Solution Set 6 (Fall 2011)

6.1

C=10μf

i(t)= 10 Cos10^3t (mA) = (10m). Cos10^3t (A)

i(t<0)=0 , v(t<0)=0

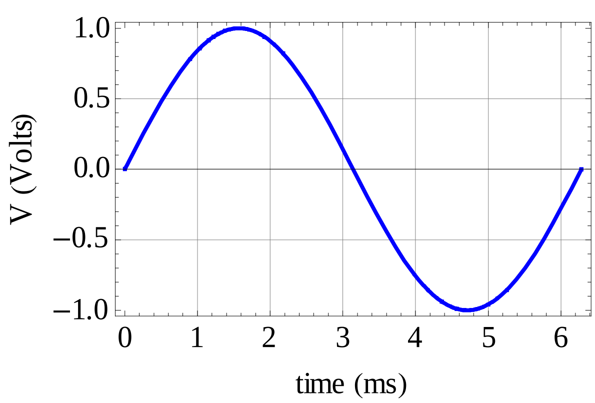
(a) Find the expression for the voltage across the capacitors.

$$V = \frac{1}{c}\int_0^t idt = \frac{1}{10\mu}\int_0^t (10m)\cos 10^3 t dt$$

$$= \frac{10m}{10\mu}.\frac{1}{10^3} \text{ Sin}10^3\text{t}$$

= Sin10^3t (v)

(b) Sketch the voltage across the capacitor:



(c)Find the expression for power.

P = i.v = (5m). Cos10^3t * 0.5 Sin10^3t

=1.25 Sin(2*10^3)t (mW)

6.2

i= 2 mA . $\frac{dv}{dt} = \frac{5v}{10v}$

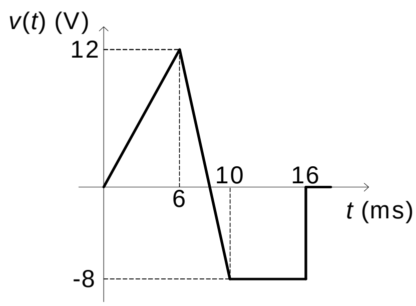
Find c:

$$i = c\ \frac{dv}{dt}$$

$$2\ m = c\ .\frac{5}{10}$$

C = 4 mF

6.3



C = 2 μF

$$i\ = c\ \frac{dv}{dt}$$

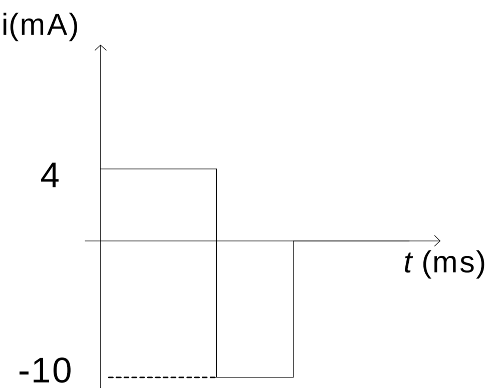
0 < t < 6 ms , $\frac{dv}{dt} = \frac{12}{6m} = 2\ k$

