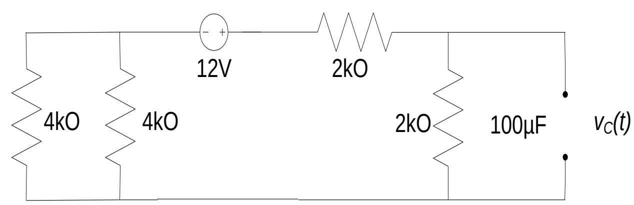
Solution Set 10 (Fall 2011)

Use our step-by-step approach (Irwin pp. 202-204) (do not use diff'l eq'ns) for all of the following:

<u>RC Circuits:</u>

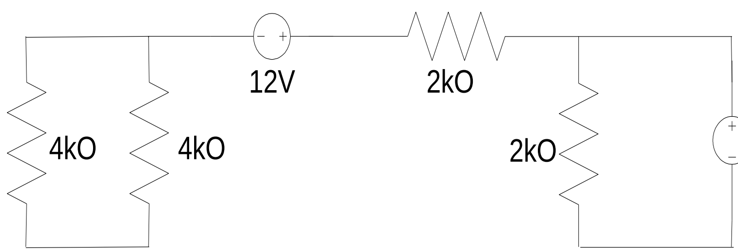
10.1



Step1 : $V_c(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

Step2 : $t = 0^-$

Using voltage divider, $V_c(t = 0^-) = \frac{12v}{6k\Omega} \cdot 2\ k\Omega = 4\ v$



$4v = v_c(t=0^+)$

Step3 : $t = 0^+$

$V_c(t = 0^+) = V_c(t = 0^-) = 4\ v$

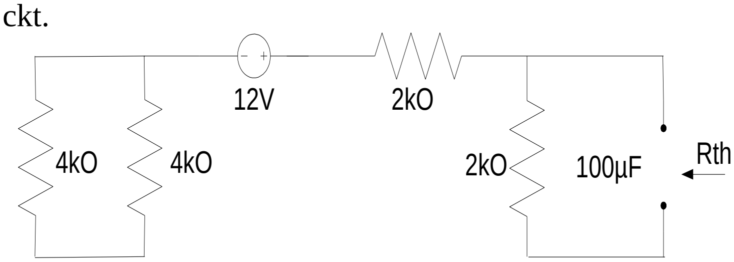
Step4: $t \to \infty$

Capacitor discharges through 2K resistance, $V_c(t \to \infty) = 0\ v$

Step5 : $@t \to \infty$ capacitor becomes open ckt.

