Chapter 1

Analyzing Economic Problems

Solutions to Review Questions

1. Microeconomics studies the economic behavior of individual economic decision makers, such as a consumer, a worker, a firm, or a manager. Macroeconomics studies how an entire national economy performs, examining such topics as the aggregate levels of income and employment, the levels of interest rates and prices, the rate of inflation, and the nature of business cycles.

2. While our wants for goods and services are unlimited, the resources necessary to produce those goods and services, such as labor, managerial talent, capital, and raw materials, are "scarce" because their supply is limited. This scarcity implies that we are constrained in the choices we can make about which goods and services to produce. Thus, economics is often described as the science of constrained choice.

3. Constrained optimization allows the decision maker to select the best (optimal) alternative while accounting for any possible limitations or restrictions on the choices. The objective function represents the relationship to be maximized or minimized. For example, a firm's profit might be the objective function and all choices will be evaluated in the profit function to determine which yields the highest profit. The constraints place limitations on the choice the decision maker can select and defines the set of alternatives from which the best will be chosen.

4. If the price in the market was above the equilibrium price, consumers would be willing to purchase fewer units than suppliers would be willing to sell, creating an excess supply. As suppliers realize they are not selling the units they have made available, sellers will bid down the price to entice more consumers to purchase their goods or services. By definition, equilibrium is a state that will remain unchanged as long as exogenous factors remain unchanged. Since in this case suppliers will lower their price, this high price cannot be an equilibrium.

   When the price is below the equilibrium price, consumers will demand more units than suppliers have made available. This excess demand will entice consumers to bid up the prices to purchase the limited units available. Since the price will change, it cannot be an equilibrium.

5. Exogenous variables are taken as given in an economic model, i.e., they are determined by some process outside the model, while endogenous variables are determined within the economic model being studied.

An economic model that contained no endogenous variables would not be very interesting. With no endogenous variables, nothing would be determined by the model so it would not serve much purpose.

6. Comparative statics analyses are performed to determine how the levels of endogenous variables change as some exogenous variable is changed. This type of analysis is very important since in the real world the exogenous variables, such as weather, policy tools, etc. are always changing and it is useful to know how changes in these variables affect the levels of other, endogenous, variables. An example of comparative statics analysis would be asking the question: If extraordinarily low rainfall (an exogenous variable) causes a 30 percent reduction in corn supply, by how much will the market price for corn (an endogenous variable) increase?

7. Positive analysis attempts to explain how an economic system works or to predict how it will change over time by asking explanatory or predictive questions. Normative analysis focuses on what should be done by asking prescriptive questions.

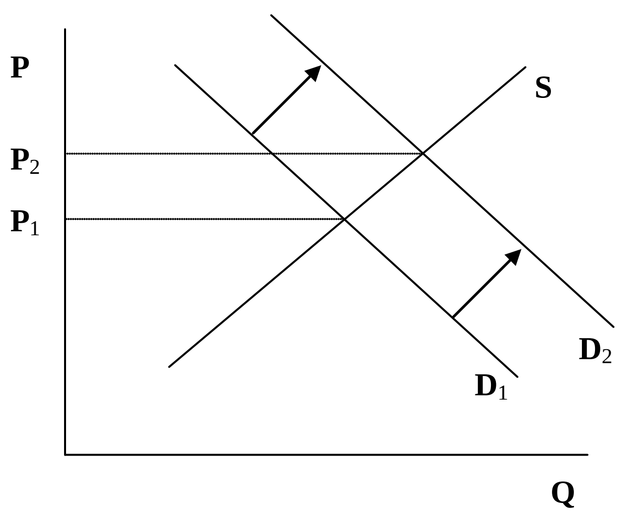
   a) Because this question asks whether dealership profits will go up or down (and by how much) – but refrains from inquiring as to whether this would be a good thing – it is an example of positive analysis.

   b) On the other hand, this question asks whether it is desirable to impose taxes on Internet sales, so it is normative analysis. Notably, this question does not ask what the effect of such taxes would be.

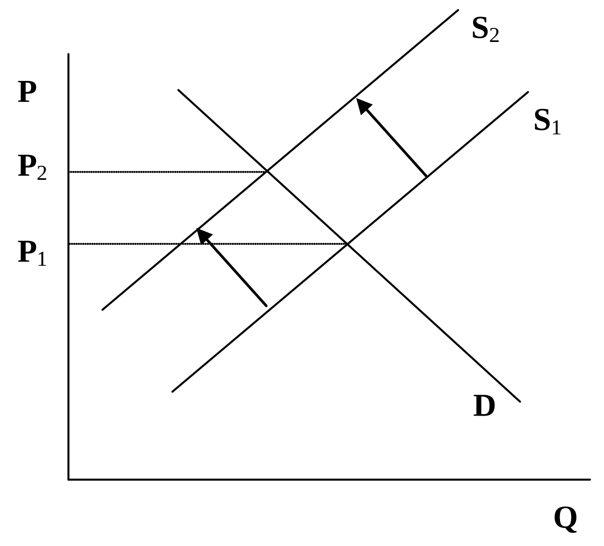
Solutions to Problems

1.1 While the claim that markets never reach an equilibrium is probably debatable, even if markets do not ever reach equilibrium, the concept is still of central importance. The concept of equilibrium is important because it provides a simple way to predict how market prices and quantities will change as exogenous variables change. Thus, while we may never reach a particular equilibrium price, say because a supply or demand schedule shifts as the market moves toward equilibrium, we can predict with relative ease, for example, whether prices will be rising or falling when exogenous market factors change as we move toward equilibrium. As exogenous variables continue to change we can continue predict the direction of change for the endogenous variables, and this is not "useless."

1.2 a) Surprisingly high export sales mean that the demand for corn was higher than expected, at D_2 rather than D_1.



b) Dry weather would reduce the supply of corn, to S_2 rather than S_1.



c) Assuming the U.S. does not import corn, reduced production outside the U.S. would not impact U.S. corn market supply. El Nino would, however, cause demand for U.S. corn to shift out, the figure being the same as in part (a) above.

1.3 a) The production manager wants to minimize total costs $TC = P_E{*}E + P_L{*}L$.

b) The constraint is to produce $Q = 200$ units, so the manager must choose E and L so that $\sqrt{EL} = 200$.

c) The endogenous variables are E and L, because those are the variables over which the production manager has control. By contrast, the exogenous variables are Q, P_E, and P_L because the production manager has no control over their values and must take them as given.

1.4 In 2003, the initial equilibrium is at price P_1 and quantity Q_1. As national income increased, demand for aluminum shifted to the right, as depicted in the graph below by the shift from D_1 to D_2. The fall in the price of electricity shifted the supply curve to the right, from S_1 to S_2. Both shifts have the effect of increasing the equilibrium quantity, from Q_1 to Q_2. However, it is unclear whether price will rise or fall – if the demand shift dominates, price would rise; if the supply shift dominates, price would fall.

1.5 When the price of gasoline abroad goes up, the supply on the domestic market decreases. Firms are willing to supply less gasoline for the same price as before. At that price the domestic demand exceeds the supply and therefore the equilibrium price in the US has to increase. When this is followed by increase in the demand – consumers are willing to buy more gasoline then before – supply would again be smaller than the demand. Hence the equilibrium price of the gasoline would increase even more.

1.6

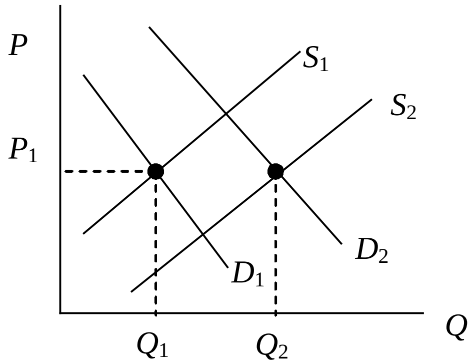
P	200	250	300	350	400
Q^d	500	450	400	350	300
Q^s	300	350	400	450	500

1.7

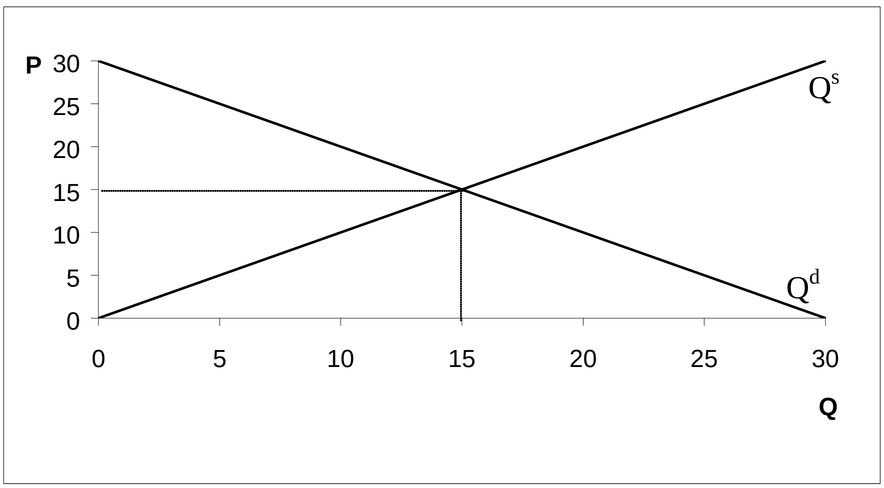
P	80	90	100	110	120
Q^d	680	640	600	560	520
Q^s	580	640	700	760	820

1.8 When the demand increases, more people are willing to buy sunglasses at the equilibrium price. Hence, the supply is insufficient to satisfy the demand and the equilibrium price has to go up. The table below confirms this.

P	80	90	100	110	120
Q^d	880	840	800	760	720
Q^s	580	640	700	760	820



1.9 a) Assuming $I = 20$ we have $Q^s = P$ and $Q^d = 30 - P$. Graphing these yields:



The equilibrium occurs at $P = 15$, $Q = 15$.

b) At a price of 18, $Q^s > Q^d$ implying an excess supply of wool. Because sellers will not be able to sell all of their wool at this price, they will need to reduce price to attract buyers. At the lower price, the suppliers will offer a lower quantity of output for sale, and consumers will want to purchase more.

c) At a price of 14, $Q^d > Q^s$, implying an excess demand for wool. Buyers will begin to bid up the price of wool until the new equilibrium is reached. At the higher price, the suppliers will offer a higher quantity of output for sale, and consumers will want to purchase less.

1.10 a) With $I_1 = 20$, we had $Q^s = P$ and $Q^d = 30 - P$, which implied an equilibrium price of 15.
With $I_2 = 24$, we have $Q^s = P$ and $Q^d = 34 - P$. Finding the point where $Q^s = Q^d$ yields

$$\begin{aligned} Q^s &= Q^d \\ P &= 34 - P \\ 2P &= 34 \\ P &= 17 \end{aligned}$$

Thus, a change in income of $\Delta I = 4$ yields a change in price of $\Delta P = 2$.

b) Plugging the result from part a) into the equation for Q^s reveals the new equilibrium quantity is $Q = 17$. Thus, a change in income of $\Delta I = 4$ yields a change in quantity of $\Delta Q = 2$.

1.11 a) Formulate each plan as a function of *V*, the number of videos to rent.

$$\begin{aligned} TC_A &= 3V \\ TC_B &= 50 + 2V \\ TC_C &= 150 + V \end{aligned}$$

Then we have

$$\begin{aligned} TC_A(75) &= 225 \\ TC_B(75) &= 200 \\ TC_C(75) &= 225 \end{aligned}$$

Plan B provides the lowest possible cost of $200 if you will purchase 75 videos.

b)

$$\begin{aligned} TC_A(125) &= 375 \\ TC_B(125) &= 300 \\ TC_C(125) &= 275 \end{aligned}$$

Plan C provides the lowest possible cost of $275 if you will purchase 125 videos.

c) In this case, the number of videos rented is exogenous because we are choosing a plan given a fixed level of videos.

d) Because you may choose the plan, the plans are endogenous. Note, though, that the details of the individual plans are exogenous.

e) Because you may choose the plan and the plans imply a total cost given a fixed level of videos, you are implicitly choosing the level of total expenditure. Total expenditures are therefore endogenous.

1.12 a) Now formulate each plan as a function of TC, the level of total expenditure on videos.

$$V_A = TC/3$$
$$V_B = TC/2 - 25$$
$$V_C = TC - 150$$

This gives

$$V_A(125) = 41.67$$
$$V_B(125) = 37.5$$
$$V_C(125) = -25$$

With plan A you could rent 41 movies, with plan B you could rent 37 movies, and with plan C you would not be able to rent any movies (because the membership fee exceeds your total budget). Plan A, therefore, will allow you to rent the most videos with a budget of $125.

b)

$$V_A(300) = 100$$
$$V_B(300) = 125$$
$$V_C(300) = 150$$

Now plan C offers the opportunity to rent the most videos.

c) The number of videos rented depends on the choice of plan. The number of videos rented is endogenous, then, since you can choose the plan.

d) As before, because you may choose any of the three plans, this choice is endogenous.

e) In this problem total expenditure is exogenous because we are choosing a plan given some fixed video rental budget.

