STAT 2507 Assignment # 2 Fall 2012

Section A due Tuesday, October 16, 2012 before 10:05am
Section B due Tuesday, October 16, 2012 before 6:05pm
Section C due Monday, October 15, 2012 before 2:35pm

Last Name ————————————————————-, **First** ————————————-

Student # ————————————————

Part I: Lab Part

Use spaces left to answer lab questions.

1. The Minitab data set 'sat' has the mean *verbal* and *math* scores of the Scholastic Aptitude Test (SAT) for the academic years 1967 to 1981. Retrieve the 'sat' data set (by choosing **file** → **Open Worksheet** (you need to click on "look in Minitab Sample Data folder" icon) and then choosing sat).

   (a) Construct a scatterplot with year marked along the horizontal axis and verbal marked along the vertical axis. Calculate the correlation coefficient: -0.981. If appropriate,fit a least squares (L-S) regression line using *year* to predict the *verbal* scores (verbal score is called the response variable). What is the equation of the L-S regression line? (READ PAGES 121-122 OF THE TEXTBOOK FIRST.) verbal=7373-3.51 year

   (b) Construct a scatterplot with year marked along the horizontal axis and math marked along the vertical axis. Calculate the correlation coefficient: -0.978. If appropriate, fit a L-S regression line using *year* to predict the *math* scores (response variable). What is the equation of the L-S regression line? Math=4955-2.27 year.

   (c) Construct a scatterplot with verbal marked along the horizontal axis and math marked along the vertical axis. Calculate the correlation coefficient: 0.993.. If appropriate,Fit a L-S regression line using *verbal* scores to predict the *math* scores (response variable). What is the equation of the L-S regression line?Math=193+0.643 verbal.

(d) What is the predicted score for *math* in 1970? 483.1. In 1983? 453.6. In 2000? 415. Which of these seem to make sense? 1970 (year).

(e) Use your answer to part (c) to predict the *math* score if the *verbal* score is 450. Answer: 482.35. Can you predict the *verbal* score if the *math* score is 480? Answer: No. Why?because the equation in (c) is meaningful only for predicting math from verbal.

2. (Conditional probability and Independence) the link click here gives an experiment of tossing two fair dice (one green and one red).
   Let C be the event that the green die shows a number less than or equal to 2.
   Let D be the event that the red die shows a number less than or equal to 3.

   (a) (Press Reset) Obtain $P(C)= 12/36$
   (b)(Press Reset) Obtain $P(D)= 18/36$
   (c)(Press Reset) By clicking on the simple events in which the green die has a number at most 2 and the red one has a number at most 3, obtain $P(C \cap D)= 6/36$
   (d)(Use parts (b) and (c)) If you know that the red die showed a number less than or equal to 3, then obtain the probability that the green shows a number less than or equal to 2, by clicking on the relevant simple events in the applet?
   I.e., what is $P(C|D) = 6/18$
   (e)What is your conclusion about the relation between the events C and D? Sol: Independent as $P(C|D) = P(C)$

3. (Mean and variance of random variables using Minitab)
   The values (1 to 10) in the first column of the Open Office file (see file: Data-lab2-25-7-F12.xls) are the values that random variable x takes (copy and paste them in C1 of your Minitab worksheet) and the second column contains $p(x)$ (copy and paste this column from Open Office file in C2 of your Minitab worksheet)

   (a) Obtain the mean of x, i.e., $\mu = E(x)$ by $Calc \rightarrow Calculator \rightarrow sum(C1 * C2)$. What is μ? 1.998047

4. by clicking $Calc \rightarrow Calculator \rightarrow sum((C1 * *2) * C2) - (sum(C1 * C2)) * *2$. obtain $Var(x) = \sigma^2$. What is σ^2 ? 1.962886 What is σ? 1.40103

5. Calculate the interval $(\mu - 2\sigma, \mu + 2\sigma)$? $(1.998047 - 2(1.40103), 1.998047 - 2(1.40103)) = (-0.804013, 4.800107)$

6. What values of x are inside the interval in part (c)? 1, 2, 3, and 4

Part II: Written Questions

7. According to Statistics Canada, the monthly earnings of workers in the mining industry was \$3840, with a standard deviation of \$240. A mine worker claims to earn \$4325. each month. Find the z-score corresponding to this worker's wage. Is the amount unusual?
   Solution:
   $z = \frac{4325-3840}{240} = 2.02$, it is not unusual since z-score is less than 3.

8. In a certain factory, Machines A, Machine B, and Machine C are all producing widgets. Widgets produced by Machine A have a 1% chance of being defective. Likewise, widgets produced by Machine B and Machine C are defective 4% and 2% of the time, respectively. Of the total production of widgets in the factory, Machine A produces 30%, Machine B produces 25%, and Machine C produces 45%. Suppose a widget is selected at random from this factory. (a) What is the probability the widget is defective? (b) If the widget is defective, what is the probability it was produced by Machine B?

   Solution:
   D: defective widgets, A: Machine A, B: Machine B, C: Machine C, so

   $$P(D|A) = 0.01, P(D|B) = 0.04, P(D|C) = 0.02, P(A) = 0.3, P(B) = 0.25, P(C) = 0.45$$

   $$P(D) = P(D|A)P(A)+P(D|B)P(B)+P(D|C)P(C) = 0.01(0.3)+0.04(0.25)+0.02(0.45) = 0.22$$

   (b) [8] If the widget is defective, what is the probability it was produced by Machine B?
   Solution:

   $$P(B|D) = \frac{P(D|B)P(B)}{P(D)} = \frac{0.04(0.25)}{0.22} = 0.045$$

9. Suppose that $P(A) = 0.6$, $P((A \cup B)^c) = 0.2$, $P(C) = 0.7$ and $P(D) = 0.4$.
   (a) [1 mark] Can C and D be mutually exclusive? Why? Explain No, since $0.7 + 0.4 = 1.1 > 1$.
   (b) [1 mark] If the events A and B are independent then P(B)= 0.5.
   Since $P(A \cup B) = 1 - 0.2 = 0.8$, if A and B are independent, then

$P(A \cup B) = P(A) + P(B) - P(A)P(B) \rightarrow 0.8 = 0.6 + P(B) - 0.6P(B) \rightarrow P(B) = 0.5$

(c) [1 mark] If the events A and B are mutually exclusive then P(B)= 0.2.
Since $P(A \cup B) = P(A) + P(B) - 0 \rightarrow 0.8 = 0.6 + P(B) \rightarrow P(B) = 0.2$.

10. Suppose that a random variable X follows the distribution given by:

a	-2	-1	0	1	2
$P(X = a)$	1/8	1/4	1/4	?	1/8

Let $Y = X^2$. Find $P(Y \geq 1)$ and $E(Y)$.
Solution:
$P(X = 1) = \frac{1}{4}$ so when $Y = X^2$

a	0	1	4
$P(Y = a)$	1/4	1/2	1/4

$P(Y \geq 1) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, and $E(Y) = 0(1/4) + 1(1/2) + 4(1/4) = 1/2 + 1 = 3/2$