs \frac{1}{1 + Z_R / Z_x} = Vs(1/(1+0))$

$$Va = Vs$$

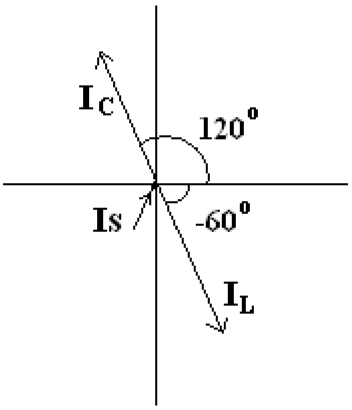
LC is in resonance

This is very large impedance (Zx) –it is like an open circuit

Since $Va = Vs$ there is no current through the resistor

$$I_L = Vs / Z_L = \frac{10\angle 30^\circ}{j100} = \frac{10\angle 30^\circ}{100\angle 90^\circ} = 0.1\angle -60^\circ$$

$$I_C = Vs / Z_C = \frac{10\angle 30^\circ}{-j100} = \frac{10\angle 30^\circ}{100\angle -90^\circ} = 0.1\angle -120^\circ$$



$$Is = \frac{Vs - Va}{R}(A) = 0 / R(A) = 0(A)$$

Note: the currents are identical in magnitude, but 180º out of phase. This means the Is is zero. This is the same value as the current which passes through the resistor.

The currents flow between L and C, but never escape the resonant network. The energy is exchanged between the electric field of the capacitor and the magnetic field of the inductor.