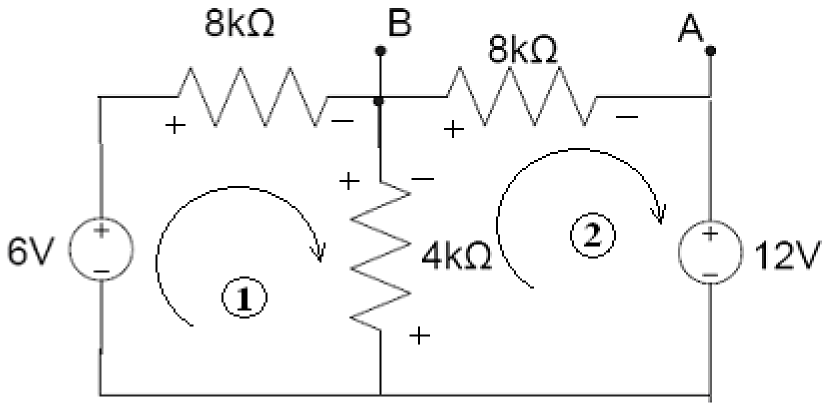
Solution Set 5 (Fall 2008)

5.1 Use Thevenin's theorem using KVL around 2 loops to find V_{ab} in the following network.

Solution:



To find let's consider V_{ab}

Loop 1: $-6V + I_1 * 8k\Omega + (I_1 - I_2) * 4k\Omega = 0$, (1)

Loop 2: $I_2 * 8k\Omega + (I_2 - I_1) * 4k\Omega + 12V = 0$, (2)

(1): $I_1 * 12k\Omega - I_2 * 4k\Omega = 6V$, $I_1 = \frac{1}{12k\Omega}(I_2 * 4k\Omega + 6V)$.

(2): $I_2 * 12k\Omega - I_1 * 4k\Omega = -12V$

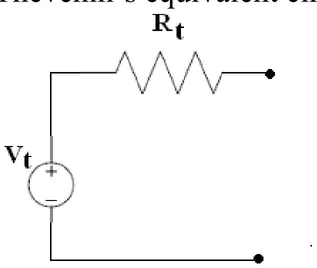
By substituting (1) into (2):

$I_2 * 12k\Omega - \frac{4k\Omega}{12k\Omega}(I_2 * 4k\Omega - 6V) = -12V$,

$I_2 = -15/16mA$;

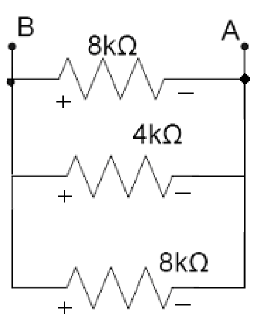
$V_{AB} = -I_2 * 8k\Omega = 7.5V$;

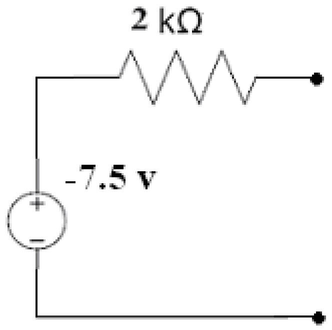
Thevenin's equivalent circuit can be presented as:



To find R_t set all voltage sources to be equal zero (short circuit) and all current sources to be equal zero (open circuit):