R_{th}=2k

$\tau = R_{th}.C = 2k.100 \times 16^{-6} = 0.2s$

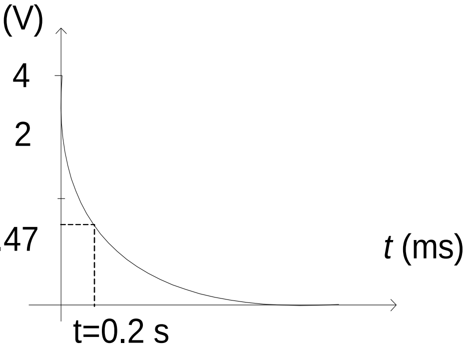


Step6 $v_c(t = 0^+) = K_1 + K_2 = 4$

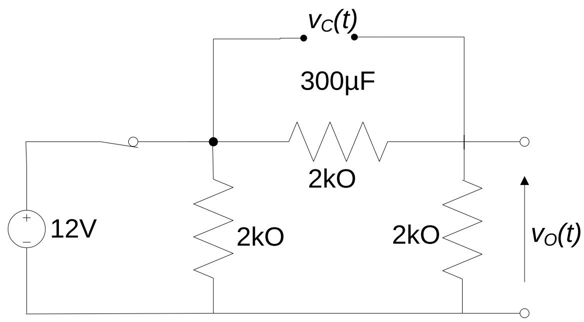
$v_c(t \to \infty) = K_1 = 0$

This gives $K_1=0$; $K_2=4$

$v_c(t) = 4e^{\frac{-t}{0.2}}V$



10.2

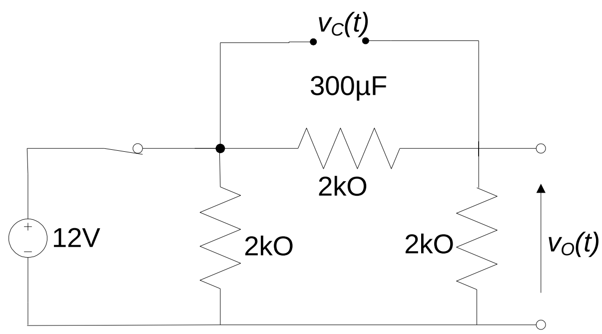


Step1 : $V_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

Step2 : $t = 0^-$

Using voltage divider, $V_c(t = 0^-) = \dfrac{12v}{2k + 2k\Omega} \cdot 2\ k\Omega = 6$ v

Step3 : $t = 0^+$

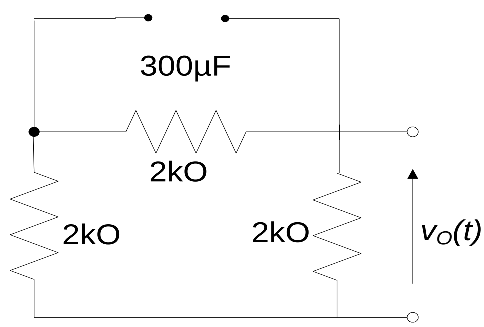


$V_c(t = 0^+) = V_c(t = 0^-) = 6$ v

$V_0(t = 0^+) = \dfrac{-V_c(t = 0^+)}{2k + 2k\Omega} \cdot 2\ k\Omega = -3$ v

Step4: $t \to \infty$

capacitor becomes open ckt, $V_c(t\to\infty) = 0$ v



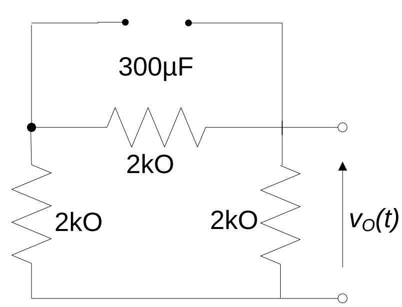
Step5 : $R_{th}=2k//(2k+2k)=\frac{4}{3}k\Omega$