1.13 a) The objective function is the number of new SUVs sold, which we can denote by $Q(F, G)$.

b) The constraint is that total spending must be less than or equal to $2million, or $TS \leq$ $2 million.

c) The constrained optimization problem is

$$\max_{(F,G)} Q(F,G) \quad \text{subject to } TS(F,G) \le \$2 \text{ million}$$

d) The following table shows all possible combinations of spending on football games and golf events:

(*F*, *G*)	New sales from *F*	New sales from *G*	Total new sales
(0, 2)	0	9	9
(0.5, 1.5)	10	8	18
(1, 1)	15	6	21
(1.5, 0.5)	19	8	27
(2, 0)	20	0	20

The table indicates that new SUV sales are maximized when (*F*, *G*) = (1.5, 0.5), that is, when the manufacturer spends $1.5 million on football and $0.5 million on golf.

1.14 When $R = 1$, the equilibrium occurs where $Q^d = Q^s$, or $100 - 4P^* = P^*$, or $P^* = 20$. The equilibrium quantity can be found from either supply or demand; using the latter we have $Q^* = 100 - 4(20) = 20$. When $R = 2$, $Q^d = Q^s$ implies $100 - 4P^* = 2P^*$ or $P^* = 16.67$ and $Q^* = 33.33$. Similarly, we can fill out the rest of the table:

R	1	2	4	8	16
Q^*	20	33.33	50	8.33	80
P^*	20	16.67	12.5	66.67	5

1.15 a)

L	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**
W	**90**	**80**	**70**	**60**	**50**	**40**	**30**	**20**	**10**
A	**900**	**1600**	**2100**	**2400**	**2500**	**2400**	**2100**	**1600**	**900**

L	20	30	40	50	60	70	80	90	100
W	100	90	80	70	60	50	40	30	20
A	2000	2700	3200	3500	3600	3500	3200	2700	2000

The length L of the optimally designed fence increases by 10 ($\Delta F / 4$).

b) As in b), the length L of the optimally designed fence increases by 10 ($\Delta F / 4$).

c) When $\Delta F = 40$, $\Delta A = 1100$. The area in this problem is an endogenous variable. The farmer may choose values for L and W and choices for these variables imply a value for A. So, implicitly, the farmer is choosing the area of the pen.

1.16 a) Positive analysis – this statement indicates what the consequences of the U.S. action will be, ignoring any value judgment when making the claim.

b) Positive analysis – again this statement simply indicates the consequences of a change in an exogenous variable on the market, ignoring any value judgments.

c) Normative analysis – here the author implies that there are two possible solutions to providing additional revenues for public schools and suggests, based on a value judgment, which of the alternatives is better.

d) Normative analysis – again the author makes a claim based upon his own value judgment, namely that telephone companies offering cable TV service would be a good thing.

e) Positive analysis – The author is making a positive statement. The author is predicting the effect of a policy change on the price in a market.

f) Normative analysis – here the author is making a prescriptive statement about what should be done. This is a value judgment about the policy to subsidize farmers.

g) Positive analysis – the author is making a prediction about what will happen if the tax on cigarettes is increased. While the claim may not be accurate, the statement is predictive and made without the author imposing any value judgments on the prediction.

Chapter 2

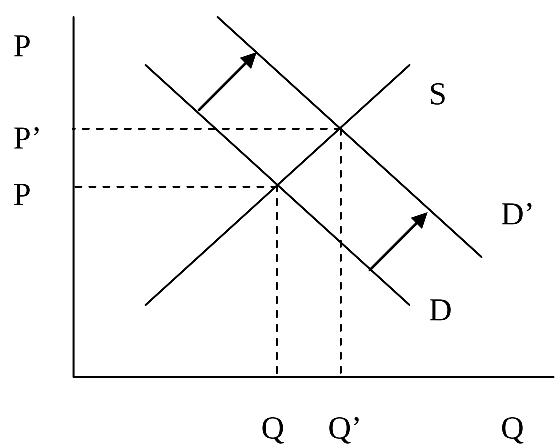
Supply and Demand Analysis

Solutions to Review Questions

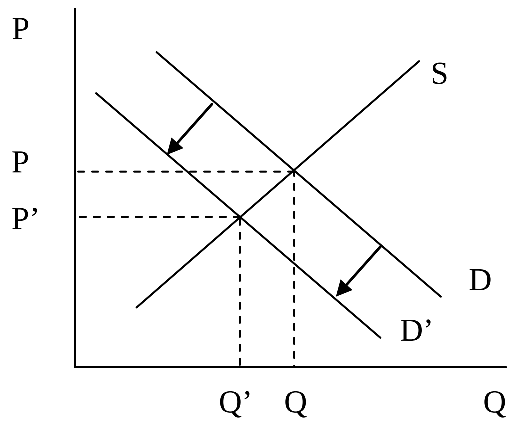
1. Excess demand occurs when price falls below the equilibrium price. In this situation, consumers are demanding a higher quantity than is being made available by suppliers. This creates pressure for the price to increase. As the price increases, quantity demanded will fall as quantity supplied increases returning the market to equilibrium.

   Excess supply occurs when price is above the equilibrium price. Suppliers have made available more units than consumers are willing to purchase at the high price. This creates pressure for the price to decrease. As the price decreases, the quantity demanded will go up while at the same time the quantity supplied will decrease, returning the market to equilibrium.

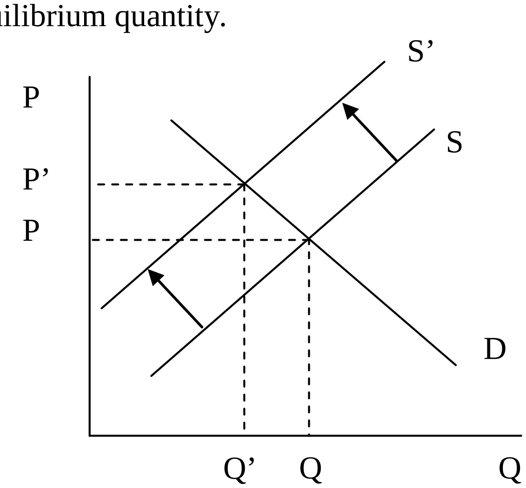
2. An increase in the price of a substitute, such as tea, will increase demand for coffee, raising the market equilibrium price and quantity.



a) This study will reduce demand for caffeine drinks, lowering the market equilibrium price and quantity.

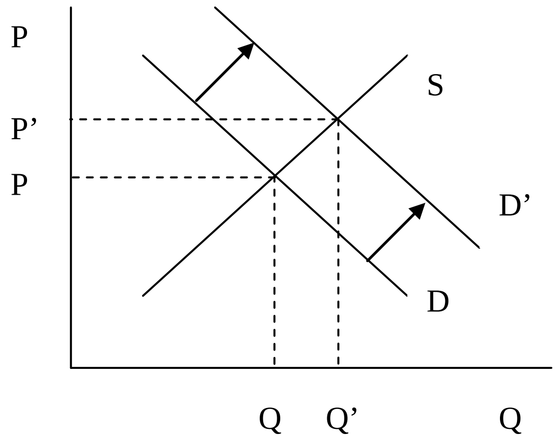


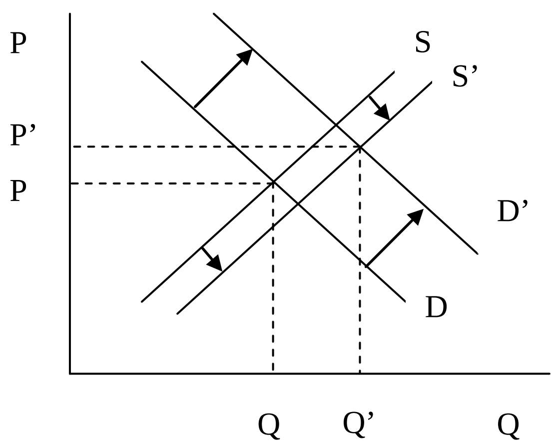
b) The frost will reduce supply raising the equilibrium price while lowering the equilibrium quantity.



c) Increasing the price of an input for a cup of coffee will reduce supply, increasing market price and reducing market quantity. This will result in the same figure as that for part c).

3. Any factor increasing demand and leaving the remainder of the market unchanged will increase both market price and quantity sold. If demand were to increase at the same time as supply changed, both market price and quantity sold could increase if the change in demand is large relative to the change in supply.





4. $$\varepsilon_{Q,P} = \frac{\%\Delta Q}{\%\Delta P} = \frac{-8}{10} = -0.80$$

5. The choke price is the price where $Q = 0$. Using the given demand curve we have

$$\begin{aligned} Q &= 50 - 100P \\ 0 &= 50 - 100P \\ 100P &= 50 \\ P &= \$0.50 \end{aligned}$$

6. Speedboats could probably be categorized as a luxury item whereas light bulbs are more likely categorized as a necessity. For the necessity, the change in quantity demanded will be relatively small for any percent change in price. The change in quantity demanded may be quite large, however, for a luxury item. Since the percent change in quantity demanded is likely higher for the luxury item for any given percent change in price, the elasticity of demand would be less (more negative).

7. Because business travelers receive reimbursement for expenses, they will probably be less sensitive to price changes than the vacation traveler who pays out of her own pocket. This implies the price elasticity for vacationers would be less (more negative) than for business travelers.

8. If the prices for a particular product, such as Dannon, within a product category changes (say it increases) then it is easy for a consumer to switch to another brand, implying a relatively high percent change in quantity demanded for the product. On the other hand, if prices for the entire product category change, substitutes are not as easily found and the percent change in quantity demanded for the category will be relatively lower. This implies the elasticity for the entire product category will be higher (less negative) than the elasticity for a single product.

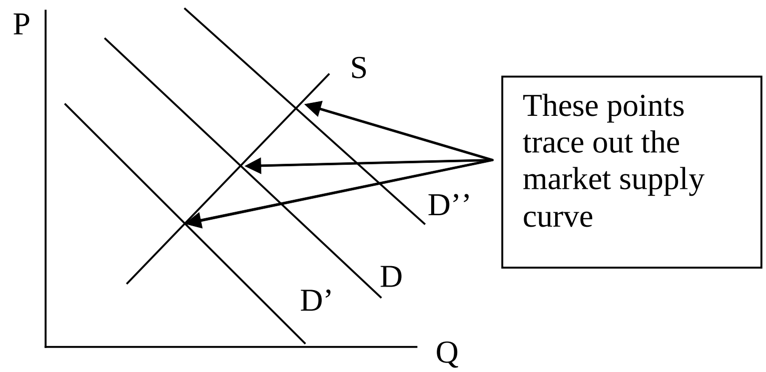
9. When the cross-price elasticity is positive we have

$$\frac{\%\Delta Q_A}{\%\Delta P_B} > 0$$

Either a) both Q_A and P_B increased or b) they both decreased. Since they are moving in the same direction, the product must be substitutes. Take coffee and tea for example; if the price of tea increases, the quantity of coffee demanded will increase.

When the cross-price elasticity is negative, Q_A and P_B are moving in the opposite direction, implying the products are complements. Take coffee and cream for example; if the price of cream increases, the quantity of coffee demanded will decrease.

10.



As the demand curve shifts, the market will reach a new equilibrium. Each new equilibrium occurs at a new price and quantity. These price/quantity combinations trace out the market supply curve. Thus, in order to identify the market supply curve one needs to observe shifts in the demand curve.

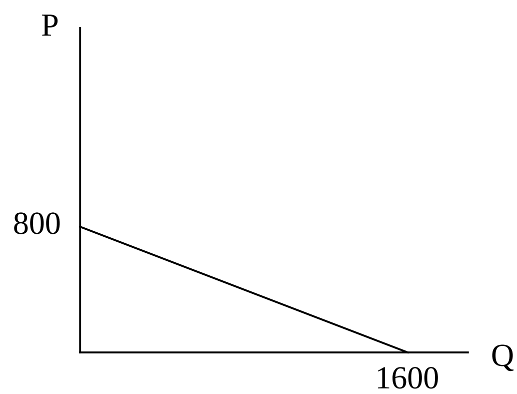
Solutions to Problems

2.1

a) When the price of nuts goes up, quantity demanded falls for all levels of price (demand shifts left). Beer and nuts are demand complements.

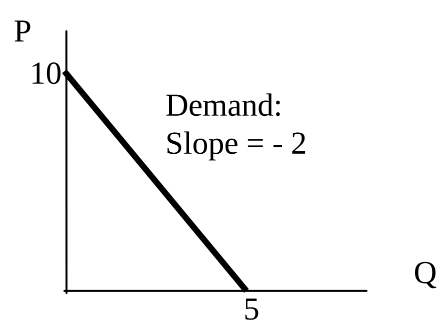
b) When income rises, quantity demanded increases for all levels of price (demand shifts rightward).

c)



2.2

a) The graph is shown below:



b) We know that the value of the price elasticity of demand is given by

$$\varepsilon_{Q,P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = -b\frac{P}{Q}$$

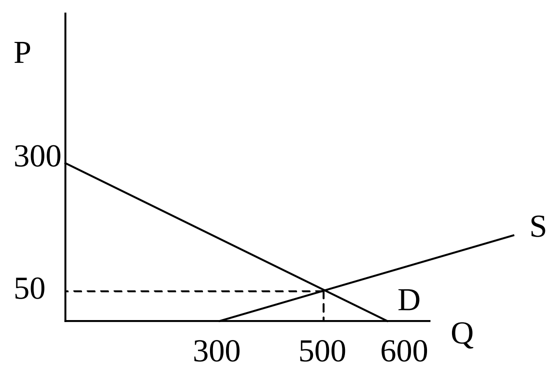
Here, –b = –1/2. For demand to be unitary elastic it must be that

$$-\tfrac{1}{2}\left[\frac{P}{5-\frac{P}{2}}\right]=-1$$

which implies that P = 5.

