MAT 1007B **Term test #3 – solution** **1 November 2012**

Total marks: 30 marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT	First name	Last name	Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

1. [2+2 marks] Compute the following limits.

a. $\lim_{x\to 6^+} \frac{3x+2}{2x-12} \xrightarrow{D.S.} \frac{3(6^+)+2}{2(6^+)-12} = \frac{20^+}{12^+-12} = \frac{20^+}{0^+} = \frac{(+)}{(+)} \to +\infty$

b. $\lim_{x\to -1^-}\left(\frac{12-x}{x^2-1}\right) = \lim_{x\to -1^-}\left[\frac{12-x}{(x-1)(x+1)}\right] \xrightarrow{D.S.} \frac{12-(-1^-)}{(-1^--1)(-1^-+1)} = \frac{\overset{12+1^+}{13^+}}{(-2^-)(0^-)} =$

$\dots = \frac{(+)}{(-)(-)} = +\infty$

2. [4] Let f be a function such that: $f(x) = \begin{cases} x^2 & \text{if } x<2 \\ 3x-2 & \text{if } x \geq 2 \end{cases}$

Is f is differentiable at x=2? Justify your answer using the **basic definitions** for $f_-'(x)$ and $f_+'(x)$.

$f(2) = 3(2)-2 = 4$

$f_-'(2) = \lim_{h\to 0^-}\left[\frac{f(2+h)-f(2)}{h}\right] = \lim_{h\to 0^-}\left[\frac{(2+h)^2-4}{h}\right] =$

$\dots = \lim_{h\to 0^-}\left[\frac{4+4h+h^2-4}{h}\right] = \lim_{h\to 0^-}[4+h] \xrightarrow{D.S.} \boxed{4}$

$f_+'(2) = \lim_{h\to 0^+}\left[\frac{f(2+h)-f(2)}{h}\right] = \lim_{h\to 0^+}\left[\frac{3(2+h)-2-4}{h}\right] = \lim_{h\to 0^+}\left[\frac{6+3h-6}{h}\right]$

$\dots = \lim_{h\to 0^+}[3] = \boxed{3}$

$\therefore f_-'(2) \neq f_+'(2) \Rightarrow f$ not differentiable at $x=2$

3. [7] **Simple answer questions**: determine $f'(x)$ if:

$f(x)=x^{3.5} \Rightarrow f'(x) = 3.5x^{2.5}$	$f(x)=\frac{3}{x^3}+5 = 3x^{-3}+5 \Rightarrow f'(x) = \frac{-9}{x^4}$
$f(x) = -\sec x$ $\Rightarrow f'(x) = -\sec x\tan x$	$f(x) = -2e^x+\cos x$ $\Rightarrow f'(x) = -2e^x - \sin x$
$f(x)=(x+2)^2 = x^2+4x+4$ $\Rightarrow f'(x) = 2x+4$	$f(x) = 4\sqrt{\frac{2}{x^3}} - \frac{1}{2}\sqrt{\frac{x^3}{4}} = 4\sqrt{2}x^{-3/2} - \frac{1}{4}x^{3/2}$ $\Rightarrow f'(x) = -6\sqrt{2}x^{-5/2} - \frac{3}{8}x^{1/2}$ $= -\frac{3}{8}\sqrt{x} - 6\sqrt{\frac{2}{x^5}}$

[1mark each, the last one is worth **2 marks**]

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4. [3] What is the **rule of the tangent line** to the curve $y = f(x) = x^3 + 3x$ at $x = 1$?

Slope of line : $f'(1)$, with $f'(x) = 3x^2 + 3 \Rightarrow f'(1) = 6$

$\therefore y = 6x + b \Rightarrow$ equation of line

But $f(1) = 4 = 6(1) + b \Rightarrow b = -2$

$\therefore y = 6x - 2 \Rightarrow$ equation of line

5. [12] Determine $f'(x)$ using all the differentiation rules learned so far. **<u>Do not simplify.</u>**

a. $f(x) = \dfrac{3x-5}{\sin^2 x} = \dfrac{u}{v}$

$u = 3x-5$	$v = \sin^2 x$
$u' = 3$	$v' = 2\sin x \cos x$

$$\therefore f'(x) = \frac{vu' - uv'}{v^2} = \frac{3\sin^2 x - 2\sin x\cos x\,(3x-5)}{\sin^4 x}$$

b. $f(x) = \tan(\cos x) = \tan u$, $u = \cos x$

$\dfrac{du}{dx} = u' = -\sin x$

$$\therefore f'(x) = \sec^2 u \cdot (-\sin x)$$
$$= -\sin x \cdot \sec^2(\cos x)$$

c. $f(x) = e^{x^2}\sqrt{3x+4} = uv$,

$u = e^{x^2}$	$v = \sqrt{3x+4}$
$u' = 2xe^{x^2}$	$v' = \dfrac{3}{2\sqrt{3x+4}}$

$$\therefore f'(x) = uv' + vu'$$
$$= \frac{3e^{x^2}}{2\sqrt{3x+4}} + 2xe^{x^2}\sqrt{3x+4}$$

d. $f(x) = (5x^2+4x)^2\,(8-10x^2)^4 = uv$

$u = (5x^2+4x)^2$

$u' = 2(5x^2+4x)(10x+4)$

$v = (8-10x^2)^4$

$v' = 4(8-10x^2)^3(-20x)$

$v' = -80x(8-10x^2)^3$

$$f'(x) = uv' + vu' = -80x(8-10x^2)^3(5x^2+4x)^2 + 2(5x^2+4x)(10x+4)(8-10x^2)^4$$