$\tau = R_{th}.C = \frac{4}{3}k\Omega.300\mu = 0.4s$

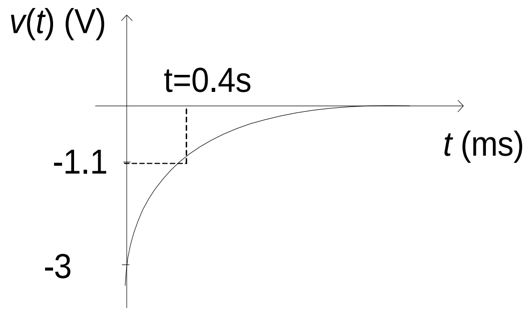
Step6 $v_c(t = 0^+) = K_1 + K_2 = -3$

$v_c(t \to \infty) = K_1 = 0$

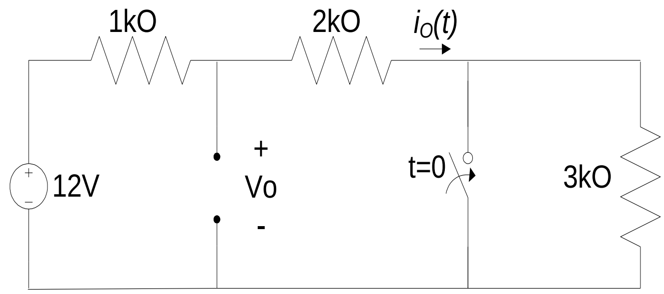
This gives $K_1=0$; $K_2=-3$



$v_c(t) = -3e^{\frac{-t}{0.4}}V$



10.3



Step1 : $i_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

Step2 : $t = 0^-$

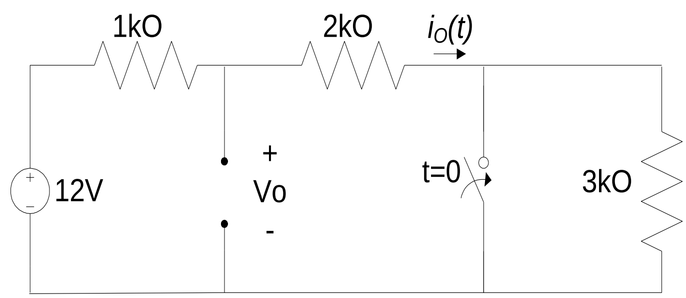
$$i_0(t = 0^-) = \frac{12v}{6k\Omega} = 2 \text{ mA}$$

Voltage across the capacitor , $V_c(t = 0^-) = 2\text{mA } 5 \text{ k}\Omega = 10 \text{ v}$

Step3 : $t = 0^+$

The 3K resistor is shorted

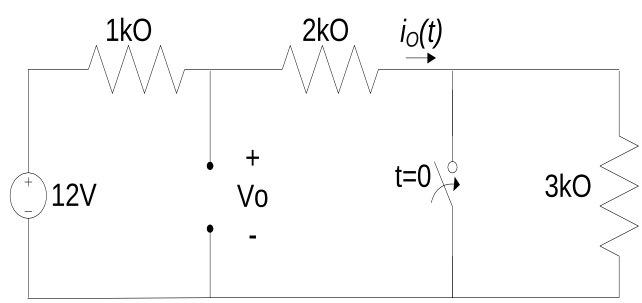
$$i_0(t = 0^+) = \frac{10v}{2k\Omega} = 5 \text{ mA}$$



Step4: $t \to \infty$

Capacitor becomes open circuit, $i_0(t \to \infty) = \frac{12v}{1k + 2k} = 4 \text{ mA}$

Step5 : .



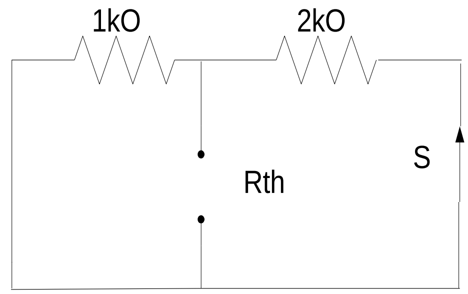
$$R_{th} = 1k//2k = \frac{2}{3} k\Omega$$

$$\tau = R_{th}.C = \frac{2}{3} k\Omega.400\mu = \frac{0.8}{3} = 0.2667s$$

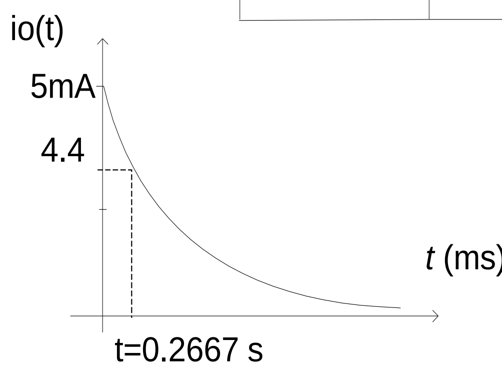
Step6 $i_0(t = 0^+) = K_1 + K_2 = 5mA$

$i_0(t \to \infty) = K_1 = 4mA$

This gives K_1=4mA; K_2=1mA



$$i_0(t) = 4 + e^{\frac{-t}{0.2667}} mA$$



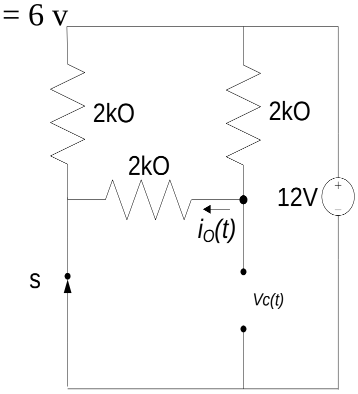
10.4

Step1 : $i_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

Step2 : $t = 0^-$

Using voltage divider, $V_c(t = 0^-) = \frac{12v}{2k\Omega + 2k\Omega} \cdot 2\,k\Omega = 6$ v