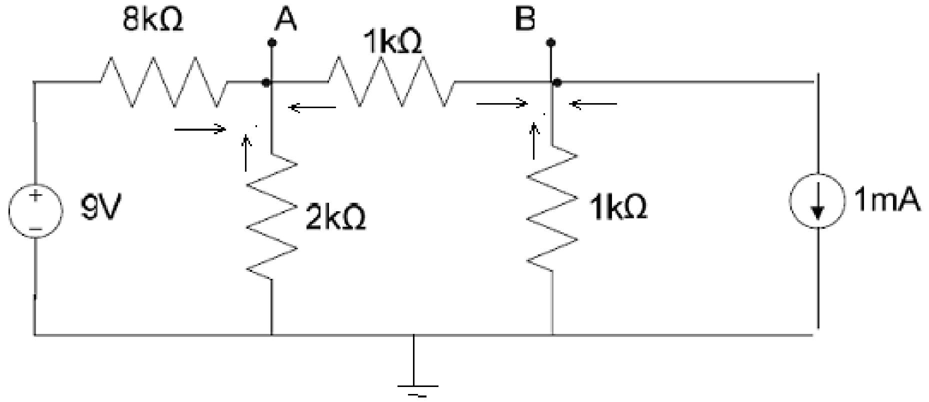
$R_t = 8k\Omega \| 4k\Omega \| 8k\Omega = 2k\Omega$;





So the Thevenin's circuit is

5.2 Use Thevenin's theorem using KCL at nodes A and B to find V_{ab} in the following network. (check the solution at node B)))



Solution:

KCL at the node A: $\frac{9V - V_A}{8k\Omega} + \frac{0 - V_A}{2k\Omega} + \frac{V_B - V_A}{1k\Omega} = 0$, (1)

KCL at the node B: $-1mA + \frac{0 - V_B}{1k\Omega} + \frac{V_A - V_B}{1k\Omega} = 0$, (2)

Let's express V_a from (1): $V_A = 2V_B + 1V$,

And substitute it into (2):

$9V - V_A + -V_A * 4k\Omega + V_B * 8k\Omega - V_A * 8k\Omega = 0$,

$8V_B = 13V_A - 9V$,

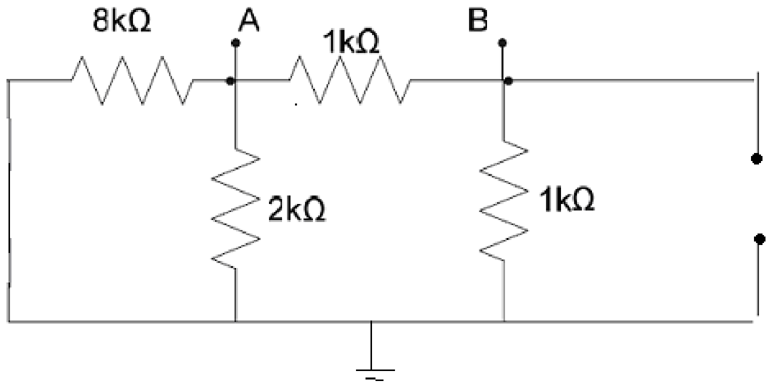
$8V_B = 13(2V_B + 1V) - 9V$

$V_B = -4/18V$,

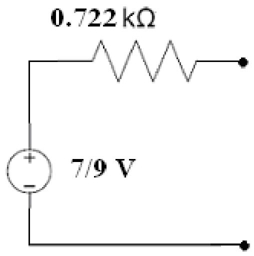
Hence $VA = 5/9V$;

$V_{AB} = V_A - V_B = 5/9V + 4/18V = 7/9V$

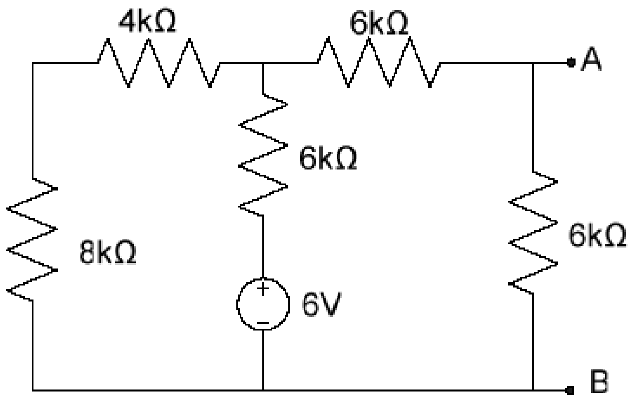
To find R_t set all voltage sources to be equal zero (short circuit) and all current sources to be equal zero (open circuit):