2.3

a)



b)

$$600-2P=300+4P$$
$$300=6P$$
$$50=P$$

Plugging $P=50$ back into either the supply or demand equation yields $Q=500$.

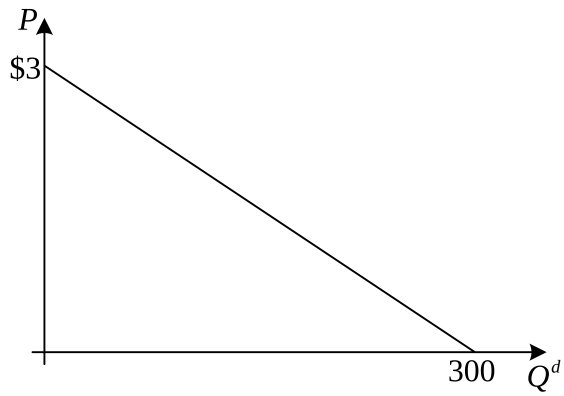
2.4.

P	0.10	0.45	0.50	0.55	2.50
Q^d	290	255	250	245	50
$\varepsilon_{Q,P}$	–0.035	–0.176	–0.2	–0.225	–5

We can find elasticities of demand using the following formula

$$\varepsilon_{Q,P}=\frac{\Delta Q^d}{\Delta P}\frac{P}{Q^d}=-100\cdot\frac{P}{300-100\cdot P}=\frac{P}{P-3}.$$

This demand curve is linear.



Observe that for price $1.50 the elasticity of demand is equal to

$$\varepsilon_{Q,P} = \frac{1.5}{1.5-3} = -1.$$

For all prices below $1.50, the demand is inelastic, while for all prices above $1.50, the demand is elastic.

2.5. Using the data from the problem we can graph the demand curve. The slope of the demand curve is equal to

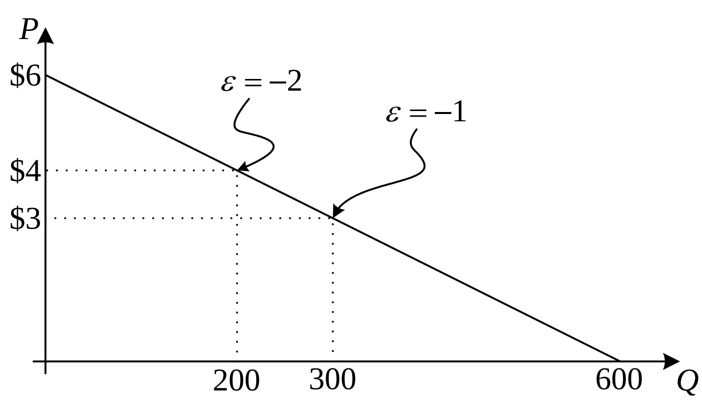
$$\Delta P / \Delta Q = -1/100$$

So the equation of the demand curve is P = A – 0.01Q.

We can find the vertical intercept A substituting P = 3 and Q = 300.

3 = A – 0.01(300), so A = 6. The vertical intercept (choke price) is P = $6.

The equation of the demand curve is then P = 6 – 0.01Q.



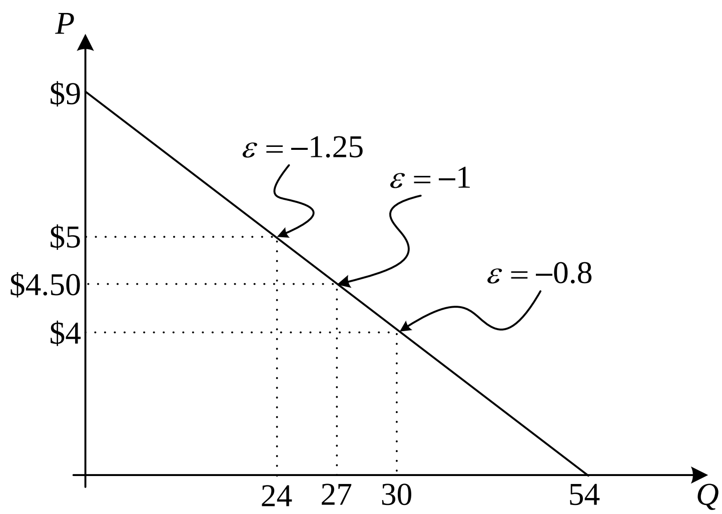
Elasticity of demand can be computed using formula

$$E_{Q,P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = -100\frac{P}{Q} = -100\frac{6-0.01Q}{Q}$$

When the elasticity is -1, Q = 300 and P = \$3.

Thus demand is unitary elastic at a price P = \$3.

2.6. The demand for apple pies is $Q^d = 54 - 6P$.



To find the equation of the demand curve, observe that when she drops the price by \$0.50, she sells 3 more pies. So, movement along the demand occurs so that

$$\Delta Q / \Delta P = -3/0.5 = -6$$

The demand curve then has the form $Q^d = A - 6P$, where A is a constant. We can determine the value of A using any one of the three data points on the demand curve. For example, if we use the point P = 5 and Q = 24, we see that 24 = A – 6(5), so that A = 54. So the demand curve can be described by the equation $Q^d = 54 - 6P$.

To find elasticity of demand at any point on the demand curve, we use formula

$$E_{Q,P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = -6\frac{P}{Q}$$

2.7 a) Since the price is being bid up above the official price, quantity demanded must exceed quantity supplied at the official price. This is a situation of excess demand and the official price must be below the equilibrium price.

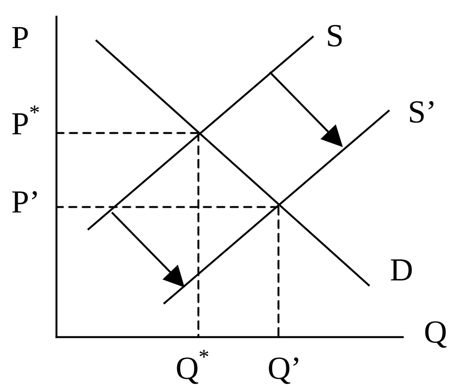
b) Lowering the official price would increase the amount of excess demand, but would have no effect on the demand or supply curves. Thus the equilibrium price would remain unchanged.

2.8 This could occur as a result of the demand curve shifting to the right, increasing both equilibrium price and quantity. This would not contradict what was learned regarding downward sloping demand curves.

2.9 The law of demand states that, holding other factors fixed, there is an inverse relationship between price and quantity demanded, i.e. that an increase in price decreases quantity and vice versa. If a good has a positive price elasticity of demand, it must be that an increase in the price of that good leads to an increase in the quantity demanded. Therefore, such a good violates the law of demand.

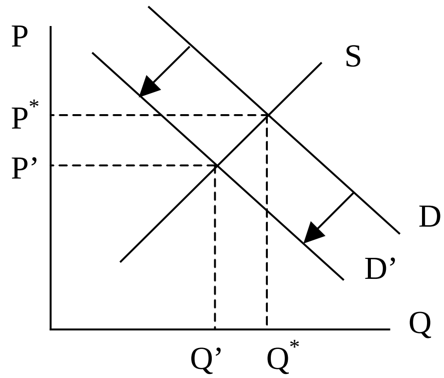
2.10

a)



An increase in rainfall will increase supply, lowering the equilibrium price and increasing the equilibrium quantity.

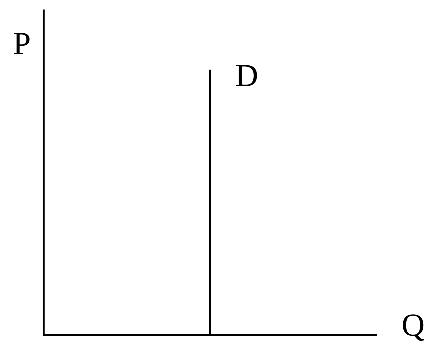
b)



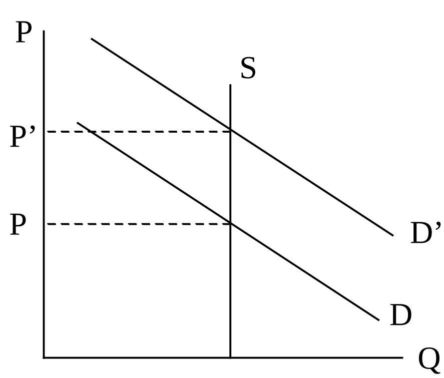
A decrease in disposable income will reduce demand, shifting the demand schedule left, reducing both the equilibrium price and quantity.

2.11

a) A perfectly inelastic demand curve will be vertical.



b) The renewed interest will shift demand to the right, raising the equilibrium price. Since supply is perfectly inelastic (and therefore vertical) there will be no change in the quantity supplied; the quantity is fixed.



2.12

a)

$$Q = 350 - 7P$$
$$7P = 350 - Q$$
$$P = 50 - \tfrac{1}{7}Q$$

b) The choke price occurs at the point where $Q = 0$. Setting $Q = 0$ in the inverse demand equation above yields $P = 50$.

c) At $P = 50$, the choke price, the elasticity will approach negative infinity.

2.13 Recall that for an elastic good, a higher price charged by the firm leads to a decrease in total revenue. Therefore, the firm should expect a level of output such that its revenue at a price of $102 is less than $70,000. Only if the output level is 400 or 600 is this possible (102*400 = $40,800) and (102*600 = $61,200). At the other quantities the revenue would rise.

2.14 Gina's expenditure on ice-cream is *P***Q*, where *P* is the price and *Q* is the number of units of ice cream that she buys. We know that *P***Q* increases as *P* decreases which can only mean that *Q* increases at a faster rate than the rate at which *P* decreases. This is equivalent to saying that demand is very sensitive to price changes, or that her demand for ice cream is quite elastic ($\varepsilon_{Q,P} < -1$). More generally, recall that when price and total revenue (*P***Q*) move in opposite directions, it is because demand is elastic over that price range.

2.15

a) More elastic in the long run as the theatre owner can increase space or add another screen if the price remains high, but cannot easily adjust the number of seats at short notice.

b) More elastic in the short run as people can be relatively flexible about when to undergo an eye exam, but in the long run the need for eye exams is fixed.

c) More elastic in the long run. Cigarettes tend to be addictive and so smokers are less likely to be able to reduce their demand in response to short term fluctuations in price. However if the price remains high for a long time they will consider giving up the habit as it becomes too expensive.

2.16

a) Substituting the values of *R* and *T*, we get

$$Demand: Q^d = 70 - 2P$$
$$Supply: Q^s = -14 + 5P$$

In equilibrium, 70 – 2*P* = –14 + 5*P*, which implies that *P* = 12. Substituting this value back, *Q* = 46.

b) Elasticity of Demand = –2(12/46), or –0.52. Elasticity of Supply = 5(12/46) = 1.30.

c) $\varepsilon_{golf,titanium} = -2(\frac{10}{46}) = -0.43$. The negative sign indicates that titanium and golf balls are complements, i.e., when the price of titanium goes up the demand for golf balls decreases.

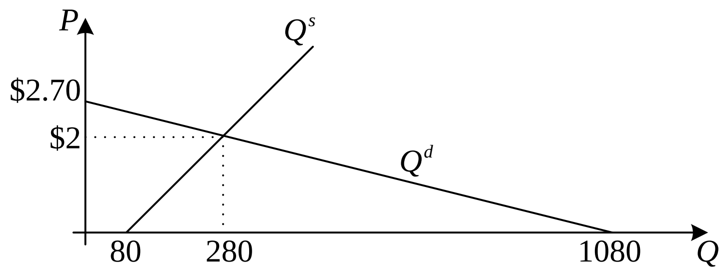
2.17

a) When the price of gasoline goes up, it becomes more expensive to drive a private automobile; because private automobiles and taxis are substitutes, the demand for taxi service should increase (shift to the right). On the other hand, when the average speed of a trip by automobile increases, commuters are more likely to use their cars instead of public transportation; the demand for taxi service should shift to the left. On the supply side, a higher price of gasoline increases to cost of providing taxi service; the supply curve for taxi service should shift to the left.

b) Substituting *G* = 4 and *E* = 30 into equations for the supply and demand curves we have

$$Q^d = 1080 - 400 \cdot P,$$
$$Q^s = 80 + 100 \cdot P.$$

Solving equation $Q^d = Q^s$ we have *P* = 2, *Q* = 280. Supply and demand curves are graphed below.



c) In equilibrium $Q^d = Q^s$. When we

$$P = \frac{1}{125}\left(200 - E + 20 \cdot G\right).$$

The equilibrium taxi fare goes up as gasoline price increases and goes down when it private automobiles can travel faster.

2.18

a) Since the two goods are rather close substitutes for each other, you would expect that the demand for Tylenol would go up if the price of Advil increases and vice versa. Therefore, the cross price elasticity will be positive.

b) Similar to part (a). Although VCRs and DVD players are not very close substitutes, if the price of VCRs were to go up substantially, potential buyers would probably decide to pay a little bit more and get the higher-end DVD player. Similarly if the latter becomes expensive, some consumers will not be able to afford it and will switch to the VCR instead. The elasticity will be positive.

c) Since the two usually go together, a sharp increase in the price of one will lead to a decline in the demand for the other, and the cross-price elasticity will be negative.