$i_0(t = 0^-) = \frac{6v}{2k\Omega} = 3$ mA



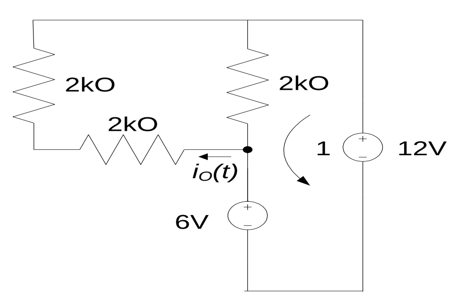
Step3 : $t = 0^+$

$V_c(t = 0^+) = V_c(t = 0^-) = 6$ v

KVL for loop 1: V_1+6V-12V=0

V_1=6V

$i_0(t = 0^+) = \frac{-6v}{4k\Omega} = -1.5$ mA



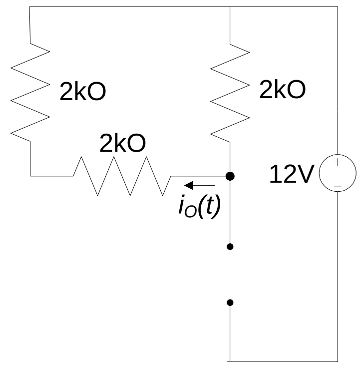
Step4: $t \to \infty$

$i_0(t \to \infty) = 0$

Step5 : .

R_{th}=2k//(2k+2k)=$\frac{4}{3}k\Omega$

$$\tau = R_{th}.C = \frac{4}{3}k\Omega.100\mu = \frac{0.4}{3}$$

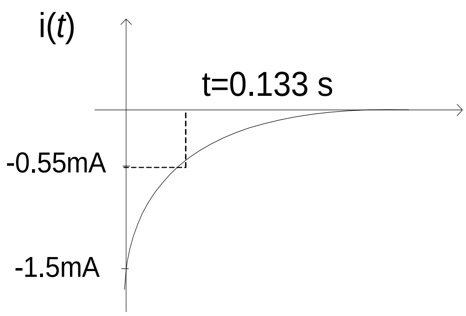


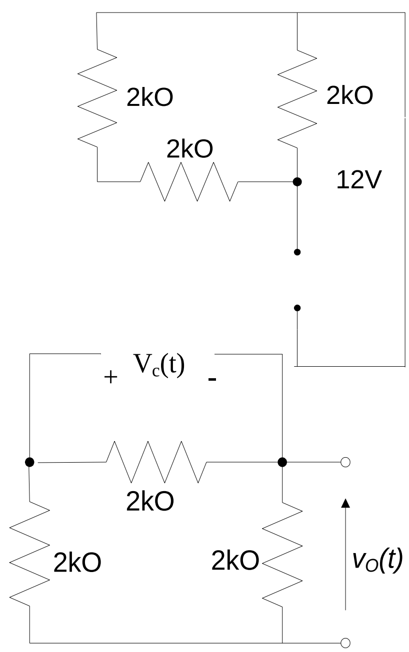
Step6 $i_0(t = 0^+) = K_1 + K_2 = -1.5mA$

