MATH 1007B **Term test #2 – solution** **18 October 2012**

Total marks: 32 marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ______________________ ______________

First name Last name Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

1. Determine the following limits.

   a. [**3marks**] $\lim_{x\to -3} \frac{x^2+6x+9}{x+3} \xrightarrow{D.S.} \frac{(-3)^2+6(-3)+9}{(-3)+3} \left(= \frac{0}{0}\right)$

   $\ldots = \lim_{x\to -3} \frac{(x+3)(x+3)}{(x+3)} = \lim_{x\to -3} x+3 \xrightarrow{D.S.} \boxed{0}$

   b. [2] $\lim_{x\to 6} \frac{x^2+4x-12}{x+6} \xrightarrow{D.S.} \frac{(6)^2+4(6)-12}{6+6} =$

   $\ldots = \frac{36+24-12}{12} = \boxed{4}$

   c. [4] $\lim_{h\to 0} \frac{\frac{3}{\sqrt{h+4}} - \frac{3}{2}}{h} \xrightarrow{D.S.} \frac{\frac{3}{\sqrt{0+4}} - \frac{3}{2}}{0} = \frac{0}{0}$

   $= \lim_{h\to 0} \left[\frac{1}{h} \cdot \frac{6-3\sqrt{h+4}}{2\sqrt{h+4}}\right] = \lim_{h\to 0} \left[\frac{1}{h} \cdot \frac{6-3\sqrt{h+4}}{2\sqrt{h+4}} \cdot \frac{6+3\sqrt{h+4}}{6+3\sqrt{h+4}}\right]$

   $\ldots = \lim_{h\to 0} \left[\frac{1}{h} \cdot \frac{36-9(h+4)}{2\sqrt{h+4}\cdot(6+3\sqrt{h+4})}\right] = \lim_{h\to 0} \left[\frac{1}{h} \cdot \frac{-9h}{2\sqrt{h+4}\cdot(6+3\sqrt{h+4})}\right]$

   $\ldots = \lim_{h\to 0} \left[\frac{-9}{2\sqrt{h+4}\cdot(6+3\sqrt{h+4})}\right] \xrightarrow{D.S.} \frac{-9}{2\cdot 2\cdot(6+6)} = \boxed{-\frac{3}{16}}$

   d. [3] $\lim_{y\to 5} \frac{y^2+2y-35}{y-5} \xrightarrow{D.S.} \frac{5^2+2(5)-35}{5-5} = \frac{0}{0}$

   $\ldots = \lim_{y\to 5} \frac{(y+7)(y-5)}{(y-5)} = \lim_{y\to 5} (y+7) = 12$

   e. [3] $\lim_{z\to 0} \frac{\sin 2z}{7z} \xrightarrow{D.S.} \frac{0}{0}$

   $\ldots = \lim_{z\to 0} \left[\frac{2}{2} \cdot \frac{\sin 2z}{7z}\right] = \lim_{z\to 0} \left[\frac{2}{7} \cdot \frac{\sin 2z}{2z}\right]$

   $\ldots = \lim_{z\to 0} \left[\frac{2}{7}\right] \cdot \lim_{z\to 0} \left[\frac{\sin 2z}{2z}\right] = \frac{2}{7} \cdot 1 = \boxed{\frac{2}{7}}$

   (because as $z\to 0$, so does $2z$)

2. [2] Let *dom* (*f*) = [*a*, *b*]. List the conditions for *f* to be continuous at the end point *x*=*a*.

   - $f(a)$ is defined
   - $\lim_{x\to a^+} f(x) = L$ $(L \in \mathbb{R})$
   - $f(a) = \lim_{x\to a^+} f(x)$

   $x\to a^+$ is necessary because a is an endpoint

NOTE to TA's: if $x\to a^+$ is missing, then give 1/2 marks

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3. [2] What is $\lim_{z\to 2}(5x)$? Explain. $\lim_{z\to 2}(5x) = 5x$ because the expression $5x$ does not depend on z. It is a constant w.r.t. z.

4. Determine the following limits if they exist. If a limit does not exist, then determine whether it is +, –, or neither.

   a. [3] $\lim_{x\to+\infty}\sqrt{9x^2+12}-3x \xrightarrow{D.S.} \infty - \infty$

$$= \lim_{x\to+\infty}\left[\left(\sqrt{9x^2+12}-3x\right)\cdot\frac{\sqrt{9x^2+12}+3x}{\sqrt{9x^2+12}+3x}\right] = \lim_{x\to+\infty}\left[\frac{9x^2+12-9x^2}{\sqrt{9x^2+12}+3x}\right]$$

$$\ldots = \lim_{x\to+\infty}\left[\frac{12}{\sqrt{9x^2+12}+3x}\right] \xrightarrow{D.S.} \frac{12}{\infty} = \boxed{0}$$

   b. [2] $\lim_{x\to-\infty}\left(\frac{5x^3+5x^2+5x+5}{2-2x-2x^2-2x^3}\right) \xrightarrow{D.S.} \frac{\infty}{\infty}$ $\therefore \lim = \boxed{-\frac{5}{2}}$

This is the form $\frac{P(x)}{Q(x)}$, with $\deg P = \deg Q$.

   c. [3] $\lim_{x\to\infty}\frac{\sqrt{6x^2+11}}{2x+9} \xrightarrow{D.S.} \frac{\infty}{\infty}$

$$= \lim_{x\to\infty}\left[\frac{\sqrt{6x^2+11}\div x}{2x+9\div x}\right] = \lim_{x\to\infty}\left[\frac{\frac{\sqrt{6x^2+11}}{\sqrt{x^2}}}{2+9/x}\right] =$$

$$\ldots = \lim_{x\to\infty}\left[\frac{\sqrt{6+11/x^2}}{2+9/x}\right] = \boxed{\frac{\sqrt{6}}{2}}$$

5. Let f be a function such that $f(x)=\begin{cases}\sqrt{9-x}+3, & x\le -16\\ 3x^2-2x, & -16<x\le 0\\ 4\ln(x+1), & x>0\end{cases}$. **Determine:**

   a. [2] $\lim_{x\to-16^-} f(x)$

$$\Rightarrow = \lim_{x\to-16^-} f(x) = \lim_{x\to-16^-}\left[\sqrt{9-x}+3\right] \xrightarrow{D.S.}$$

$$\ldots = \left(\sqrt{9-(-16)}+3\right) = \boxed{8}$$

   b. [3] ...whether f is continuous or not at $x = 0$. If it is not continuous, explain the reason.

- $f(0) = 3(0)^2 - 2(0) = 0$
- $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(3x^2-2x) \xrightarrow{D.S.} 0$
- $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(4\ln(x+1)) \xrightarrow{D.S.} 4\ln(1) = 0$

$\therefore \lim_{x\to 0} f(x) = 0$, and $f(0) = \lim_{x\to 0} f(x) = 0$

$\therefore f$ is continuous at $x = 0$