2.19

a) Assuming red and black umbrellas are substitutes, we would expect the cross-price elasticity of demand to be positive.

b) Coca-cola and Pepsi are substitutes. We would expect the cross-price elasticity of demand to be positive.

c) Grape jelly and peanut butter are typically complements (people want both on their sandwiches!). We would expect the cross-price elasticity of demand to be negative.

d) Chocolate chip cookies and milk are typically complements (people want to consume them together). We would expect the cross-price elasticity of demand to be negative.

e) Computers and software are complements (consumers want to use them together). We would expect the cross-price elasticity of demand to be negative.

2.20

a)
$$Q_U^d = 10000 - 100(300) + 99(300)$$
$$Q_U^d = 9700$$

Using $P_U = 300$ and $Q_U^d = 9700$ gives

$$\varepsilon_{Q,P} = -100\left(\frac{300}{9700}\right) = -3.09$$

b) Market demand is given by $Q^d = Q_U^d + Q_A^d$. Assuming the airlines charge the same price we have

$$Q^d = 10000 - 100P_U + 99P_A + 10000 - 100P_A + 99P_U$$
$$Q^d = 20000 - 100P + 99P - 100P + 99P$$
$$Q^d = 20000 - 2P$$

When $P = 300$, $Q^d = 19400$. This implies an elasticity equal to

$$\varepsilon_{Q,P} = -2\left(\frac{300}{19400}\right) = -.0309$$

2.21 We know that along a linear demand curve

$$\varepsilon_{Q,P} = -b\left(\frac{P}{Q}\right)$$

Using the given information this implies

$$-.5 = -b\left(\frac{.05}{10,000,000}\right)$$
$$b = 100,000,000$$

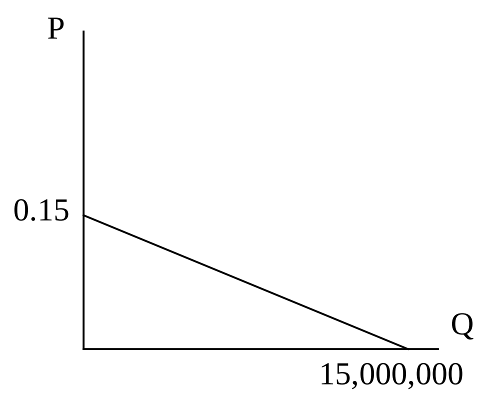
Plugging this result into a demand equation using the known price and quantity then implies

$$Q^d = A - bP$$
$$10,000,000 = A - 100,000,000(.05)$$
$$A = 15,000,000$$

So a demand equation that fits this information is given by

$$Q^d = 15,000,000 - 100,000,000P$$

Graphically, the demand curve looks like



2.22

a) In case of the linear demand $Q = A - bP$, we know that $\varepsilon_{Q,P} = -b\frac{P}{Q} = -1$

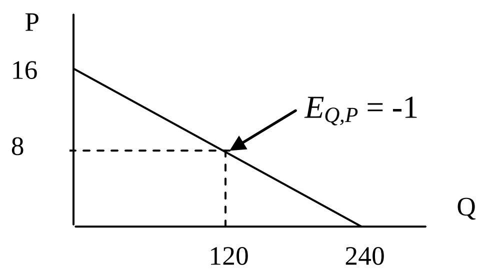
Using the values of P and Q given in the problem we have

$$-1 = -b\frac{8}{120} \quad \Rightarrow \quad b = \frac{120}{8} = 15.$$

Now we can solve for the second parameter of the linear demand curve

$$120 = a - 15(8) \quad \Rightarrow \quad a = 240.$$

Hence the linear demand curve is given by equation $Q^d = 240 - 15P$.



b) There exist several linear demand curves for which the demand is equal to 120 at price of \$8. Information about elasticity of demand lets us determine exactly one of those. More formally, we need second equation to solve for both parameters of the linear demand curve.

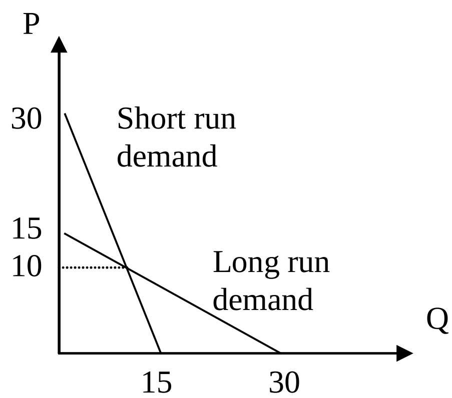
2.23

a) Butter has some reasonably close substitutes such as margarine or cheese, while eggs have no immediate substitutes. Therefore we would expect the demand for butter to be more elastic.

b) Vacation trips are sensitive to price because leisure travelers can be relatively flexible about when to fly. Your congressman, however, has fixed dates on which to be in Washington and would be prepared to pay more to ensure that he flies on the day of his choosing. Therefore, demand for vacation trips is likely to be more elastic (i.e. the price elasticity will be more negative) than the demand for trips by your congressman.

c) As discussed in the chapter, market level elasticities tend to be lower (less negative) than the elasticity of a particular brand. Thus, expect the demand for Tropicana to be more elastic than the demand for generic orange juice.

2.24 First, consider each demand curve in its "inverse" form: long run demand is $P = 15 - 0.5Q$, and short run demand is $P = 30 - 2Q$. Thus, the slope of the long run demand is –0.5, which is closer to zero than that of the short run demand, –2. Thus, long run demand is flatter. Second, consider the graph below:



Again, long run demand is flatter and thus more sensitive to changes in price. Consider, for instance a price of $10. Quantity demanded is equal in both the long and short runs at $P = 10$. However, consider increasing the price to, say, $15. Although this will reduce quantity demanded in the short run by a little, it would reduce quantity demanded all the way to zero in the long run.

2.25 The scare in 1999 would shift demand to the left, identifying a second point on the supply curve. The information implies that price fell $0.50 while quantity fell 1.5 million. This implies

$$b = \frac{-.5}{-.15} = \frac{1}{3}$$

Using a linear supply curve we then have

$$P = a + \frac{1}{3}Q^s$$
$$5 = a + \frac{1}{3}(4)$$
$$a = \frac{11}{3}$$

Finally, plugging these values for a and b into the supply equation results in

$$P = \frac{11}{3} + \frac{1}{3}Q^s$$
$$3P = 11 + Q^s$$
$$Q^s = -11 + 3P$$

The floods in 2000 will reduce supply. The shift in supply will identify a second point along the demand curve. Because the scare of 1999 is over, assume that demand has returned to its 1998 state. The change in price and quantity in 2000 imply that price increased \$3.00 and that quantity fell 0.5 million.

Performing the same exercise as above we have

$$-b = \frac{3}{-0.5} = -6$$

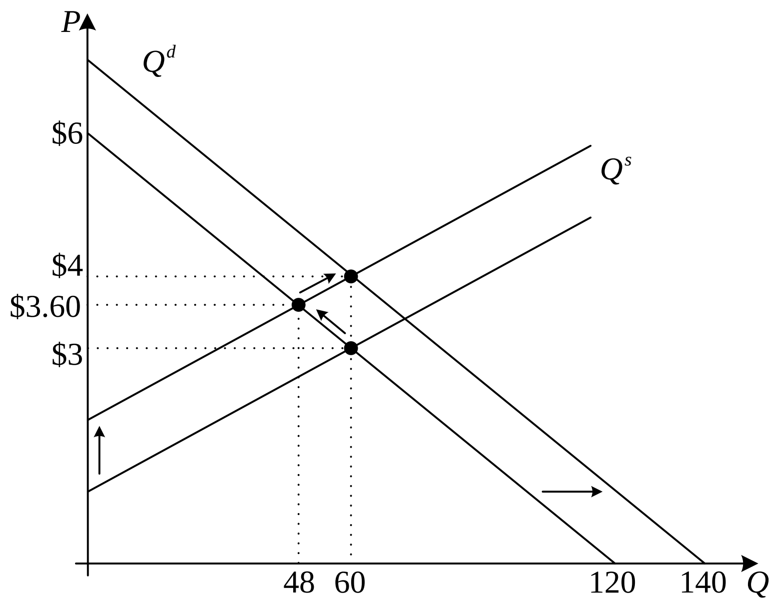
Using the 1998 price and quantity information along with this result yields

$$\begin{aligned} P &= a - bQ^d \\ 5 &= a - 6(4) \\ a &= 29 \end{aligned}$$

Finally, plugging these values for a and b into a linear demand curve results in

$$\begin{aligned} P &= 29 - 6Q^d \\ 6Q^d &= 29 - P \\ Q^d &= \frac{29}{6} - \frac{1}{6}P \end{aligned}$$

2.26 The equilibrium price in January is equal to $P = 3$ and equilibrium quantity is equal to $Q = 60$. We find equilibrium price by solving $Q^s = Q^d$, which is $30 \cdot P - 30 = 120 - 20 \cdot P$. When we have equilibrium price we can substitute it to either the demand function or supply function, since they have to give the same quantity at that price, and obtain equilibrium quantity equal to $Q = 60$. After the supply decreases in February, new equilibrium price is per mile is equal to $P =$ \$3.60, while the demanded quantity is equal to $Q = 48$. When the demand goes up in March, the quantity in equilibrium is the same as in January but price is even higher and equal to $P =$ \$4. All those changes are illustrated on the graph below.


P
Q^d
$6
Q^s
$4
$3.60
$3
48
60
120
140
Q

Chapter 3

Preferences and Utility

Solutions to Review Questions

1. A basket is a collection of goods and services that an individual might consume.

2. By requiring preferences to be complete, economists are ensuring that consumers will not respond indecisively when asked to compare two baskets. A consumer will always be able to state that either A is preferred B, B is preferred to A, or that she is indifferent between A and B.

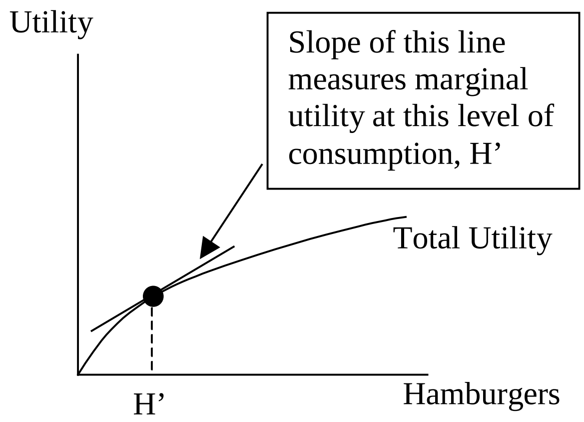
3. Unfortunately, it is impossible to say definitively whether D, H, or J is the least preferred basket. Since more is better, baskets to the northeast are more preferred and baskets to the southwest are less preferred. In this case, H has more clothing but less food than D, while J has more food but less clothing than D. Without more information regarding how the consumer feels about clothing relative to food, we cannot state which of these baskets is the least preferred.

4. If a consumer states that A is preferred to B and that B is preferred to C, but then states that C is preferred to A, she will be violating the assumption of transitivity. The third statement is inconsistent with the first two.

5. If more is better, then the marginal utility of a good must be positive. That is, total utility must increase if the consumer consumes more of the good.

6. An ordinal ranking simply orders the baskets, but does not give any indication as to how much better one basket is when compared with another; only that one is better. A cardinal ranking not only orders the baskets, but also provides information regarding the intensity of the preferences. For example, a cardinal ranking might indicate that one basket is twice as good as another basket.

7.



Marginal utility would be measured as the slope of a line tangent to the total utility curve in the graph above.

8. The two cannot be plotted on the same graph because utility and marginal utility are not measured in the same dimensions. Total utility has the dimension U, while marginal utility has the dimension of utility per unit, or $\Delta U / \Delta y$ where y is the number of units purchased.

9.

a) Yes, we can determine the MRS as

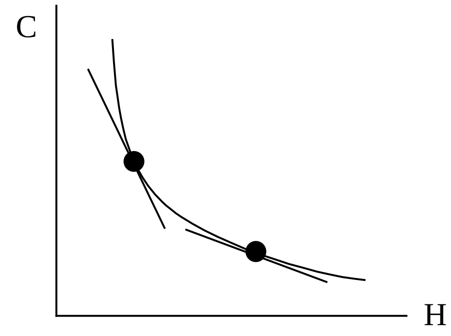
$$MRS_{h,f} = \frac{MU_h}{MU_f}$$

b) No, when we know the MRS, all we know is the ratio of the marginal utilities. We cannot “undo” that ratio to determine the individual marginal utilities. For example, if we know that $MRS_{h,f} = 5$, it could be the case that $MU_h = 5$ and $MU_f = 1$, but it could equivalently be the case that $MU_h = 10$ and $MU_f = 2$. Clearly, there are countless combinations of MU_h and MU_f that could lead to some particular value of $MRS_{h,f}$, and we have no way of inferring which is the right one.