So $R_T = 1k\Omega \| (1k\Omega + 2k\Omega \| 8k\Omega) = 0.722k\Omega$;
The Thevenin's equivalent is:



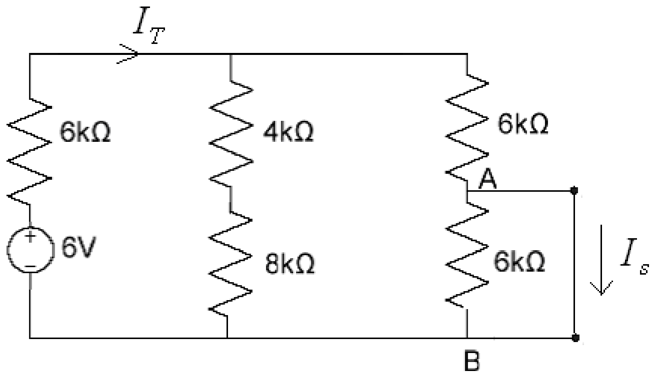
5.3 Find the Norton equivalent source for each of the following circuits



Solution:

1. Find I_S

Redraw circuit:



(The $6k\Omega$ resistor shorted out)

$$I_T = \frac{6v}{R_T} = \frac{6v}{6k\Omega + (4k\Omega + 8k\Omega) \parallel 6k\Omega}$$

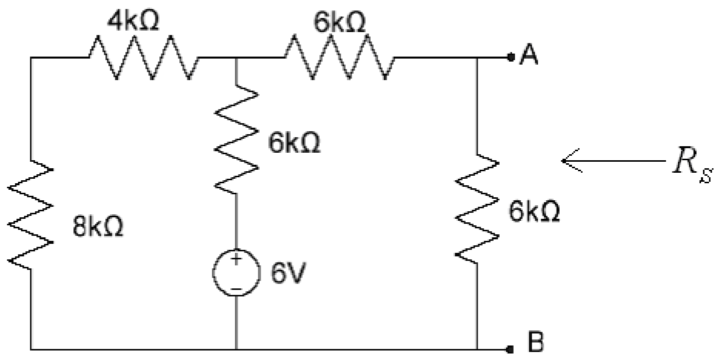
$$= \frac{6v}{6k\Omega + 4k\Omega} = 0.6mA$$

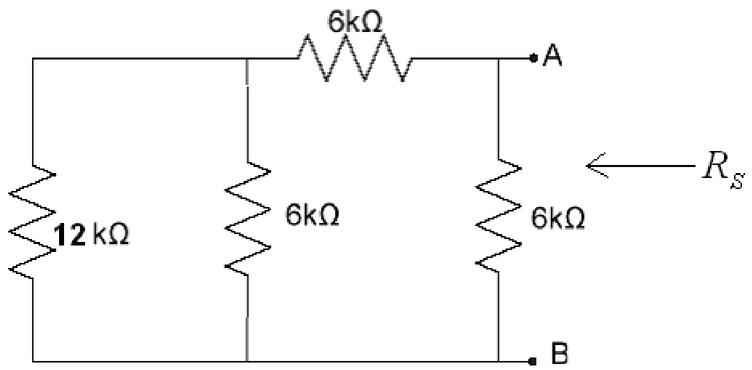
$$I_S = I_T \cdot \frac{4k\Omega + 8k\Omega}{(4k\Omega + 8k\Omega) + 6k\Omega}$$ (Current divider)

$$= 0.6mA \cdot \frac{12k\Omega}{18k\Omega} = 0.4mA$$

2. Find R_S, set Vsource = 0

Redraw circuit:



