

# STAT 2507 Assignment #3, Fall 2012

Section A due Tuesday, October 30, 2012 before 10:05am

Section B due Tuesday, October 30, 2012 before 6:05pm

*Last Name* \_\_\_\_\_ *First Name* \_\_\_\_\_ *Student Number* \_\_\_\_\_

## Part I Lab Questions

**Question 1.** Suppose that  $X$  has a binomial distribution with  $n=25$  and  $p=0.8$ . Use minitab to simulate 25 values of  $X$ :

```
random 25 c1;  
binomial 25 0.8.
```

- (a) [1 **MARK**] How many of your values are less than 21?
- (b) [1 **MARK**] How many of your values are between 21 and 24 inclusive?
- (c) [2 **MARKS**] MINITAB has a *cdf* command that computes  $P(X \leq k)$  for each possible value of  $k$ . Enter the commands:

```
cdf;  
binomial 25 0.8.
```

Use the resulting output to compute  $\Pr(X < 21)$  and  $\Pr(21 \leq X \leq 24)$ . How do these probabilities compare to the relative frequencies you observed in parts (a) and (b)?

- (d) [1 **MARK**] If you simulate 10000 values of  $X$ , what would be the expected number of values (among the 10000 values) that are less than 21?

### Solution:

- (a) Answers will vary.
- (b) Answers will vary.
- (c)  $\Pr(X < 21) = \Pr(X \leq 20) = 0.579$  and  $\Pr(21 \leq X \leq 24) = \Pr(X \leq 24) - \Pr(X \leq 20) = 0.996 - 0.579 = 0.417$ .
- (d)  $10000 * 0.579 = 5790$ .

**Question 2.** Suppose that  $X$  has a Poisson distribution with  $\mu = 20$ .

- (a) [2 **MARKS**] Use the resulting output of following MINITAB commands:

```
cdf;  
poission 20.
```

to calculate  $P(X < 18)$  and  $P(18 \leq X \leq 22)$ .

- (b) [1 MARK] If you simulate 10000 values of  $X$ , what would be the expected number of values (among the 10000 values) that are less than 18?

**Solution:**

- (a)  $\Pr(X < 18) = \Pr(X \leq 17) = 0.297$  and  $P(18 \leq X \leq 22) = \Pr(X \leq 22) - \Pr(X \leq 17) = 0.721 - 0.297 = 0.424$ .
- (b)  $10000 * 0.297 = 2970$ .

**Question 3.** Suppose that  $X$  has a Normal distribution with  $\mu = 35$  and  $\sigma = 2$ .

- (a) [3 MARKS] To find  $\Pr(X \leq a)$ , where  $a$  is any fixed constant, we can use following MINITAB commands:

```
cdf a;
normal 35, 2.
```

Use these commands to find  $\Pr(X \leq 33)$ ,  $\Pr(X > 36)$  and  $\Pr(33 \leq Y \leq 36)$ .

- (b) [1 MARK] To find '?' that satisfies  $\Pr(X \leq ?) = b$ , where  $b$  is a fixed know number between 0 and 1, we can use following MINITAB commands:

```
invcdf b;
normal 35, 2.
```

Use these commands to find the value of  $c$  in  $\Pr(X \leq c) = 0.25$ ?

**Solution:**

- (a)  $\Pr(X < 33) = \Pr(Z \leq \frac{33-35}{2}) = 0.1587$ ,  
 $\Pr(X > 36) = \Pr(Z \geq \frac{36-35}{2}) = 1 - \Pr(Z < 0.5) = 1 - 0.6915 = 0.3085$  and  
 $P(33 \leq X \leq 36) = \Pr(X \leq 36) - \Pr(X \leq 33) = 0.6915 - 0.1587 = 0.5328$ .
- (b) Since  $0.25 = \Pr(X \leq c) = \Pr(Z \leq \frac{c-35}{2})$  so  $\frac{c-35}{2} = -0.675$ . Then  $c = 35 - 1.35 = 33.65$ .

## Part II Written Questions

**Question 4.** During a major disaster, for example, a hurricane, earthquake. It is very important to have certain recommended household items on hand. In a recent survey, only 40% of Americans had a three-day supply of bottled water in case of emergency. Suppose a sample of 20 Americans is selected at random. Let  $X$  be the number of Americans sampled who have a three-day supply of bottled water on hand.

- (a) [3 MARKS] Does the random variable  $X$  have a binomial distribution? If not, why not? If so, what are the values of  $n$  and  $p$ .
- (b) [1 MARK] What is the probability that exactly 12 Americans have a three-day supply of bottled water on hand?

- (c) [1 MARK] What is the probability that at least 12 Americans have a three-day supply of bottled water on hand?
- (d) [1 MARK] What is the probability that more than 12 Americans have a three-day supply of bottled water on hand?
- (e) [1 MARK] What is the probability that at most 12 Americans have a three-day supply of bottled water on hand?
- (f) [1 MARK] What is the probability that less than 12 Americans have a three-day supply of bottled water on hand?
- (g) [3 MARKS] What is the probability that between 12 and 14 Americans(inclusive) have a three-day supply of bottled water on hand?
- (h) [2 MARKS] What are the mean and variance of  $X$ ?