$i_0(t \to \infty) = K_1 = 0$

This gives K_1=0mA; K_2=-1.50mA

$$i_0(t) = -1.5e^{\frac{-t}{0.133}}mA$$





10.5

@t=0.5s , switch closes again

Step1 : $V_0(t) = K_1 + K_2 e^{\frac{-(t-0,5)}{\tau}}$

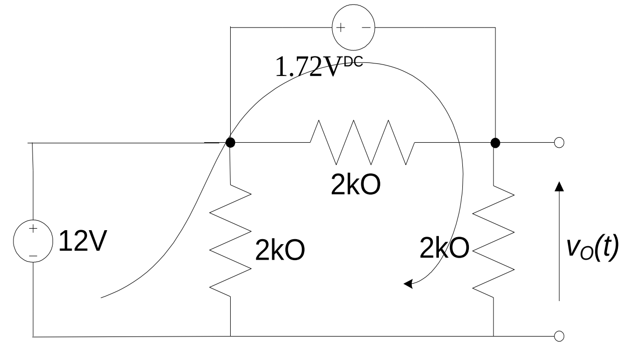
Step2 : $t = 0.5^-$

$V_0(t = 0.5^-) = -3e^{\frac{-(0,5)}{0.4}} = -0.86$v

$V_c(t = 0.5^-) = -2\ V_0(t = 0.5^-) = 1.72$ v

Step3 : $t = 0.5^+$

$V_c(t = 0.5^+) = V_c(t = 0{,}5^-) = 1.72$ v
KVL for outside loop : $-12 + 1.72 + V_0 = 0$
$V_0(t = 0.5^+) = 10.28$ v

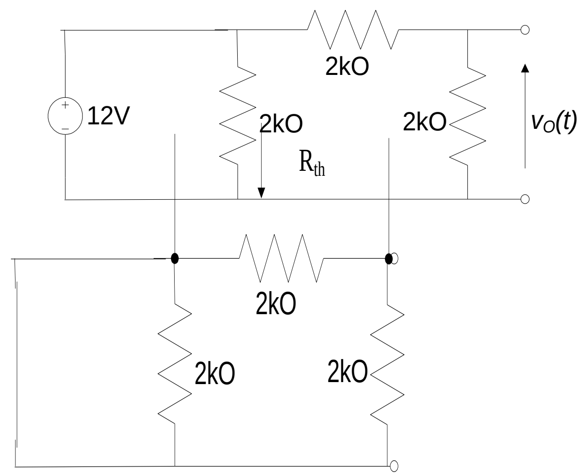


Step4: $t \rightarrow \infty$

$$V_o(t\rightarrow\infty) = 12\text{v}.\frac{2k}{2k+2k} = 6\text{v}$$

Step5 :

R_{th}=2k//2k=1k
$\tau = R_{th}.C = 1k.300\mu = 0.3s$



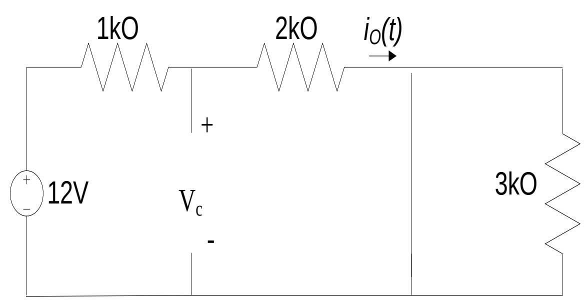
Step6: $V_0(t = 0.5^+) = K_1 + K_2 = 10.28v$
$v_0(t \rightarrow \infty) = K_1 = 6$

This gives K_1=6; K_2=4.28

$$v_c(t) = \begin{Bmatrix} -3e^{\frac{-t}{0.4}}V(0 < t < 0.5) \\ 6 + 4.28e^{\frac{-(t-0.5)}{0.3}}(t > 0.5s) \end{Bmatrix}$$