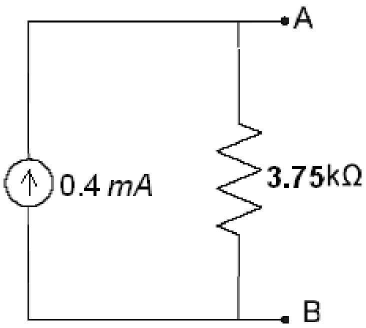


$$R_S = 6k\Omega \parallel [(12k\Omega \parallel 6k\Omega) + 6k]$$

$$= 6k\Omega \parallel 10k\Omega$$

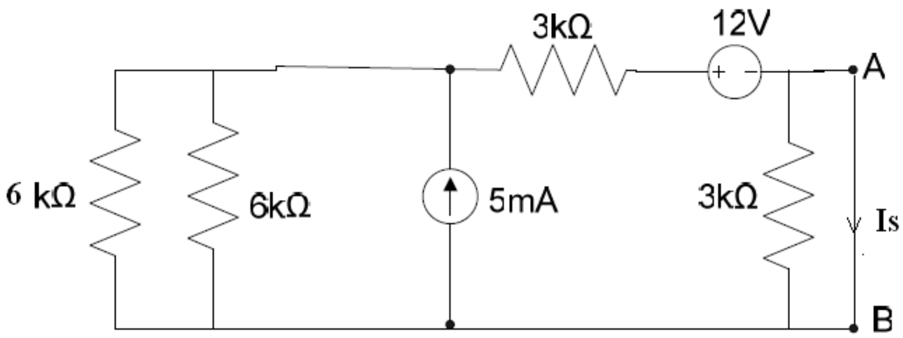
$$= 3.75k\Omega$$

3. Draw the Norton equivalent source:

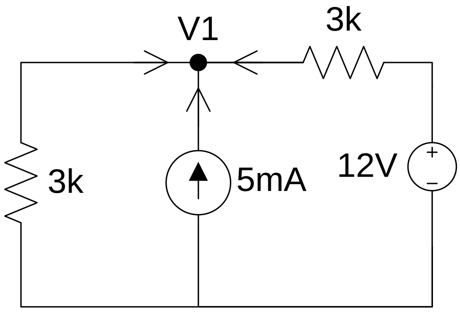


5.4 Find Norton equivalent source for the following circuit.

Solution:
1. The original circuit can be redrawn as:



To find I_s let's apply KCL at the node V1:

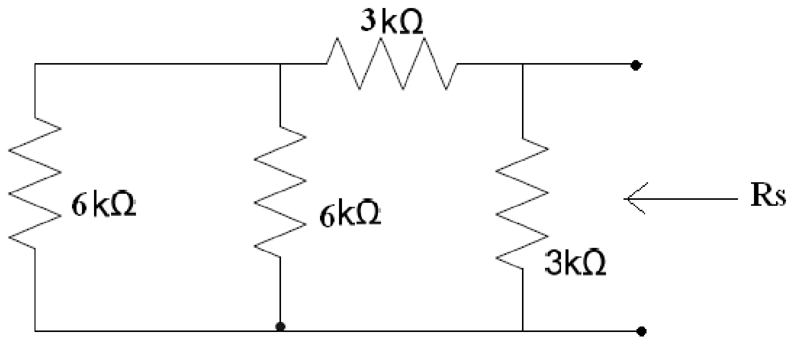


$$\frac{0-V1}{3k\Omega}+5mA+\frac{12V-V1}{3k\Omega}=0,$$

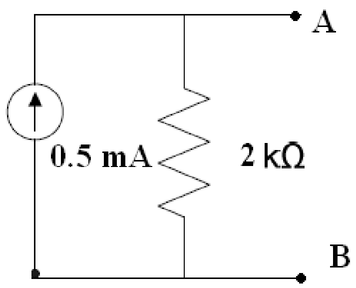
$$V1=13.5V$$

$$Is=\frac{V1-12V}{3k\Omega}=0.5mA,$$

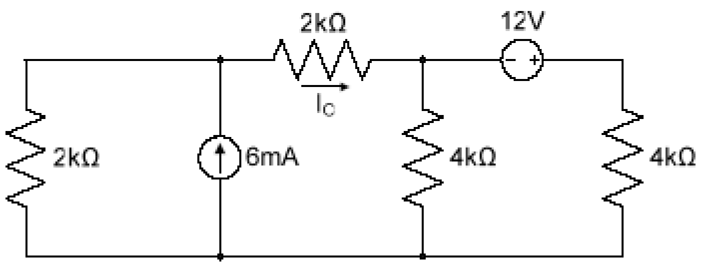
2. Find Rs (set all voltage sources to be equal zero (short circuit) and all current sources to be equal zero (open circuit):



