MATH 1007B | **Term test #4 – solution** | **15th November 2012**

Total marks: 30 marks

Calculators are NOT permitted. This test is closed book. **Supply your answers on this sheet**, but TA's have extra paper if you need it.

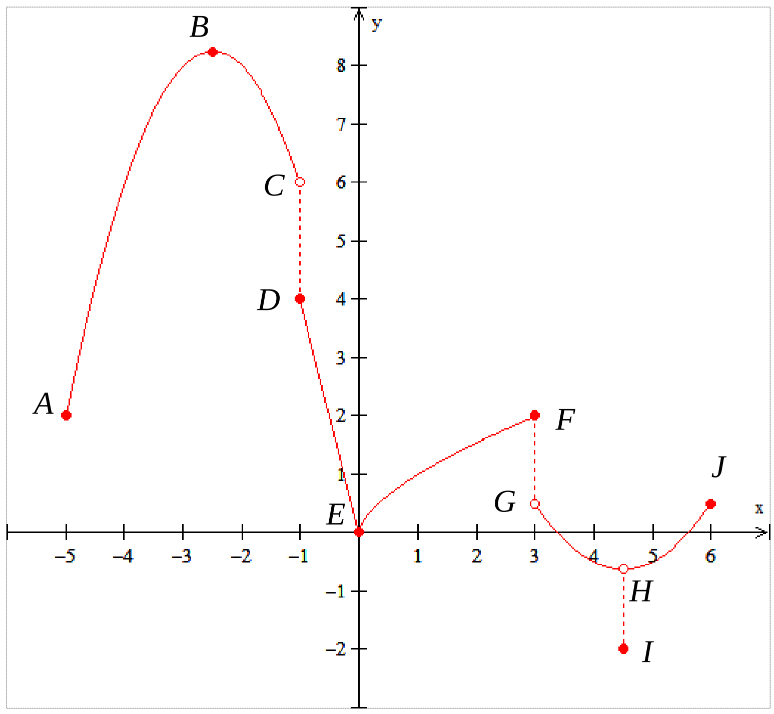
PLEASE PRINT ______________________ First name Last name ____________ Student number

Please show your work where appropriate!

1. [5] Consider the graph of a function, defined on [–5, 6]. Place each label in the appropriate category listed in the table. **Please note that a point can belong to more than one category**.

Category	Label(s)
Global maximum	B
Global minimum	I
Local maximum	B, F, J
Local minimum	A, E, I
Nothing	C, D, G, H

Please note that there are 10 points, labelled *A* to *J*.



2. [3] Determine the **equation of the line** ($y = mx+b$) tangent to $y^2 = 5x^4 - x^2$ at the point $(1, 2)$.

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(5x^4 - x^2)$$
$$2yy' = 20x^3 - 2x$$
$$y' = \frac{10x^3 - x}{y}$$
$$(1,2) \Rightarrow y'\Big|_{(1,2)} = \frac{10(1)^3 - 1}{2} = \frac{9}{2} = m$$

$$\therefore y = \frac{9}{2}x + b$$
$$(1,2) \Rightarrow 2 = \frac{9}{2}(1) + b$$
$$\therefore b = -\frac{5}{2}$$
$$\boxed{\therefore y = \frac{9}{2}x - \frac{5}{2}}$$

3. [3] Estimate the expression $e^{0.01}$, using the linearization of a function, at a chosen value $x=a$.

Use $f(x) = e^x$, $x = a = 0$, and determine $L(x)$:

$f'(x) = e^x \Rightarrow f'(a) = f'(0) = e^0 = 1$; and $f(a) = f(0) = 1$

$\therefore L(x) = f(a) + f'(a)\cdot(x-a) = 1 + 1\cdot(x-0) = 1+x$

$\therefore e^{0.01} = f(0.01) \approx L(0.01) = 1 + 0.01 = 1.01$

4. [3] An invertible function f is such that $f(1) = 0$ and $f'(1) = 2/5$. Calculate $(f^{-1})'(0)$.

Recall: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))} \Rightarrow (f^{-1})'(0) = \dfrac{1}{f'(f^{-1}(0))}$

Now: $f(1) = 0 \Rightarrow f^{-1}(0) = 1 \Rightarrow (f^{-1})'(0) = \dfrac{1}{f'(1)} = \dfrac{1}{2/5} = \boxed{\dfrac{5}{2}}$

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5. [4] Determine the absolute minimum and absolute maximum of the function $f(x) = x^4 - 8x^2 + 6$ in the interval [–1, 3].

$f(-1) = (-1)^4 - 8(-1)^2 + 6 = \boxed{-1}$
$f(3) = (3)^4 - 8(3)^2 + 6 = \boxed{15}$ } Endpoints

CRITICAL POINTS:

⇓

$f'(x) = 4x^3 - 16x$
$= 4x(x^2 - 4)$
$= 4x(x-2)(x+2)$

$f'(x) = 0 \Rightarrow$ $x = 0$ ✓, $x = 2$ ✓, $x = -2$ IGNORE

$\rightarrow f(0) = \boxed{6}$ and $f(2) = 2^4 - 8(2)^2 + 6 = \boxed{-10}$

$\therefore$ Absolute min. : -10
" max. : 15

6. [3+3+3+3] Differentiate the following functions whose rule is given below. **Do not simplify**.

a. $f(x) = \arctan\left(\sqrt{x}\right)$	b. $f(x) = (\log_4 x)(\sec x)$	c. $f(x) = \tan\,(5x^2 + 5^x)$	d. $f(x) = (x^2+1)^{(\cos x)}$

a. $f'(x) = \dfrac{1}{1+(\sqrt{x})^2} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{1}{1+x} \cdot \dfrac{1}{2\sqrt{x}}$

b. $f'(x) = \left(\dfrac{1}{\ln 4}\right) \cdot \left(\dfrac{1}{x}\right) \cdot \sec x + \log_4 x \sec x \tan x$

c. $f'(x) = \sec^2(5x^2 + 5^x) \cdot (10x + \ln 5 \cdot 5^x)$

d. $y = (x^2+1)^{(\cos x)} \Rightarrow \ln y = (\cos x) \cdot \ln(x^2+1)$

$\Rightarrow \dfrac{1}{y} \cdot y' = \cos x \cdot \dfrac{2x}{x^2+1} - \sin x \cdot \ln(x^2+1)$

$\therefore y' = f'(x) = (x^2+1)^{(\cos x)} \cdot \left[\cos x \cdot \dfrac{2x}{x^2+1} - \sin x \cdot \ln(x^2+1)\right]$