10.6

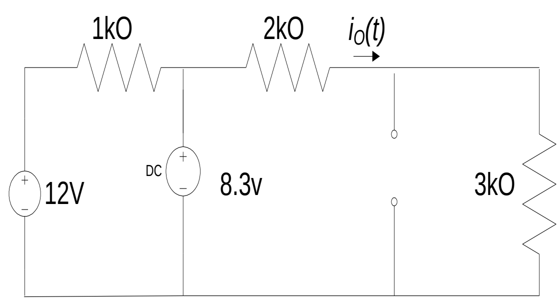
@t=0.5s , switch closes again



Step1 : $i_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

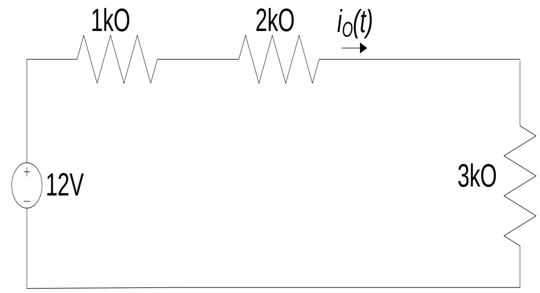
Step2 : $t = 0.5^-$

$i_0(t = 0.5^-) = 4 + e^{\frac{-0.5)}{0.267}} = 4.15$ mA
$V_c(t = 0.5^-) = i_0\ .2k = 8.3$v



Step3 : $t = 0.5^+$
$V_c(t = 0.5^+) = V_c(t = 0.5^-) = 8.3$ v

i_0 (t = 0.5^+) = $\frac{8.3v}{5k\Omega}$ = 1.66 mA



Step4: $t \to \infty$

Capacitor becomes open circuit, i_0 ($t \to \infty$) = $\frac{12v}{6k}$ = 2 mA

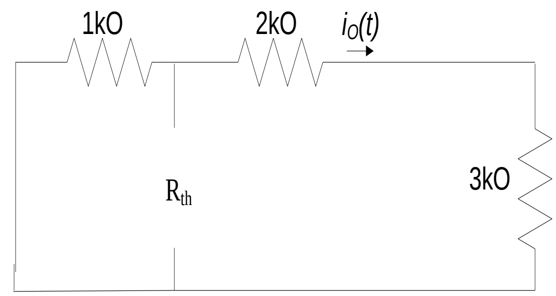
Step5 : .

R_{th}=1k//5k=0.83kΩ

$\tau = R_{th}.C = 0.83k\Omega.400\mu = 0.33s$

Step6 $i_0(t = 0^+) = K_1 + K_2 = 1.66mA$

$i_0(t \to \infty) = K_1 = 2mA$



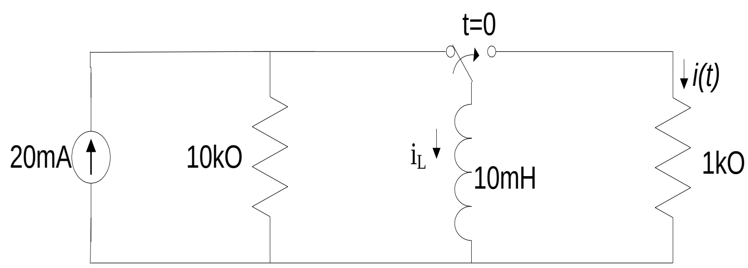
This gives K_1=2mA; K_2=-0.34mA

$$i_o(t) = \begin{cases} 4 + e^{\frac{-t}{0.2667}} mA(0 < t < 0.5) \\ 2 - 0.34e^{\frac{-(t-0.5)}{0.33}} mA(t > 0.5s) \end{cases}$$

<u>RL Circuits:</u>

10.7

Step1 : $i_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$



Step2 : $t = 0^-$

i_0 ($t = 0^-$) = 0 mA

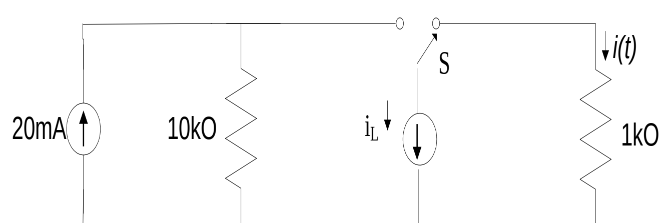
At dc inductor is short ckt.(Z_L=jωL=0)