$Rs = 3k\Omega \| [3k\Omega + 6k\Omega \| 6k\Omega] = 2k\Omega$;
Hence the Norton equivalent source is:

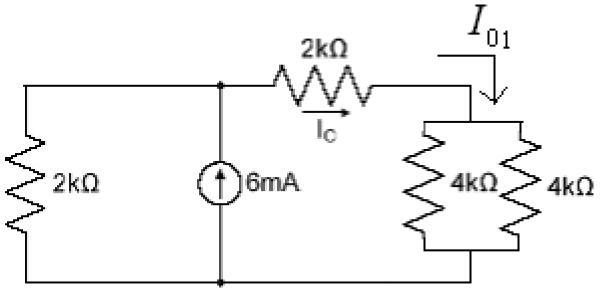


5.5 Find I_0 in the following network using superposition.



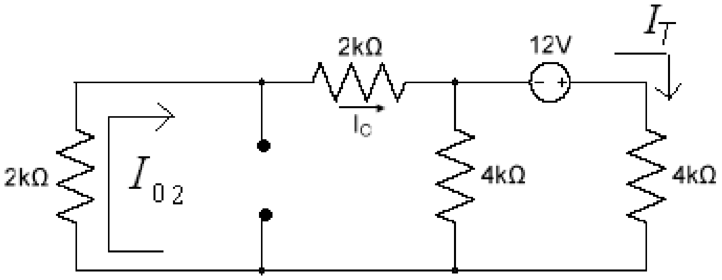
Solution:

Consider the current source (voltage source = short)
Redraw without the voltage source



$$I_{01} = 6mA \cdot \frac{2k\Omega}{2k\Omega + (2k\Omega + 4k\Omega \parallel 4k\Omega)}$$
$$= 6mA \cdot \frac{2k\Omega}{2k\Omega + 2k\Omega + 2k\Omega}$$
$$= 6mA \cdot \frac{2k\Omega}{6k\Omega}$$
$$= 2mA$$

Consider the voltage source (Current source = open)
Redraw without the current source.

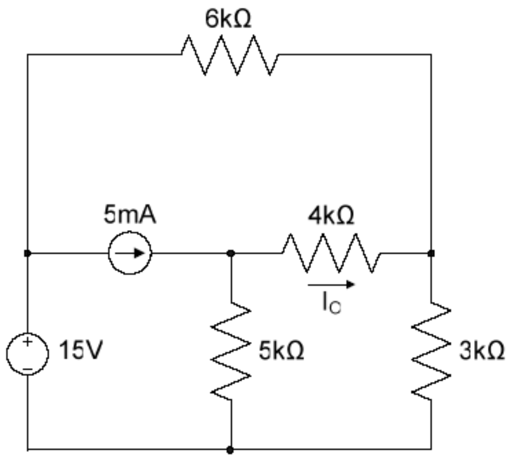


$$I_T = \frac{12v}{4k\Omega + (4k\Omega \parallel (2k\Omega + 2k\Omega))}$$
$$= \frac{12v}{4k\Omega + 2k\Omega}$$
$$= 2mA$$