**Solution:**

(a)  $X$  is a binomial random variable with  $n = 20$  and  $p = 0.4$  or  $X \sim B(20, 0.4)$ .

(b)

$$\Pr(X = 12) = C_{12}^{20} 0.4^{12} (1 - 0.4)^8 = 0.036.$$

(c)

$$\begin{aligned} \Pr(X \geq 12) &= \sum_{k=12}^{20} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\ &= 1 - \sum_{k=0}^{11} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\ &= 1 - 0.943 \\ &= 0.057. \end{aligned}$$

(d)

$$\begin{aligned} \Pr(X > 12) &= \Pr(X \geq 13) \\ &= \sum_{k=13}^{20} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\ &= 1 - \sum_{k=0}^{12} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\ &= 1 - 0.979 \\ &= 0.021. \end{aligned}$$

(e)

$$\begin{aligned} \Pr(X \leq 12) &= \sum_{k=0}^{12} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\ &= 0.979. \end{aligned}$$

(f)

$$\begin{aligned} \Pr(X < 12) &= \Pr(X \leq 11) \\ &= \sum_{k=0}^{11} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\ &= 0.943. \end{aligned}$$

(g)

$$\begin{aligned}
 \Pr(12 \leq X \leq 14) &= \sum_{k=12}^{14} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\
 &= \sum_{k=0}^{14} C_k^{20} 0.4^k (1 - 0.4)^{20-k} - \sum_{k=0}^{11} C_k^{20} 0.4^k (1 - 0.4)^{20-k} \\
 &= 0.998 - 0.943 \\
 &= 0.055.
 \end{aligned}$$

(h)  $\mu = np = 20 \times 0.4 = 8$ ,  $\sigma^2 = np(1 - p) = 20 \times 0.4 \times (1 - 0.4) = 4.8$ .

**Question 5.** During off hours, cars arrive at a highway service station at an average rate of 5 cars per 10 minutes. The number of cars arriving at the service station is distributed according to a Poisson distribution.

- (a) [1 MARK] What is the probability that during the next minute three cars will arrive?
- (b) [1 MARK] What is the probability that during the next five minutes three cars will arrive?
- (c) [1 MARK] What is the probability that during the next five minutes *at least* three cars will arrive?
- (d) [1 MARK] What is the probability that during the next five minutes *at most* three cars will arrive?
- (e) [1 MARK] What is the probability that during the next five minutes *less than* three cars will arrive?
- (f) [1 MARK] What is the probability that during the next five minutes *more than* three cars will arrive?

**Solution:**

- (a) Let  $X$  represent the number of cars arriving during the next minute. The average rate is  $\mu = 5/10 = 0.5$  car per minute. Then, the probability that during the next minute three cars will arrive is:

$$\Pr(X = 3) = \frac{0.5^3}{3!} e^{-0.5} = 0.012.$$

- (b) Let  $X$  represent the number of cars arriving during the next five minutes. The average rate is  $\mu = 5/10 \times 5 = 2.5$  car per five minutes. Then, the probability that during the next five minutes three cars will arrive is:

$$\Pr(X = 3) = \frac{2.5^3}{3!} e^{-2.5} = 0.214.$$

- (c) Let  $X$  represent the number of cars arriving during the next five minutes. The average rate is  $\mu = 5/10 \times 5 = 2.5$  car per five minutes. Then, the probability that during the next five minutes at least three cars will arrive is:

$$\Pr(X \geq 3) = \sum_{k=3}^{\infty} \frac{2.5^k}{k!} e^{-2.5} = 1 - \sum_{k=1}^2 \frac{2.5^k}{k!} e^{-2.5} = 1 - 0.544 = 0.456.$$

- (d) Let  $X$  represent the number of cars arriving during the next five minutes. The average rate is  $\mu = 5/10 \times 5 = 2.5$  car per five minutes. Then, the probability that during the next five minutes at most three cars will arrive is:

$$\Pr(X \leq 3) = \sum_{k=0}^3 \frac{2.5^k}{k!} e^{-2.5} = 0.758.$$

- (e) Let  $X$  represent the number of cars arriving during the next five minutes. The average rate is  $\mu = 5/10 \times 5 = 2.5$  car per five minutes. Then, the probability that during the next five minutes less than three cars will arrive is:

$$\Pr(X < 3) = \sum_{k=0}^2 \frac{2.5^k}{k!} e^{-2.5} = 0.544.$$

- (f) Let  $X$  represent the number of cars arriving during the next five minutes. The average rate is  $\mu = 5/10 \times 5 = 2.5$  car per five minutes. Then, the probability that during the next five minutes more than three cars will arrive is:

$$\Pr(X > 3) = \sum_{k=4}^{\infty} \frac{2.5^k}{k!} e^{-2.5} = 1 - \sum_{k=1}^3 \frac{2.5^k}{k!} e^{-2.5} = 1 - 0.758 = 0.242$$

**Question 6.** Suppose you are going to select a sample of 5 cards from a randomized standard deck of 52 cards. Let the random variable  $X$  represent the total number of diamonds that you obtain.