10.

a) $MRS_{H,C} = \frac{MU_H}{MU_C}$

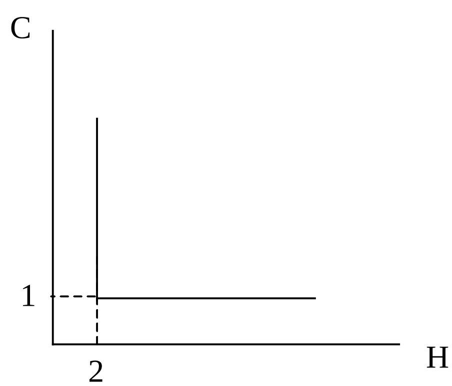
b)



The indifference curve in this case will be convex toward the origin. The marginal rate of substitution is measured as the absolute value of the slope of a line tangent to the indifference curve. As can be seen in the graph above, this slope becomes less negative as we move down the indifference curve, implying a diminishing MRS.

c) If the MRS was constant, this would imply that at any consumption level the consumer would be willing to trade a fixed amount of one good for a fixed amount of the other. This occurs with perfect substitutes.

d) If the consumer wishes to always consume goods in a fixed ratio, then the goods are perfect complements. In this case, the indifference curves will be L-shaped.



11. Marginal utility is defined as the change in total utility relative to a change in consumption for a particular good. In order to accurately measure the change in total utility, the levels of the other goods would need to be held constant. If they were not, the change in total utility would occur as a result of multiple goods changing and it would be impossible to determine what portion of the change in total utility should be assigned to each good.

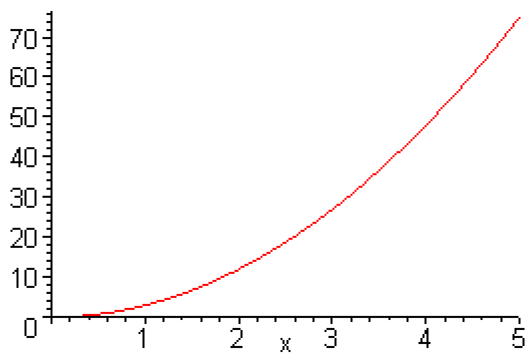
Solutions to Problems

3.1 By plugging in ever higher numerical values of x and ever higher numerical values of y, it can be verified that U increases whenever x or y increases.

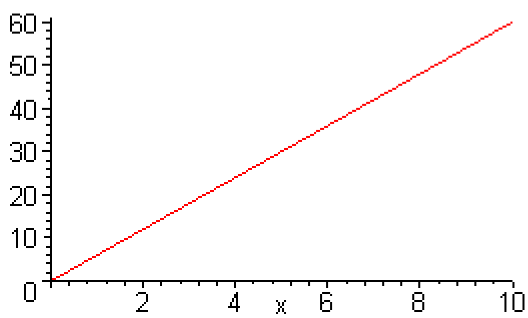
3.2 The two graphs are shown below. It can be seen from both graphs that this function does not satisfy the law of diminishing marginal utility. The first figure shows that utility increases with x, and moreover, that it increases at an increasing rate. For example, an increase in x from 2 to 3, increases utility from 12 to 27 (an increase of 15), while an increase in x from 3 to 4 induces an increase in utility from 27 to 48 (an increase of 21).

This fact is easier to see in the second figure. The marginal utility is an increasing function of x. Higher values of x imply a greater marginal utility. Therefore this function exhibits *increasing* marginal utility.

$U(x) = 3x^2$



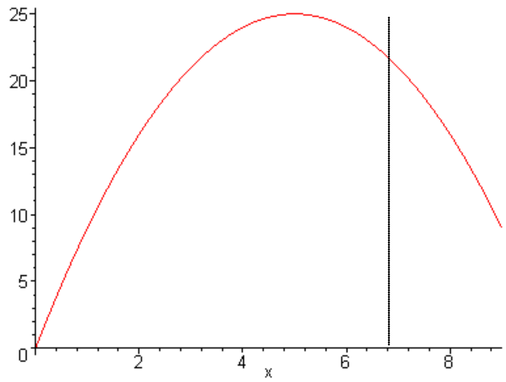
$MU_x = 6x$



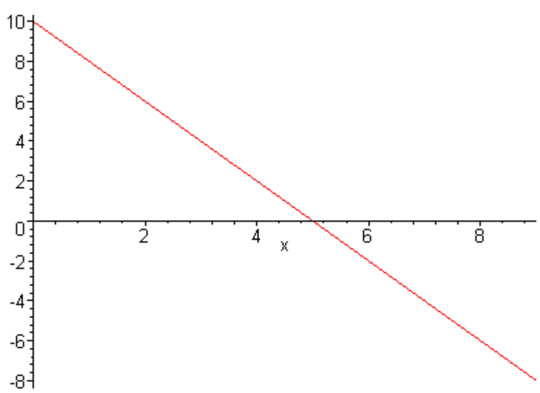
3.3 The first figure below shows Jimmy's utility function for hotdogs. You can see that the point at which $H = 5$ corresponds to the flat portion of the utility function, i.e. the point at which the marginal utility of hotdogs is zero, and beyond which the marginal utility is negative. Alternatively using the second graph it is clear that the point $H = 5$ is when the marginal utility intersects the x-axis, and beyond which it is negative. Both graphs tell you that to maximize his utility Jimmy should only consume 5 hotdogs and not more.

To answer this question algebraically, you should first recognize from the marginal utility function that Jimmy has a diminishing marginal utility of hotdogs. Therefore the point at which he should stop consuming hotdogs is the point at which $MU_H = 0$, or $10 - 2H = 0$. This gives $H = 5$.

$U(H) = 10H - H^2$



$MU_H = 10 - 2H$



3.4 a) Since U increases whenever x or y increases, more of each good is better. This is also confirmed by noting that MU_x and MU_y are both positive for any positive values of x and y.

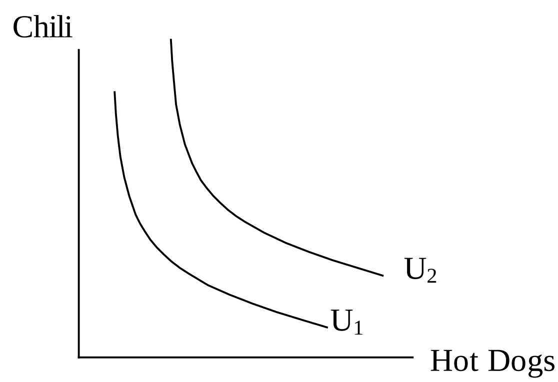
b) Since $MU_x = y/2\sqrt{x}$, as x increases (holding y constant), MU_x falls. Therefore the marginal utility of x is diminishing. However, $MU_y = \sqrt{x}$. As y increases,

MU_y does not change. Therefore the preferences exhibit a constant, not diminishing, marginal utility of *y*.

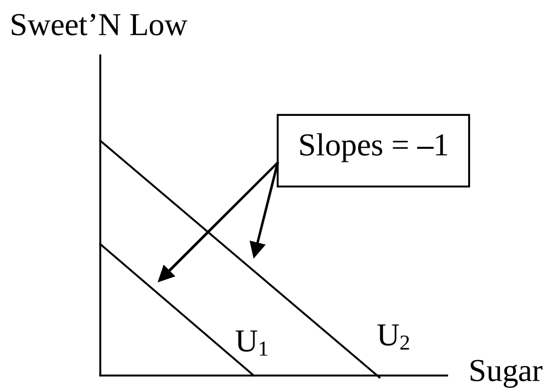
3.5 a) By plugging in ever higher numerical values of *x* and ever higher numerical values of *y*, it can be verified that Carlos' utility goes up whenever *x* or *y* increases.

b) First consider the marginal utility of *x*, MU_x. Since x does not appear anywhere in the formula for MU_x, MU_x is independent of *x*. Hence, the marginal utility of movies is independent of the number of movies seen, and so the marginal utility of movies does not decrease as the number of movies increases. Next consider the marginal utility of *y*, MU_y. Notice that MU_y is an increasing function of *y*. Hence, the marginal utility of operas does not decrease in the number of operas seen. In this case, neither good, movies or operas, exhibits diminishing marginal utility.

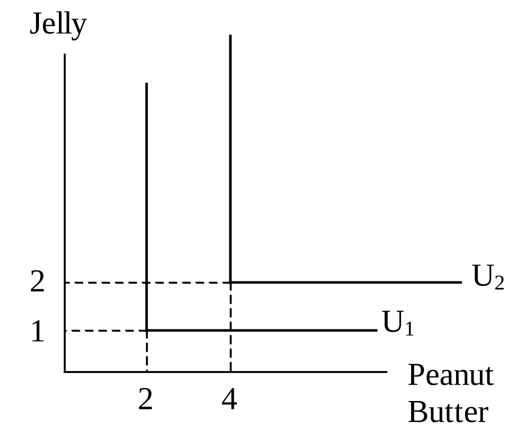
3.6 a)



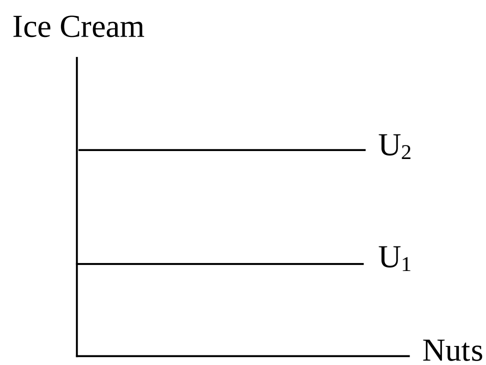
b)



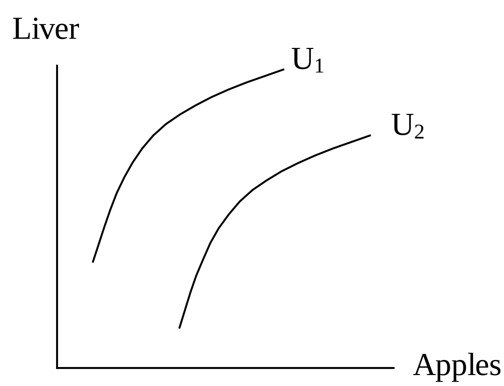
c)



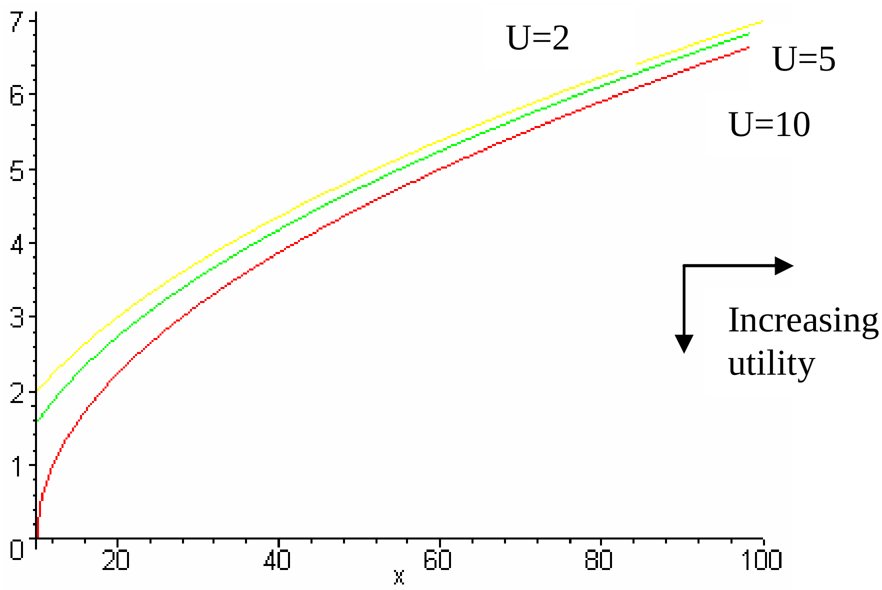
d)



e)



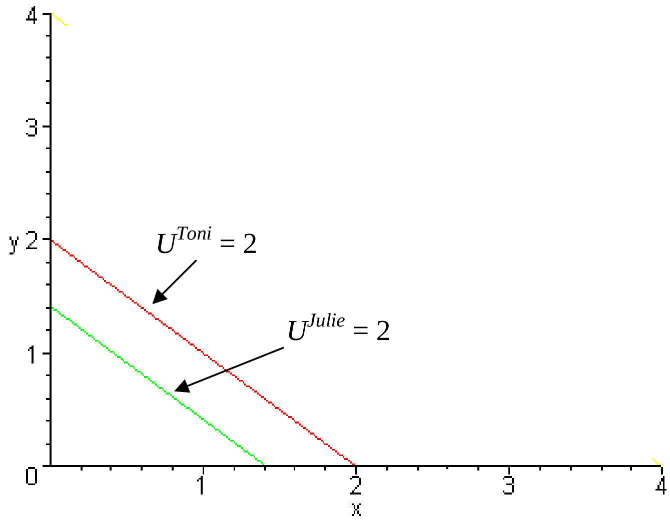
3.7 a) Three indifference curves corresponding to U = 2, 5 and 10 are shown in the figure. The direction of increasing utility is down and to the right.