$$i = c\ \frac{dv}{dt} = (2\mu)\ .\ (2k) = 4\ mA$$

6ms < t < 10 ms , $\frac{dv}{dt} = \frac{-20}{4m} = -5k$

$$i = c\ \frac{dv}{dt} = (2\mu)\ .\ (-5k)$$

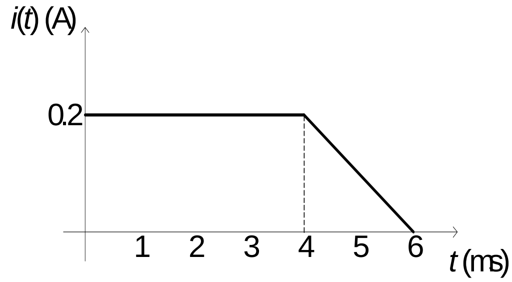
= -10 mA



10ms < t < 16ms , $\frac{dv}{dt} = 0$

$$i = c\frac{dv}{dt} = 0$$

6.4



C = 10µF

v_o = -0.1 v

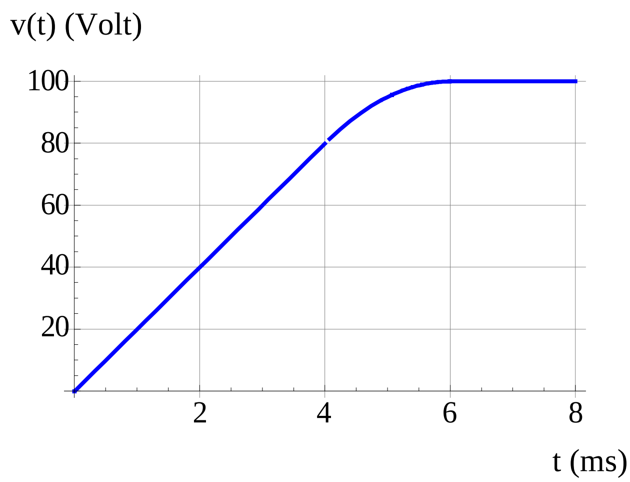
$$v(t_2) - v(t_1) = \frac{1}{c}\int_{t1}^{t2} i\,dt \ .$$

0 < t < 4 ms,   v(t) = v_o = -0.1 v

i = 0.2 A

$$v = \frac{1}{10\mu}\int_{0}^{t}(0.2)dt\ + (-0.1)$$

$$= (20 * 10^3)\ t - 0.1\ (v)$$



4ms < t < 6ms , $v(t_1) = (20*10^3)\ (4m) - 0.1$

= 0.8 - 0.1= 79.9 v

i = -0.1 t

$$v = \frac{1}{10\mu}\int_{4m}^{t}(-100t + 0.6)dt\ + 79.9$$

$$= 10^5[\frac{-100t^2}{2} + 0.6t]_{4m}^{t}\ + 79.9$$

$$= (5*10^6)\ t^2\ + 60*10^3 t - 80.1\ (v)$$

t ≥ 6 ms, $v(t_1 = 6ms) = (5*10^6)\ (6*10^{-3}) = 99.9\ (v)$

6.5

$$\frac{di}{dt} = \frac{100m}{2m} \text{ , } v = 100 \text{ mv}$$

Find L:

$$v = L\ \frac{di}{dt}$$

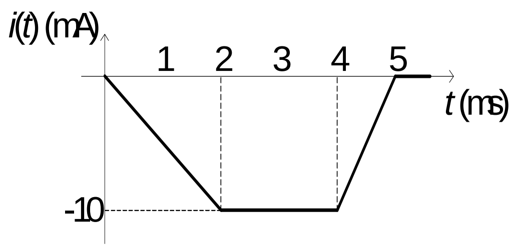
$$100\text{m} = L\ .\ \frac{100m}{2m}$$

L=2 mH

6.6

L =10 mH

$$v = L\ \frac{di}{dt}$$



$0 < t < 2\text{ms} : \frac{di}{dt} = -5$

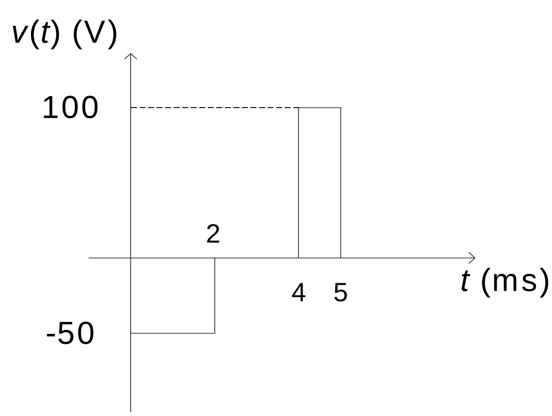
$$v = L\ \frac{di}{dt} = -50 \text{ mv}$$

