Instructor's Name (**Print**)

Student's Name (**Print**)

Student's signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF APPLIED MATHEMATICS

Calculus 1000B Final Examination

Code 333

Thursday, April 21, 2011 2:00 p.m. - 5:00 p.m.

INSTRUCTIONS

1. The first part of the exam (Part A) is MULTIPLE CHOICE. This part is to be answered on the SCANTRON answer sheet. As well, make sure that you circle your selected answer in the examination booklet. The answer you give on the scantron sheet is taken as being your intended choice in the event that the answer in the examination booklet is not the same. The second part (Part B) has questions to be answered in the space provided. Be sure to answer each part of a question in the space provided for that part of the question. INDICATE YOUR ANSWER CLEARLY.
2. SHOW ALL OF YOUR WORK FOR PROBLEMS IN PART B. All results must be justified unless you are instructed otherwise. Unjustified answers will receive little or no credit.
3. Print your name and your instructor's name on the SCANTRON answer sheet. Sign the answer sheet, and mark your student number, section and exam code (111, 222, or 333) on the answer sheet. USE AN HB PENCIL and mark your answers on the SCANTRON answer sheet. Use the (scantron) answer sheet for Part A.
4. DO NOT UNSTAPLE THE BOOKLET.
5. Questions start on Page 3 and continue to Page 21. There are 25 questions in Part A and 10 questions in Part B. Questions are printed on both sides of the paper. Should you require extra space for any answer, Pages 22-23 are provided for this purpose. BE SURE YOUR BOOKLET IS COMPLETE.
6. CALCULATORS ARE NOT PERMITTED.
7. Leave your scantron sheet on top of your examination booklet when turning them in.
8. TOTAL MARKS = 100.
9. Clearly fill in the top of this page.

   001 Kiriushcheva (main campus)

Student Number

Student's Name (Print)

FOR GRADING ONLY

pages	mark
PART A 3-11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
PART B 12-21	
Total	

Part A (50 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET. FOR SAFETY, ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

(2 marks) A1. Evaluate $\sin\left(2\cos^{-1}\left(\frac{1}{5}\right)\right)$

A: $\frac{2}{5}$	B: $\frac{\sqrt{24}}{5}$	C: $\frac{2}{25}$	D: $\frac{1}{25}$	E: $\frac{4\sqrt{6}}{25}$

(2 marks) A2. Determine $\lim_{x\to 0}\frac{3x-\sin 4x}{4x+\sin 3x}$.

A: $\frac{3}{4}$	B: $-\frac{1}{7}$	C: does not exist	D: -1	E: $-\frac{4}{3}$

(2 marks) A3. Determine $\lim_{x\to\infty}\frac{3x^3}{e^{3x}}$.

A: ∞	B: $\frac{3}{e^3}$	C: 1	D: 0	E: does not exist

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(2 marks) A4. Determine $\lim_{x\to -1} \dfrac{1-|x|}{x+1}$.

A: 0	B: 1	C: -1	D: $-\infty$	E: does not exist

(2 marks) A5. Evaluate $\displaystyle\sum_{k=5}^{49} \left(\frac{1}{k+1} - \frac{1}{k}\right)$.

A: $-\frac{9}{50}$	B: $\frac{49}{50}$	C: $\frac{51}{50}$	D: $\frac{9}{50}$	E: $\frac{1}{50}$

(2 marks) A6. If $x\sin y + y\sin x = 0$, use implicit differentiation to find y' at the point $\left(\frac{\pi}{2}, 0\right)$.

A: $-\infty$	B: $\frac{\pi}{2}$	C: 0	D: 1	E: $-\frac{\pi}{2}$

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(2 marks) A7. If $y = (\sin x)^x$, find $\dfrac{dy}{dx}$.

A: $x(\sin x)^{x-1}$	B: $e^{x\ln(\sin x)}$	C: $(\sin x)^x(\ln(\sin x) + x\cot x)$	D: $\ln(\sin x) + x\cot x$
E: $(\sin x)^x\left(\dfrac{\sin x}{x} + \cos x \ln x\right)$			

(2 marks) A8. Which of the following integrals equals to

$$\lim_{n\to\infty} \frac{\pi}{2n} \sum_{i=1}^{n} \ln\left(\frac{\pi}{4} + \left(\frac{i\pi}{2n}\right)^2\right) \ ?$$

A: $\displaystyle\int_0^{\pi/2} \ln(x^2)\,dx$	B: $\displaystyle 2\int_0^{\pi/4} \ln x\,dx$	C: $\displaystyle\int_{\pi/4}^{3\pi/4} \ln\left(\frac{\pi}{4} + x^2\right)dx$
D: $\displaystyle\int_0^{\pi/2} \ln\left(\frac{\pi}{4} + x^2\right)dx$	E: $\displaystyle\int_0^{\pi/2} \ln\left(\frac{\pi}{4} + x\right)dx$	

(2 marks) A9. Evaluate $\lim_{x\to 0} \dfrac{1}{x}\displaystyle\int_0^x e^{t^2}\,dt$.