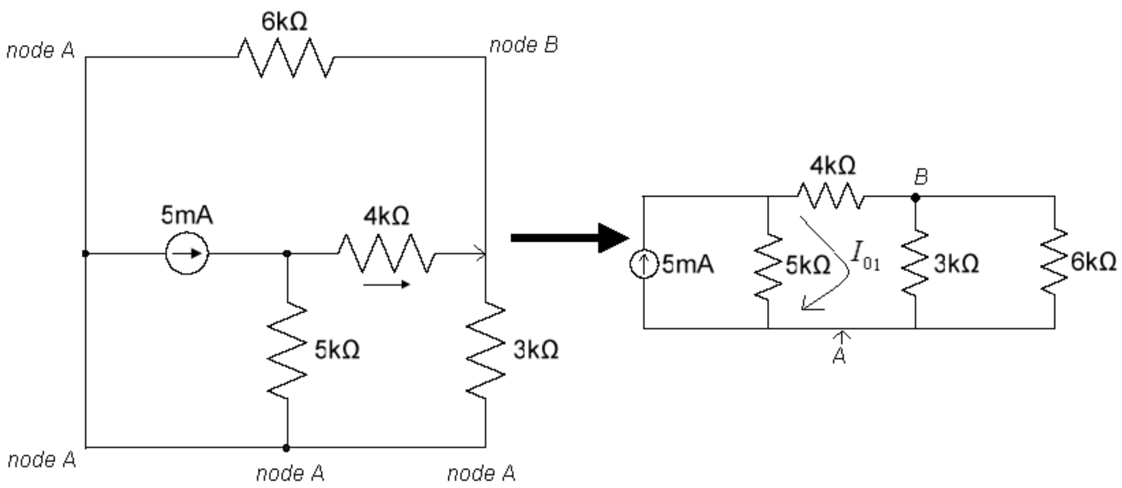
$$I_{02} = 2mA \cdot \frac{4k\Omega}{4k\Omega + (2k\Omega + 2k\Omega)}$$
$$= 2mA \cdot \frac{4k\Omega}{8k\Omega}$$
$$= 1mA$$

$$I_0 = I_{01} + I_{02}$$
$$= 2mA + 1mA$$
$$= 3mA$$

5.6 Find I_0 in the following network using superposition (Hint: At some point: $3k\Omega \parallel 6k\Omega$)



Solution:
Consider the current source (Voltage source = short)
Redraw without the voltage source

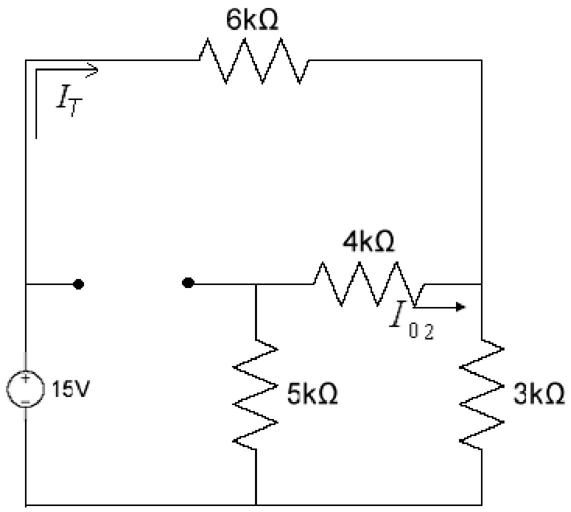


$$I_{01} = 5mA\frac{5k\Omega}{5k\Omega + (4k\Omega + 3k\Omega \| 6k\Omega)}$$

$$I_{01} = 5mA\frac{5k\Omega}{5k\Omega + 4k\Omega + 2k\Omega}$$

$$I_{01} = 5mA * 0.455 = 2.275mA$$

Consider the voltage source (current source is open):



Note: I_{02} points in the opposite direction than I_T

$$I_T = \frac{15V}{6k\Omega + (3k\Omega \parallel (4k\Omega + 5k\Omega))} = 1.82mA$$

$$I_{02} = -1.82mA * \frac{3k\Omega}{3k\Omega + (4k\Omega + 5k\Omega)} = -0.455mA$$

$$I_0 = I_{01} + I_{02} = 2.275 + (-0.455mA) = 1.82mA$$