$0 < t < 4\text{ms} : \frac{di}{dt} = 0$

v = 0

$0 < t < 5\text{ms} : \frac{di}{dt} = 10$

$$v = L\ \frac{di}{dt} = 10 \text{ m} . 10 = 100 \text{ mv}$$



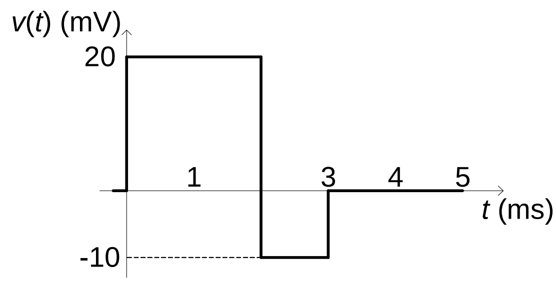
$t > 5\text{ms} : \frac{di}{dt} = 0$

v = 0

6.7

$i_o = 0$ mA

L = 20 mH



$$i(t_2) - i(t_1) = \frac{1}{L}\int_{t1}^{t2} v\,dt$$

0 < t < 2 ms : v = 20 m

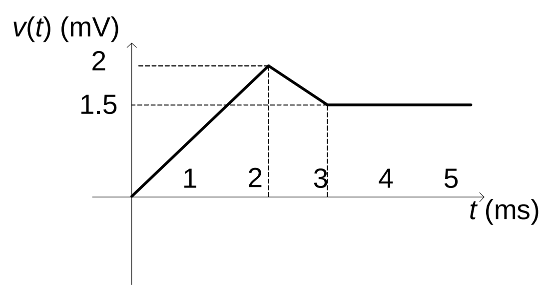
$$i(t_1) = i_0 = 0$$

$$i = \frac{1}{L}\int_{0}^{t} (20m)\,dt$$

$$= \frac{1}{20m} \,.\, (20m \,.\, t)$$

$$= t$$

2 < t < 3 ms : v = -10 m



$$i(t_1) = 2\,m$$

$$I = \frac{1}{L}\int_{2m}^{t} (-10m)\,dt + 2m$$

$$= \frac{1}{20m}\,(-10)\,(t - 2m) + 2m$$

$$= \frac{1}{20m}\,(20m^2 - 10mt) + 2m$$

$$= 3m - \frac{t}{2}$$

3 ms< t < ∞ : v = 0

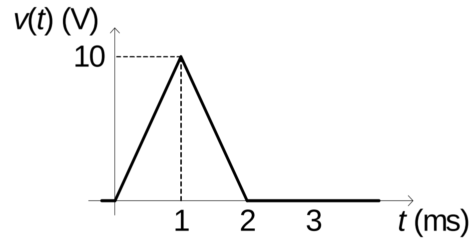
$$i(t_1) = 3m - \frac{3m}{2} = 1.5m$$

$i = i(t_1) + 0 = 1.5\ m$

6.8

$I_o = -0.4$ A

$L = 10$ mH



$$i(t_2) - i(t_1) = \frac{1}{L}\int_{t1}^{t2} v\,dt$$

$0 < t < 1\text{ms}:\ i(t_1) = i_o = -0.4$ A

$$v = \frac{10}{1m}\ t = (10k)\ t$$

$$i = \frac{1}{L}\int_{0}^{t} v\,dt + i(t_1)$$

$$= \frac{1}{10m}\ (10k)\ \frac{t^2}{2} - 0.4$$

$$= (0.5 * 10^6)\ t^2 - 0.4 \quad (A)$$

$1\text{ms} < t < 2\text{ms}:\ i(t_1) = (0.5 * 10^6)\ (1m)^2 - 0.4 = 0.1$ A

$$v = (-10k)t + 20$$

$$i = \frac{1}{L}\int_{1m}^{t} [(-10k)t + 20]dt + 0.1$$

$$= \frac{1}{10m}\ [(-10k)\ \frac{t^2 - (1m)^2}{2} + 20\ (t - 1m)] + 0.1$$

$$= (-500k)t^2 + (2k)\ t - 1.4$$

$2\text{ms} < t < \infty:\ i(t_1) = (-500k)\ (2m)^2 + (2k)(2m) - 1.4$

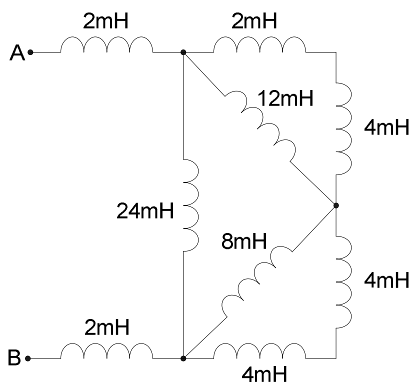
$$= -2 + 4 - 1.4$$

= 0.6 (A)