A: 2	B: ∞	C: does not exist	D: 1	E: 0

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(2 marks) A10. Find the second derivative of the function $f(x) = xe^{-x}$.

A: $e^{-x}(2x-1)$	B: xe^{-x}	C: $2e^{-x}(x-2)$	D: $-e^{-x}(x-2)$	E: $e^{-x}(x-2)$

(2 marks) A11. A particle is moving with the acceleration $a(t) = \sin t + \cos t$. Find the position $s(t)$ (in meters) of a particle at any time if the initial position and velocity are $s(0) = 0\ (m)$, $v(0) = 5\ (m/s)$.

(Recall: $v(t) = \dfrac{ds}{dt}$, $a(t) = \dfrac{d^2s}{dt^2}$.)

A: $\sin(5) + \cos(5)$	B: $-\cos t - \sin t + 5$	C: $-\cos t - \sin t + 6t + 1$
D: $-\cos t - \sin t + 1$	E: $-\cos t - \sin t + t + 5$	

(2 marks) A12. Evaluate $\dfrac{d}{dx}\displaystyle\int_0^{\ln 3} e^{2x}dx$.

A: 0	B: $\frac{1}{2}\left(2^{\ln 3} - 1\right)$	C: 4	D: $2e^{2x} + C$	E: $\frac{1}{2}e^{2x} + C$

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(2 marks) A13. Evaluate $\int \cos x \cdot \cos(\sin x)\ dx$.

A: $\cos(\sin x) + C$	B: $\sin(\cos x) + C$	C: $-\cos(\cos x) + C$
D: $\sin(\sin x) + C$	E: $\sin x \cdot \sin(\cos x) + C$	

(2 marks) A14. Evaluate $\int_0^{\pi/4} \sqrt{\tan^2 x - \sec^2 x + 1}\, dx$.

A: 1	B: 0	C: $2\sqrt{2}$	D: does not exist	E: $\frac{\pi}{4}$

(2 marks) A15. If $\int_1^4 f(x)\, dx = 3$, $\int_0^4 g(x)\, dx = 10$, $\int_0^4 [f(x) - g(x)]\, dx = 13$, find $\int_0^1 f(x)\, dx$.

A: 10	B: -6	C: 20	D: 26	E: 0

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(2 marks) A16. Evaluate $\sum_{i=0}^{3} \frac{i\,(i+1)}{2}$.

A: 10	B: 150	C: 6	D: 20	E: 3

(2 marks) A17. Evaluate $\int_1^e \frac{(\ln x)^2}{x} dx$.

A: $\frac{1}{e}$	B: $\frac{1}{e} - 1$	C: $-\frac{2}{3}$	D: $\frac{e}{3}$	E: $\frac{1}{3}$

(2 marks) A18. Find the equation of the tangent line to the graph $y = \frac{1}{\cos x - \sin x}$ at the point $(0, 1)$.

A: $y = -x + 1$	B: $y = -x - 1$	C: $y = x + 1$	D: $y = x$	E: $y = x - 1$

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(2 marks) A19. Determine $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x}$.

A: 1	B: $\frac{1}{2}$	C: 0	D: 2	E: does not exist

(2 marks) A20. If $y = \sqrt{\dfrac{1+x}{1-x}}$, find $\dfrac{dy}{dx}$.

A: $\dfrac{2\sqrt{1-x^2}}{(1-x)^{3/2}}$	B: $\dfrac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$	C: $\sqrt{\dfrac{1-x}{1+x}}$
D: $\dfrac{1}{\sqrt{1-x}\,(1+x)^{3/2}}$	E: $\dfrac{2}{(1-x)^{3/2}}$	

(2 marks) A21. Evaluate $\displaystyle\int_0^{\pi/4} e^{\tan x}\sec^2 x\, dx$.

A: $e^{\pi/4}$	B: $e^{\pi/4-1}$	C: 0	D: $e-1$	E: $e^{\pi/4}-1$

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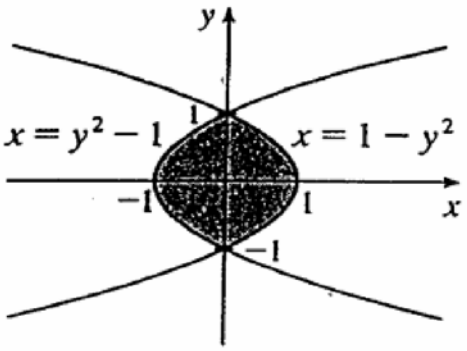
(2 marks) A22. Evaluate $\int_1^9 \frac{1+\sqrt{x}}{\sqrt{x}}\,dx$.