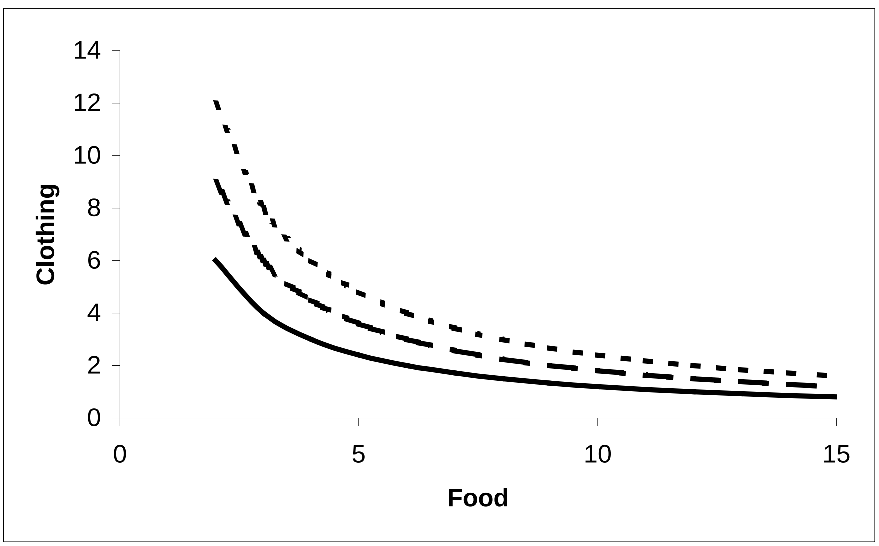
b) Note the negative sign for MU_y. This means that an increase in the consumption of *y* would *decrease* the consumer's utility. This violates the basic assumption that more is better for this utility function.

3.8 This utility function does have the property of diminishing $MRS_{x,y}$. One way to verify this is to graph several indifference curves. Another way to tell is to use algebra. Recall that $MRS_{x,y} = \frac{MU_x}{MU_y}$. Applying that general formula to this case gives us $MRS_{x,y} = 2\sqrt{y}$. As we move "down" the indifference curve, *x* increases and *y* decreases. As *y* decreases, $2\sqrt{y}$ decreases. Thus, $MRS_{x,y}$ decreases.

3.9 Indifference curves corresponding to $U = 2$ are shown for both Julie and Toni in the graph below. Notice that the indifference curves are parallel everywhere – indeed, $MRS_{x,y} = 1$ for both Julie and Toni, for all values of *x* and *y*. Toni's indifference curve for the utility level $U^{Toni} = 2$ is the same as Julie's indifference curve for the utility level $U^{Julie} = 4$. So whenever Julie ranks bundle A higher than bundle B, Toni would have the same ranking, and vice-versa. So Julie and Toni will have the same ordinal ranking of bundles of *x* and *y*. (Julie will associate each bundle with a higher utility *level* than Toni will, but that is a *cardinal* ranking.)



3.10 a)

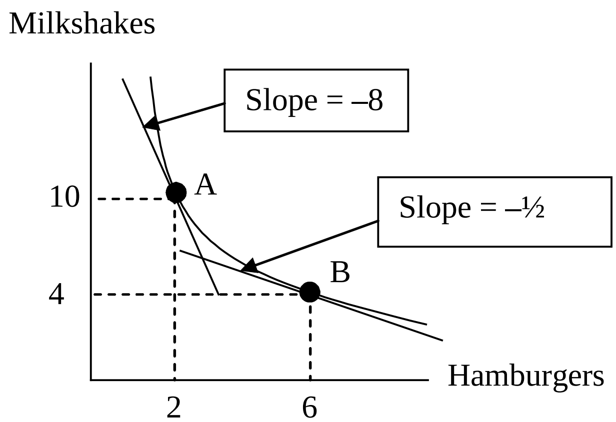


b) Yes, since the indifference curves are bowed in toward the origin we know that $MRS_{F,C}$ declines as F increases and C decreases along an indifference curve.

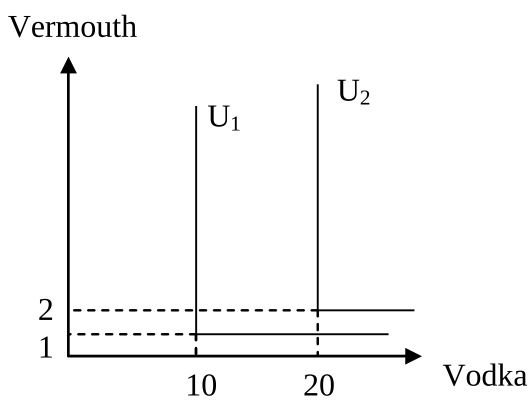
c) $$MRS_{F,C} = \frac{MU_F}{MU_C} = \frac{C}{F}$$

When $F = 2$ and $C = 6$, $MRS_{F,C} = 3$. The slope of the indifference curve is –3. When $F = 4$ and $C = 3$, $MRS_{F,C} = 0.75$, so the slope of the indifference curve is –0.75. Since the marginal rate of substitution falls as F increases and C decreases, she has a diminishing marginal rate of substitution.

3.11

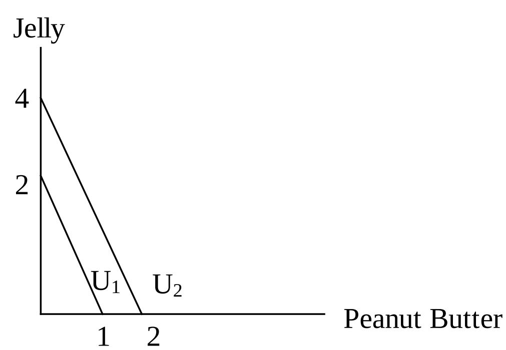


3.12

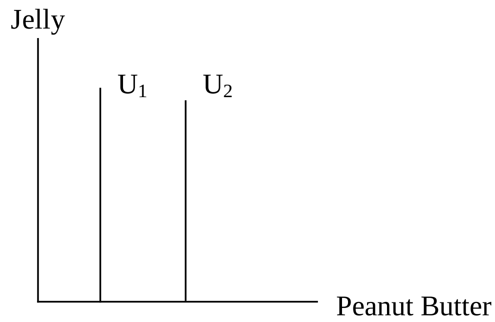


3.13 In the following pictures, $U_2 > U_1$.

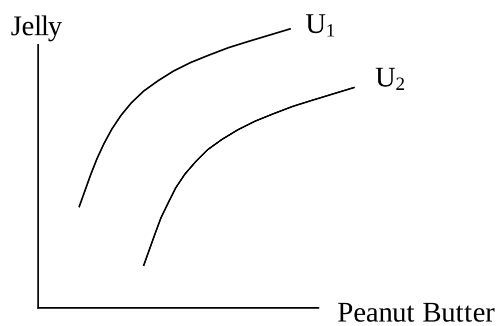
a)



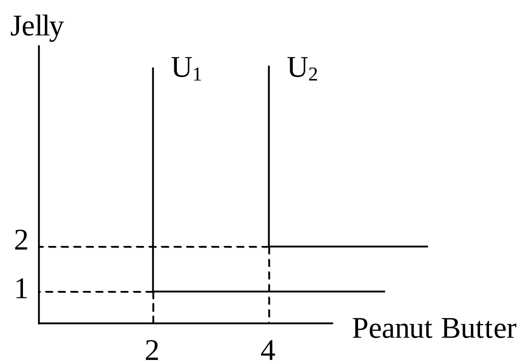
b)


Jelly
U1
U2
Peanut Butter

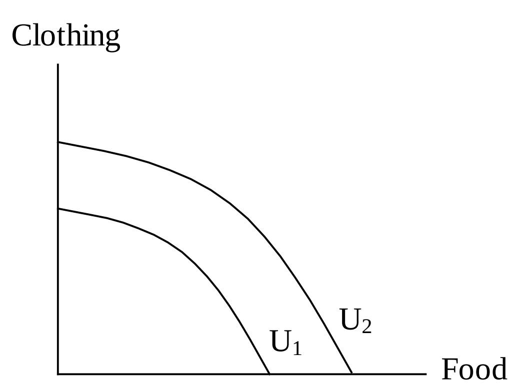
c)


Jelly
U1
U2
Peanut Butter

d)


Jelly
U1
U2
2
1
Peanut Butter
2
4

3.14


Clothing
U2
U1
Food

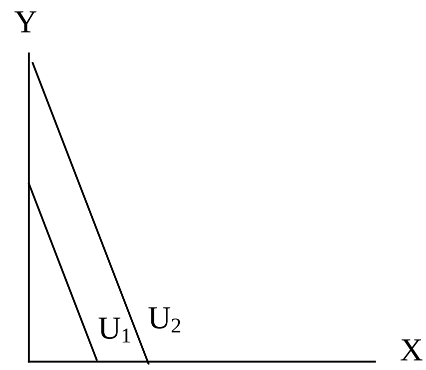
3.15 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x remains constant at 3 for all values of *x*.

c) $MRS_{x,y} = 3$

d) The $MRS_{x,y}$ remains constant moving along the indifference curve.

e & f) See figure below



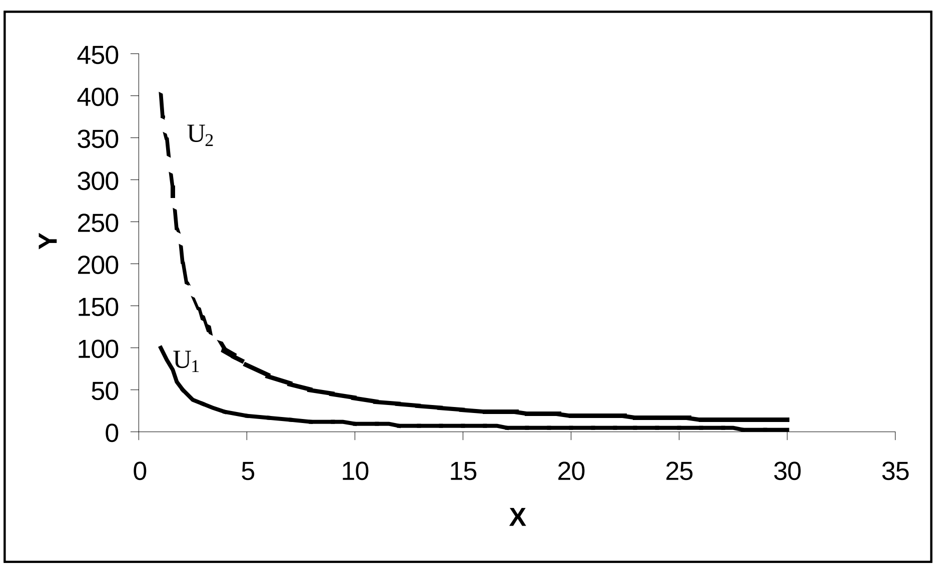
3.16 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x diminishes as the consumer buys more x.

c) $MRS_{x,y} = \left(\frac{\sqrt{y}}{2\sqrt{x}}\right)\left(\frac{2\sqrt{y}}{\sqrt{x}}\right) = \frac{y}{x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. The indifference curves will not intersect either axis.



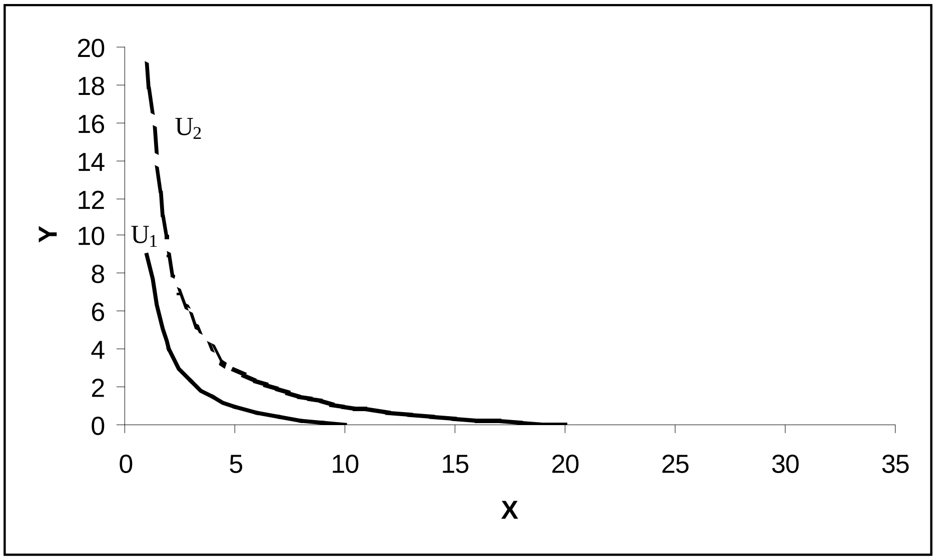
3.17 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x remains constant as the consumer buys more *x*.

c) $MRS_{x,y} = \frac{y+1}{x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. The indifference curves intersect the *x*-axis, since it is possible that *U* > 0 even if *y* = 0.



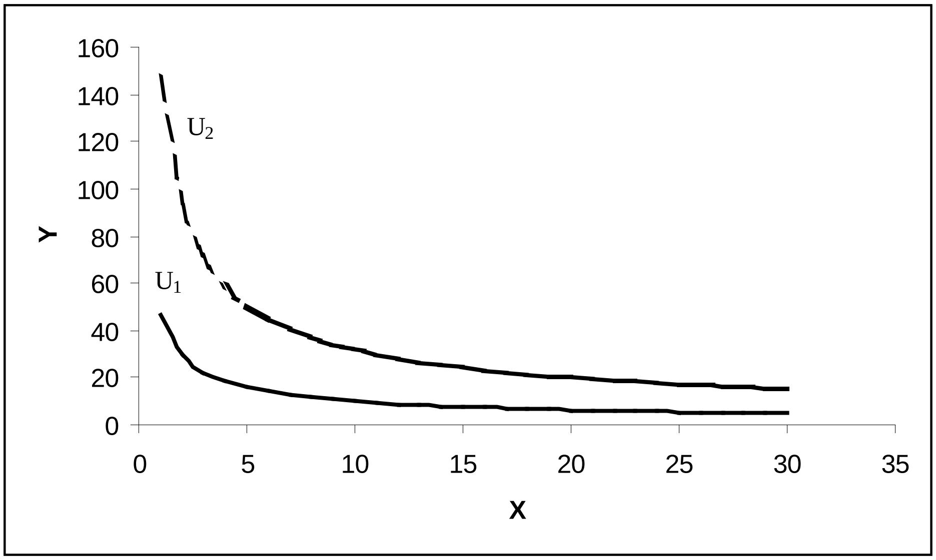
3.18 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x diminishes as the consumer buys more x.

c) $MRS_{x,y} = \frac{.4(y^{0.6} / x^{0.6})}{.6(x^{0.4} / y^{0.4})} = \frac{0.4y}{0.6x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. The indifference curves do not intersect either axis.