*Note: When applying equations $v(t_2) - v(t_1) = \frac{1}{c}\int_{t1}^{t2} i\,dt$, $i(t_2) - i(t_1) = \frac{1}{L}\int_{t1}^{t2} v\,dt$

keep in mind that $v(t_1)$ and $i(t_1)$ refer to initial conditions at t_1 . $v(t_2)$ and $i(t_2)$ are functions of t - *not* a specific value at time t_2 . Therefore, in the integral, the upper limit t_2 should be a variable of t.

6.9



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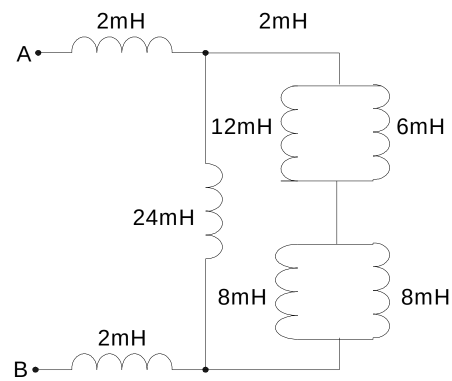

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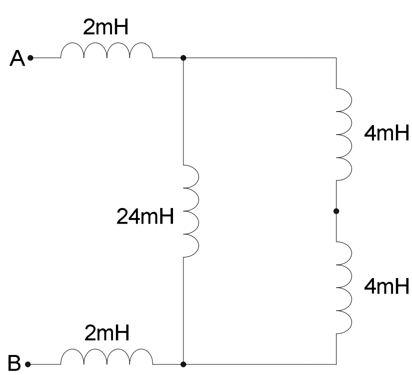

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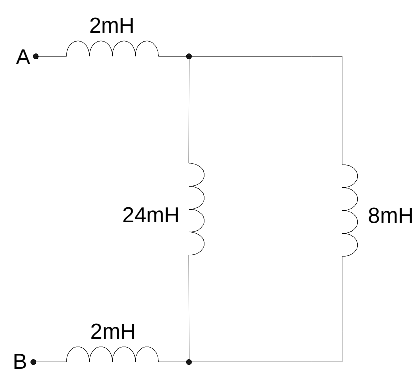
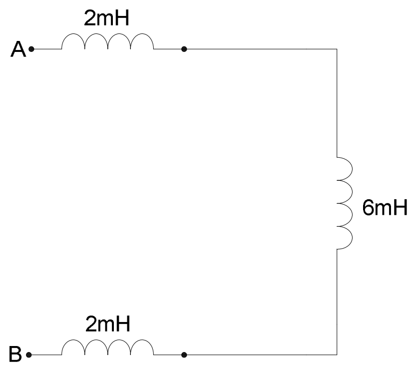
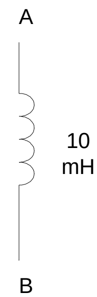
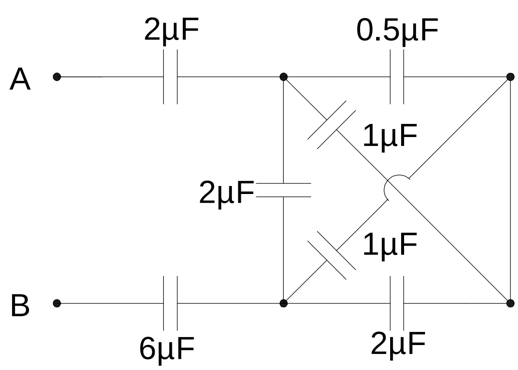