A: 12	B: 8	C: 0	D: 2	E: 6

(2 marks) A23. Choose the integral which represents the area of the shaded region.



A: $\int_{-1}^{1} \left[\sqrt{1-y^2} - \sqrt{y^2-1}\right] dy$
B: $\int_{-1}^{1} [(1-y^2) - (y^2-1)]\,dy$
C: $\int_{-1}^{1} \left[\sqrt{1+x} - \sqrt{1-x}\right] dx$
D: $\int_{-1}^{1} \left[(1+x)^2 - (1-x)^2\right] dx$
E: $\int_{-1}^{1} [(1+x) - (1-x)]\,dx$

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In problems 24 and 25, the region R, shown below, is bounded by $y = x^2$, $y = 4$ and $x = 0$.

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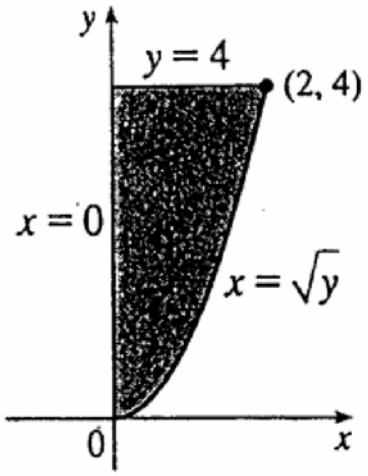

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(2 marks) A24. Choose the integral which represents the volume of the solid generated by rotating R, shown above, about y-axis.

A: $\int_0^4 \pi (x^2)^2 \, dx$
B: $2\int_0^2 \pi (x^2 - 4)^2 \, dx$
C: $\int_0^4 \pi (\sqrt{y} - 4)^2 \, dy$
D: $\int_{-2}^2 \pi (x^2)^2 \, dx$
E: $\int_0^4 \pi (\sqrt{y})^2 \, dy$

(2 marks) A25. Choose the integral which represents the volume of the solid generated by rotating R, shown above, about $y = 4$.

A: $\int_0^2 \pi x^4 dx$
B: $\int_0^4 \pi y^2 dy$
C: $2\int_0^4 \pi (4 - x^2)^2 \, dx$
D: $\int_0^2 \pi (4 - x^2)^2 \, dx$
E: $\int_0^4 \pi (y^2 + 4) \, dy$

Part B (50 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B26. If $y = (\ln x)^{\sqrt{\ln x}}$, find $\dfrac{dy}{dx}$ in terms of x.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B27. Find $\frac{dy}{dx}$ if $y = \tan^2\left(\cos^{-1}\left(\ln\left(e^{3x}+1\right)\right)\right)$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B28.

(a) Find a and $f(x)$ such that $\int_a^3 f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} e^{3+3i/n}$.

(b) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} e^{3+3i/n}$ using integration.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B29. Determine $\lim_{n\to\infty} (e^x + x)^{1/x}$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(4 marks) B30. Evaluate $\int_{-1}^{3} |x-1|\,dx$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(5 marks) B31. Evaluate $\int_0^{\pi/4} \frac{\sec x \tan x}{(1+\sec x)^2} dx$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(5 marks) B32. Evaluate $\int_{\ln 2}^{\ln 3} \frac{e^{2x}dx}{e^{2x}+1}$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(7 marks) B33. Find the area of the region bounded by the curves $y = x^2$ and $y = \dfrac{2}{x^2+1}$.

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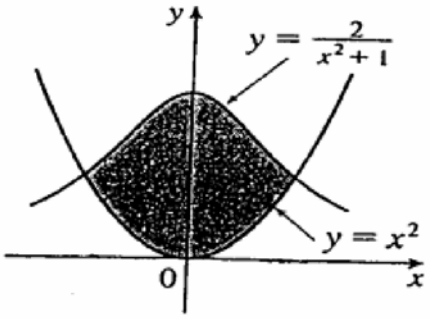

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NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(8 marks) B34. Find the volume of the solid generated by rotating the region bounded by $y = x^2$ and $x = y^2$ about the line $x = 2$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

(5 marks) B35. Find $\frac{dy}{dx}$ if $y = \int_{\sin x}^{\cos x} \sqrt{\tan^2\theta + \sec^2\theta}\, d\theta$.

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