Hence i_L ($t = 0^-$) = 20 mA

Step3 : $t = 0^+$

i_L ($t = 0^-$) = i_L ($t = 0^+$) = 20 mA

i_0 ($t = 0^+$) = -20 mA



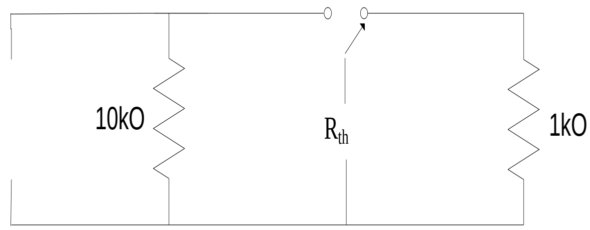
Step4: $t \to \infty$;inductor is short ckt

$i_0\ (t \to \infty) = 0$

Step5 : .

$R_{th} = 1k\Omega$

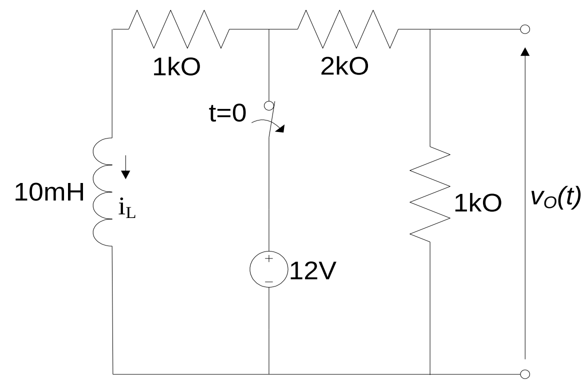
$$\tau = \frac{L}{R_{th}} = \frac{10mH}{1k} = 10\mu s$$



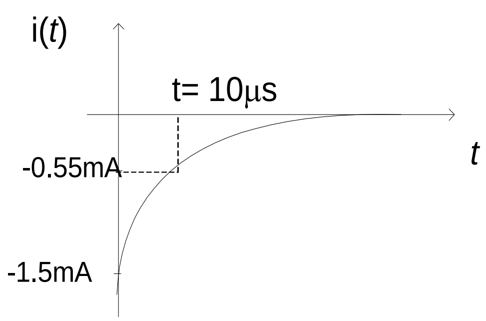
Step6 $i_0(t = 0^+) = K_1 + K_2 = -20mA$

$i_0(t \to \infty) = K_1 = 0$

This gives K_1=0mA; K_2=-20mA



$$i_0(t) = -20e^{\frac{-t}{10\mu}} mA$$



10.8

Step1 : $V_c(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

Step2 : $t = 0^-$

At dc inductor is short ckt., $Z_L = j\omega L = 0$

$$i_L\ (t = 0^-) = \frac{12v}{1k\Omega} = 12mA$$

Step3 : $t = 0^+$

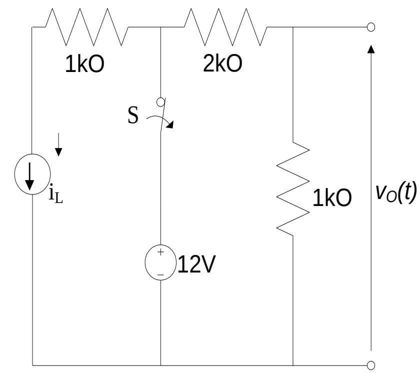
$i_L (t = 0^-) = i_L (t = 0^+) = 12$ mA

$V_0(t = 0^+)$ =-12mA.1k= -12 v

Step4: $t \rightarrow \infty$

$i_L (t \rightarrow \infty)$=0A

$V_0(t \rightarrow \infty)$=0v



Step5 :

R_{th}=4kΩ

$$\tau = \frac{L}{R_{th}} = \frac{10mH}{4k} = 2.5\mu s$$

Step6 $v_c(t = 0^+) = K_1 + K_2 = -12$

$v_c(t \rightarrow \infty) = K_1 = 0$

This gives K_1=0; K_2-12v

$$v_c(t) = -12e^{\frac{-t}{2.5\mu}}V$$