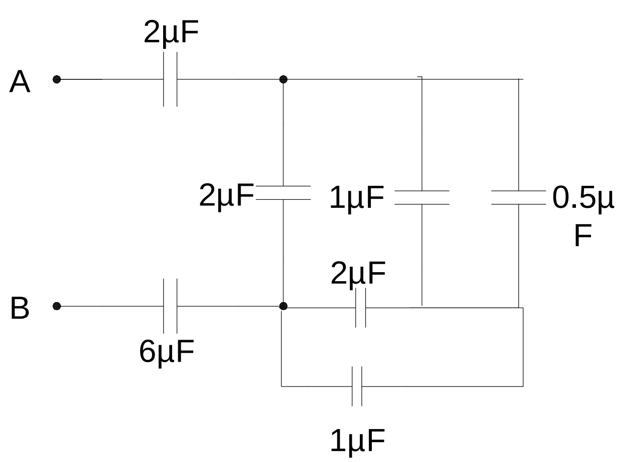
⇒ 
2mH
A
6mH
2mH
B

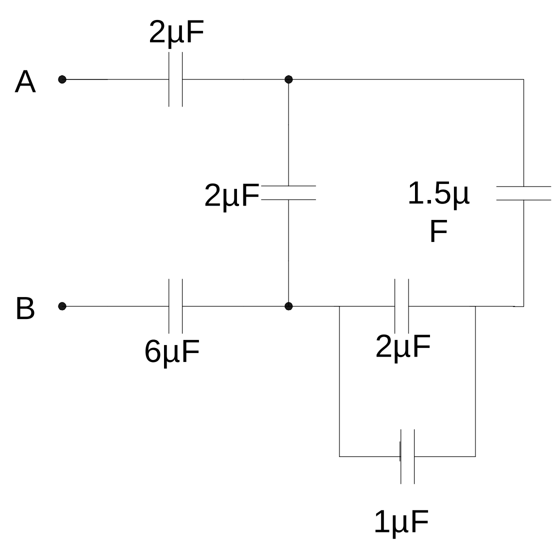
⇒ 
A
10
mH
B

6.10

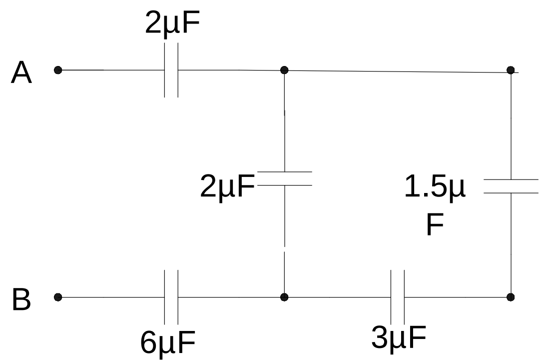

2μF
0.5μF
A
1μF
2μF
1μF
B
6μF
2μF

⇒

⇒ 
2μF
A
2μF
1μF
0.5μ
F
2μF
B
6μF
1μF

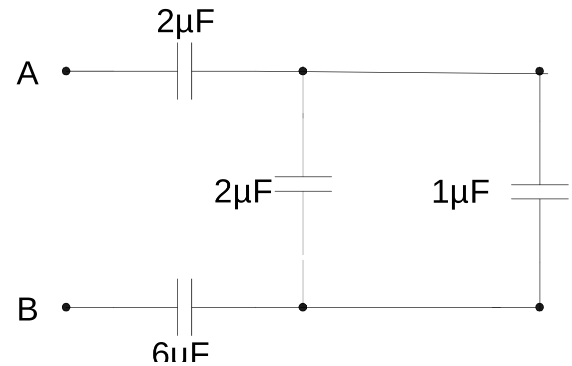
⇒ 
2μF
A
2μF
1.5μ
F
B
6μF
2μF
1μF

⇒



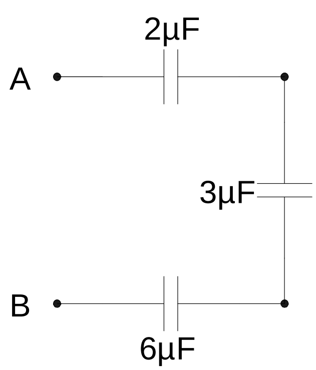
2μF
A
2μF
1.5μ
F
B
6μF
3μF

⇒



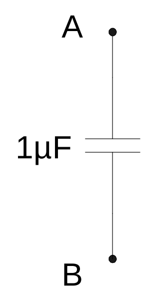
2μF
A
2μF
1μF
B

⇒



2μF
A
3μF
B
6μF

⇒



A
1μF
B