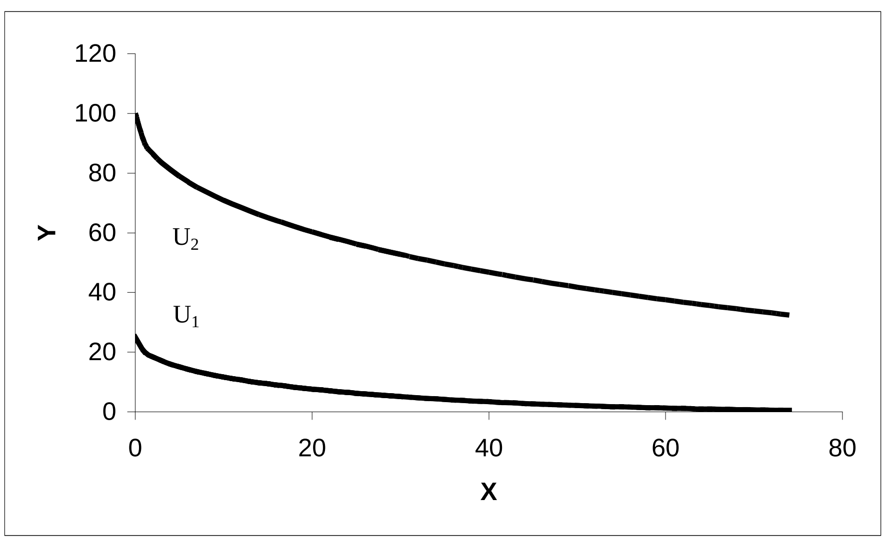
3.19 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x diminishes as the consumer buys more x.

c) $MRS_{x,y} = \dfrac{1/(2\sqrt{x})}{1/\sqrt{y}} = \dfrac{\sqrt{y}}{2\sqrt{x}}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. Since it is possible to have $U > 0$ if either $x = 0$ (and $y > 0$) or $y = 0$ (and $x > 0$), the indifference curves intersect both axes.



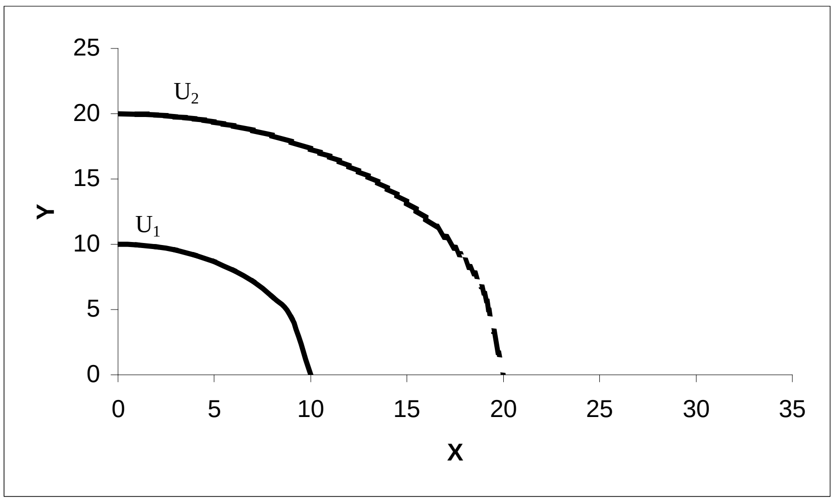
3.20 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x increases as the consumer buys more x.

c) $MRS_{x,y} = \frac{2x}{2y} = \frac{x}{y}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will increase.

e & f) See figure below. Since it is possible to have $U > 0$ if either $x = 0$ (and $y > 0$) or $y = 0$ (and $x > 0$), the indifference curves intersect both axes.



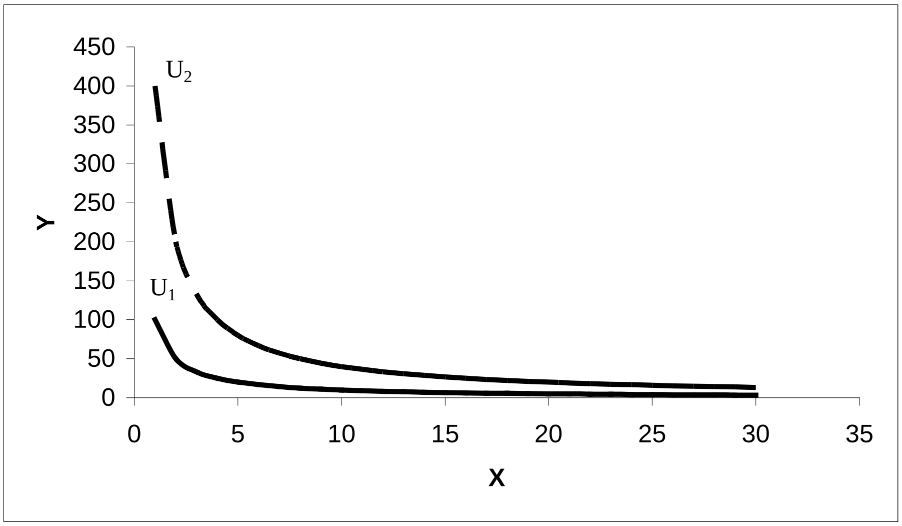
3.21 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) Since we do not know the value of α, only that it is positive, we need to specify three possible cases:
When $\alpha < 1$, the marginal utility of x diminishes as x increases.
When $\alpha = 1$, the marginal utility of x remains constant as x increases.
When $\alpha > 1$, the marginal utility of x increases as x increases.

c) $MRS_{x,y} = \frac{\alpha A x^{\alpha-1} y^{\beta}}{\beta A x^{\alpha} y^{\beta-1}} = \frac{\alpha y}{\beta x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) The graph below depicts indifference curves for the case where $A = 1$ and $\alpha = \beta = 0.5$. Thus $U(x, y) = x^{0.5} y^{0.5}$. Regardless, the indifference curves will never intersect either axis.



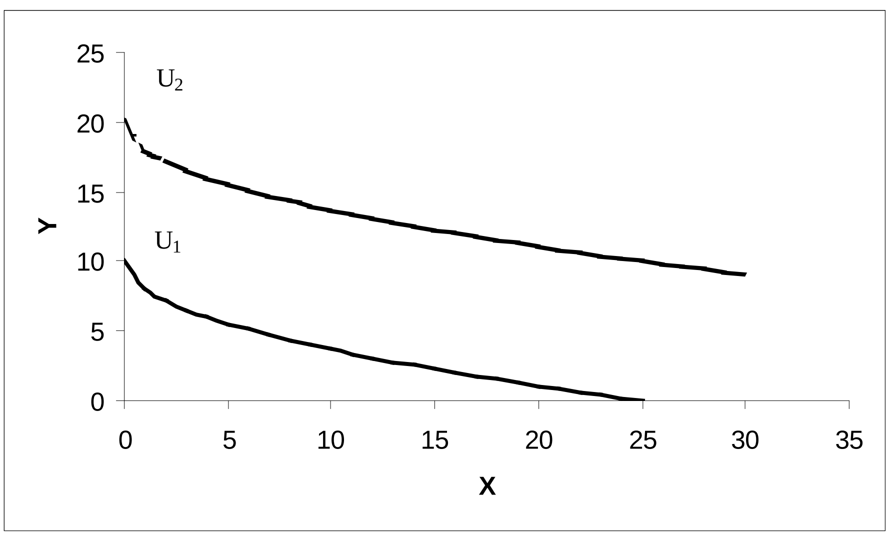
3.22 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x increases as the consumer buys more x.

c) $MRS_{x,y} = \frac{1/\sqrt{x}}{1} = 1/\sqrt{x}$

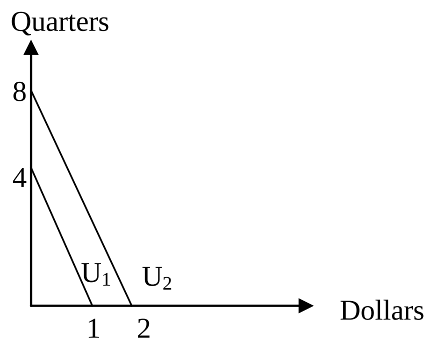
d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e) Since it is possible to have $U > 0$ if either $x = 0$ (and $y > 0$) or $y = 0$ (and $x > 0$), the indifference curves intersect both axes.

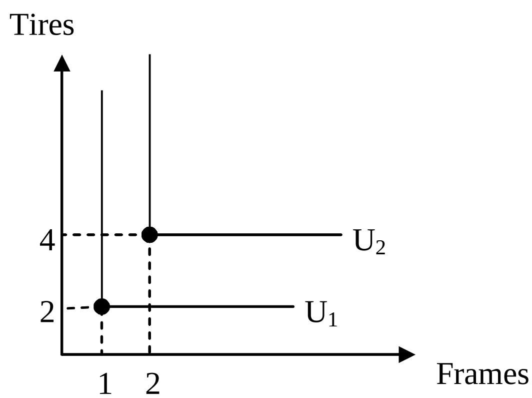


f) The slope of a typical indifference curve at some basket (x, y) is the $MRS_{x,y} = 1/\sqrt{x}$. At $x = 4$, $MRS_{x,y} = 1/\sqrt{4} = 0.5$. Note that this holds regardless of the value of y. Therefore, the slope of any indifference curve at $x = 4$ will be -0.5.

3.23 a) Quarters and dollars are perfect substitutes:



b) Tires and frames are perfect complements:



3.24 First, the expression for $MRS_{x,y}$ is

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1-\alpha)x^{\alpha} y^{-\alpha}} = \frac{\alpha}{1-\alpha}\frac{y}{x}$$

Since we know that $MRS_{x,y}$ = 4 when x = 4 and y = 8,

$$4 = \frac{\alpha}{1-\alpha}\frac{8}{4}$$

$$2 = \frac{\alpha}{1-\alpha}$$

$$2 - 2\alpha = \alpha$$

$$\alpha = \frac{2}{3}$$

3.25 Recall that $MRS_{x,y} = \frac{MU_x}{MU_y}$. Substituting in the marginal utilities given above yields $MRS_{x,y} = \frac{x^{\rho-1}}{y^{\rho-1}}$. Now, because $\rho < 1$, $x^{\rho - 1}$ decreases as x increases. By the same logic, $y^{\rho - 1}$ increases as y decreases. As we “slide down” an indifference curve, x increases and y decreases, so it follows that $MRS_{x,y}$ decreases. Thus, this utility function exhibits diminishing marginal rate of substitution of x for y.

Chapter 4

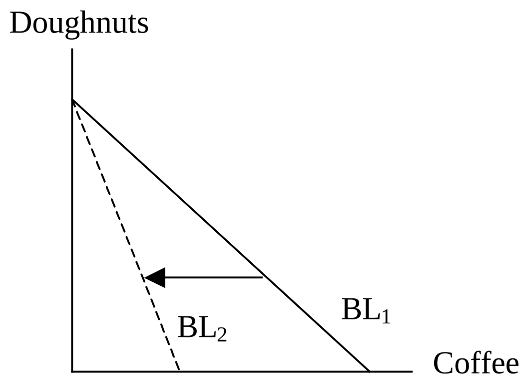
Consumer Choice

Solutions to Review Questions

1. Relative to any point on the budget line, when the consumer has a positive marginal utility for all goods she could increase her utility by consuming some basket outside the budget line. However, baskets outside the budget line are unaffordable to her, so she is constrained (as in "constrained optimization") to choosing the most preferred basket that lies along the budget line.

2. An increase in income will shift the budget line away from the origin in a parallel fashion expanding the set of possible baskets from which a consumer may choose. A decrease in income will shift the budget line in toward the origin in a parallel fashion, reducing the set of possible baskets from which a consumer may choose.

3. If the price of one of the goods increases, the budget line will rotate inward on the axis for the good with the price increase. The budget line will continue to have the same intercept on the other axis. For example, suppose someone buys two goods, cups of coffee and doughnuts, and suppose the price of a cup of coffee increases. Then the budget line will rotate as in the following diagram:



4. With an interior optimum the consumer is choosing a basket that contains positive quantities of all goods, while with a corner point optimum the consumer is choosing a basket with a zero quantity for one of the goods. The tangency condition usually does not apply at corner optima.