- (a) [6 MARKS] If you sample the cards without replacement, what is the probability distribution of  $X$ ? In this case, what is the probability that at most 2 of the cards will be diamonds?
- (b) [3 MARKS] If you sample the cards with replacement, what is the probability distribution of  $X$ ? In this case, what is the probability that at most 2 of the cards will be diamonds?

**Solution:**

- (a)  $X$  has a hypergeometric distribution with  $N = 52$ ,  $n = 5$ ,  $M = 13$ .

$$\begin{aligned} \Pr(X \leq 2) &= \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) \\ &= \frac{C_0^{13} C_5^{39}}{C_5^{52}} + \frac{C_1^{13} C_4^{39}}{C_5^{52}} + \frac{C_2^{13} C_3^{39}}{C_5^{52}} \\ &= 0.2215 + 0.4114 + 0.2743 \\ &\approx 0.907 \end{aligned}$$

- (b)  $X$  has a binomial distribution with  $n = 5$ ,  $p = 13/52 = 0.25$ .

$$\begin{aligned} \Pr(X \leq 2) &= \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) \\ &\approx 0.897 \end{aligned}$$

**Question 7.** Suppose that the waiting time for a pizza to be delivered to an individual's residence has been found to be normally distributed with a mean of 30 minutes and a standard deviation of 8 minutes. What is the probability that a randomly selected individual will have a waiting time:

- (a) [4 MARKS] Between 15 and 45 minutes?
- (b) [2 MARKS] At least 10 minutes?
- (c) [2 MARKS] At most 10 minutes?
- (d) [2 MARKS] Less than 10 minutes?
- (e) [2 MARKS] No more than 45 minutes?
- (f) [2 MARKS] More than 45 minutes?

**Solution:** Let  $X$  represent the waiting time for a pizza to be delivered to an individual's residence. And  $X$  is distributed as normal with mean 30 minutes and standard deviation 8 minutes.

(a)

$$\begin{aligned}\Pr(15 \leq X \leq 45) &= \Pr\left(\frac{15 - 30}{8} \leq Z \leq \frac{45 - 30}{8}\right) \\ &= \Pr(-1.88 \leq Z \leq 1.88) \\ &= \Pr(Z \leq 1.88) - \Pr(Z \leq -1.88) \\ &= 0.9699 - 0.0301 \\ &= 0.9398\end{aligned}$$

$$\begin{aligned}\text{OR: } &= \Pr(-1.88 \leq Z \leq 1.88) \\ &= 1 - 2\Pr(Z \leq -1.88) \\ &= 1 - 2 \times 0.0301 = 0.9398\end{aligned}$$

(b)

$$\begin{aligned}\Pr(X \geq 10) &= \Pr\left(Z \geq \frac{10 - 30}{8}\right) \\ &= \Pr(Z \geq -2.5) \\ &= 1 - \Pr(Z \leq -2.5) \\ &= 1 - 0.0062 \\ &= 0.9938\end{aligned}$$

(c)

$$\begin{aligned}\Pr(X \leq 10) &= \Pr\left(Z \leq \frac{10 - 30}{8}\right) \\ &= \Pr(Z \leq -2.5) \\ &= 0.0062\end{aligned}$$

(d)

$$\begin{aligned}\Pr(X < 10) &= \Pr(X \leq 10) \\ &= \Pr\left(Z \leq \frac{10 - 30}{8}\right) \\ &= \Pr(Z \leq -2.5) \\ &= 0.0062\end{aligned}$$

(e)

$$\begin{aligned}\Pr(X \leq 45) &= \Pr\left(Z \leq \frac{45 - 30}{8}\right) \\ &= \Pr(Z \leq 1.88) \\ &= 0.9699\end{aligned}$$

(f)

$$\begin{aligned}\Pr(X \geq 45) &= \Pr\left(Z \geq \frac{45 - 30}{8}\right) \\ &= \Pr(Z \geq 1.88) \\ &= 1 - \Pr(Z \leq 1.88) \\ &= 1 - 0.9699 \\ &= 0.0301\end{aligned}$$

$$\begin{aligned}\text{OR: } \Pr(X \geq 45) &= \Pr\left(Z \geq \frac{45 - 30}{8}\right) \\ &= \Pr(Z \geq 1.88) \\ &= \Pr(Z \leq -1.88) \\ &= 0.0301\end{aligned}$$

**Question 8.** [6 MARKS] The batting average off all players in a professional baseball league is normally distributed with the mean of 0.27 and standard deviation of 0.01. A player's contract comes up for renewal. In order to have a good bargaining position, he would like to have a batting average in the top 1% of all players. What batting average must he attain?

**Solution:**  $X \sim N(0.27, (0.01)^2)$ , we want to find  $X_0$  such that  $P(X > X_0) = 0.01$  or  $P(X < X_0) = 0.99$  then  $P\left(Z < \frac{X_0 - 0.27}{0.01}\right) = 0.99$ , by using normal table  $\frac{X_0 - 0.27}{0.01} = 2.33$ , then  $X_0 = 2.33(0.01) + 0.27 = 0.2933$ .