5. If the optimum is an interior solution, the slope of the budget line must equal the slope of the indifference curve. If these slopes are not equal at the chosen interior basket then the

"bang for the buck" condition will not hold. This condition states that at the optimum the extra utility gained per dollar spent on good x must be equal to the extra utility gained per dollar spent on good y. If this condition does not hold at the chosen basket, then the consumer could reallocate his income to purchase more of the good with the higher "bang for the buck" and increase his total utility while remaining within the given budget. Thus, if these slopes are not equal the basket cannot be optimal assuming an interior solution.

6. At an interior optimum, the slope of the budget line must equal the slope of the indifference curve. This implies

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

This can be rewritten as

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

which is known as the "bang for the buck" condition. If this condition does not hold at the chosen interior basket, then the consumer can increase total utility by reallocating his spending to purchase more of the good with the higher "bang for the buck" and less of the other good.

7. The "bang for the buck" condition will not necessarily hold at a corner solution optimum. The consumer could theoretically increase total utility by reallocating his spending to purchase more of the good with the higher "bang for the buck" and less of the other good. Since the basket is a corner point, however, he is already purchasing zero of one of the goods. This implies that he cannot purchase less of the good with a zero quantity (since negative quantities make no sense) and therefore cannot reallocate spending.

8. In the utility maximization problem, the consumer maximizes utility subject to a fixed budget constraint. At the optimum the slope of the budget line will equal the slope of the indifference curve. If we now hold that indifference curve fixed, we can solve an expenditure minimization problem in which we ask what is the minimum expenditure necessary to achieve that fixed level of utility. Since the slope of the budget line and indifference curve have not changed, when the expenditure is minimized the budget line and indifference curve will be tangent at the same point as in the utility maximization problem. The same basket is optimal in both problems.

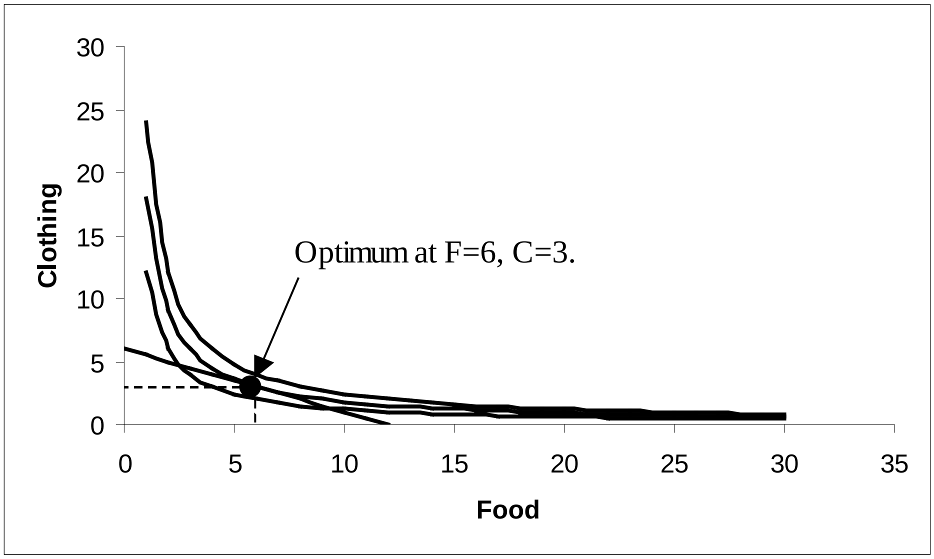
9. First, consumers typically allocate income to more than two goods. Second, economists often want to focus on the consumer's response to purchases of a single good or service. In this case it is useful to present the consumer choice problem using a two-dimensional graph. Since there are more than two goods the consumer is purchasing, however, an economist would need more than two dimensions to show the problem graphically. To

reduce the problem to two dimensions, economists often group the expenditures on all other goods besides the one in question into a single good termed a "composite good." When the problem is shown graphically, one axis represents the composite good while the other axis represents the single good in question. By creating this composite good, the problem can be illustrated using a two-dimensional graph.

10. By employing revealed preference analysis one can make inferences regarding a consumer's preferences without knowing what the consumer's indifference map looks like. For example, if a consumer chooses basket A over basket B when basket B costs at least as much as basket A, we know that basket A is at least as preferred as basket B. If the consumer chooses basket C, which is more expensive than basket D, then we know the consumer strictly prefers basket C to basket D. By observing enough of these choices, one can determine how the consumer ranks baskets even without knowing the exact shape of the consumer's indifference map.

Solutions to Problems

4.1 a)



b) The tangency condition implies that

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C}$$

Plugging in the known information results in

$$\frac{C}{F} = \frac{1}{2}$$
$$2C = F$$

Substituting this result into the budget line, $F + 2C = 12$, yields

$$2C + 2C = 12$$
$$4C = 12$$
$$C = 3$$

Finally, plugging this result back into the tangency condition implies $F = 6$. At the optimum the consumer choose 6 units of food and 3 units of clothing.

c) At the optimum, $MRS_{F,C} = C/F = 3/6 = 1/2$. Note that this is equal to the ratio of the price of food to the price of clothing. The equality of the price ration and $MRS_{F,C}$ is seen in the graph above as the tangency between the budget line and the indifference curve for $U = 18$.

d) If the consumer purchases 4 units of food and 4 units of clothing, then

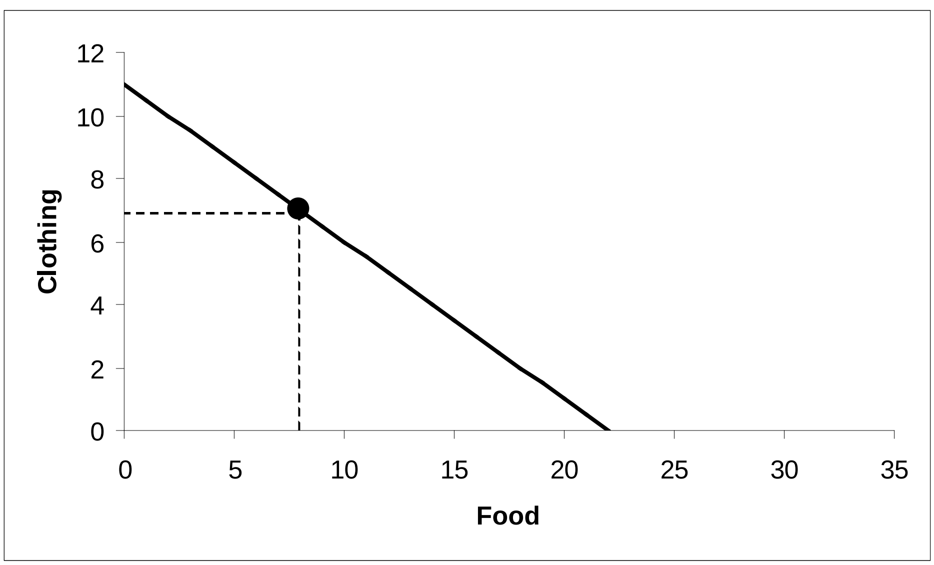
$$\frac{MU_F}{P_F} = \frac{4}{1} = 4 > \frac{MU_C}{P_C} = \frac{4}{2} = 2.$$

This implies that the consumer could reallocate spending by purchasing more food and less clothing to increase total utility. In fact, at the basket (4, 4) total utility is 16 and the consumer spent $12. By giving up one unit of clothing the consumer saves $2 which can than be used to purchase two units of food (they each cost $1). This will result in a new basket (6,3), total utility of 18, and spending of $12. By reallocating spending toward the good with the higher "bang for the buck" the consumer increased total utility while remaining within the budget constraint.

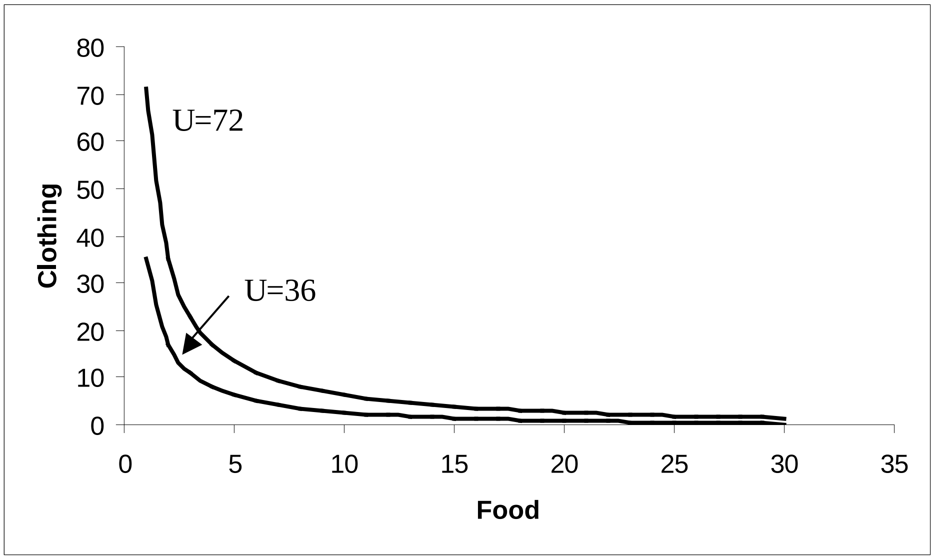
4.2 a) If Ann is spending all of her income then

$$F + 2C = 22$$
$$8 + 2C = 22$$
$$2C = 14$$
$$C = 7$$

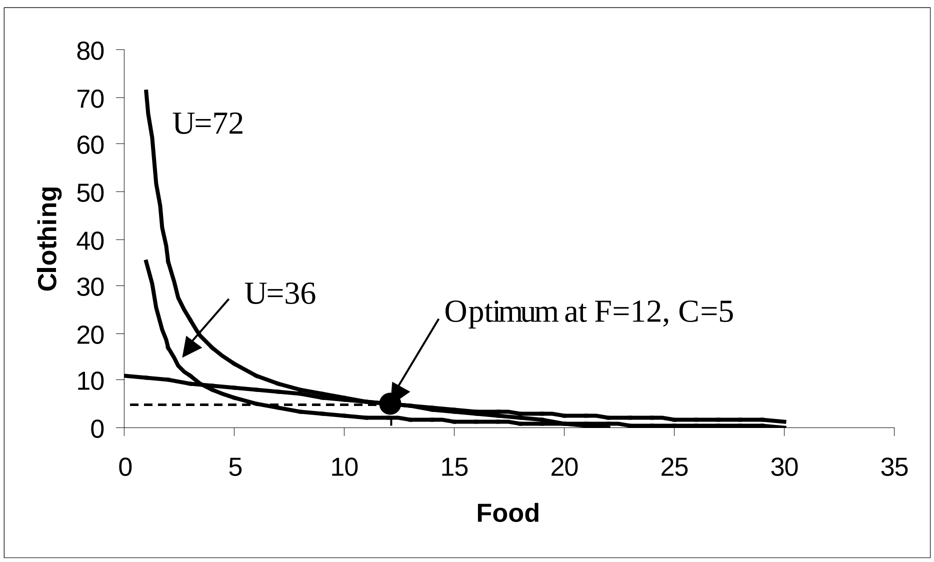
b)



c) Yes, the indifference curves are convex, i.e., bowed in toward the origin. Also, note that they intersect the *F*-axis.



d)



e) The tangency condition requires that

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C}$$

Plugging in the known information yields

$$\frac{C+1}{F} = \frac{1}{2}$$

$$2C + 2 = F$$

Substituting this result into the budget line, $F + 2C = 22$ results in

$$(2C+2)+2C=22$$
$$4C=20$$
$$C=5$$

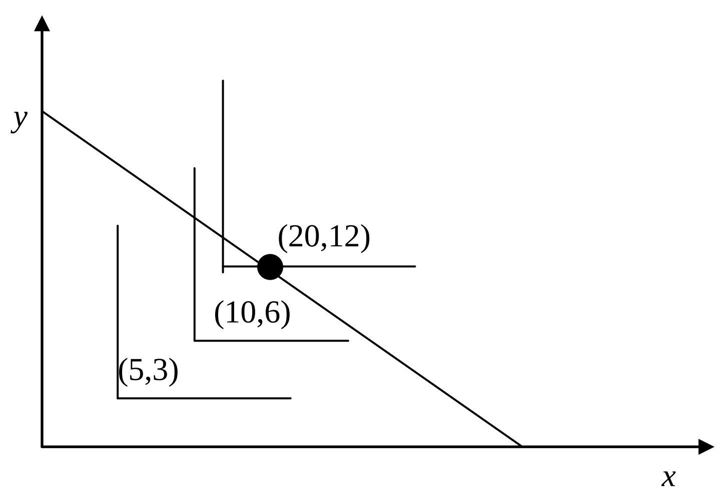
Finally, plugging this result back into the tangency condition implies that $F=2(5)+2=12$. At the optimum the consumer chooses 5 units of clothing and 12 units of food.

f) $MRS_{F,C}=\frac{C+1}{F}=\frac{5+1}{12}=\frac{1}{2}$ The marginal rate of substitution is equal to the price ratio.

g) Yes, the indifference curves do exhibit diminishing $MRS_{F,C}$. We can see this in the graph in part c) because the indifference curves are bowed in toward the origin. Algebraically, $MRS_{F,C}={}^{C+1}\!/_{F}$. As F increases and C decreases along an isoquant, $MRS_{F,C}$ diminishes.

4.3 This question cannot be solved using the usual tangency condition. However, you can see from the graph below that the optimum basket will necessarily lie on the "elbow" of some indifference curve, such as (5, 3), (10, 6) etc. If the consumer were at some other point, he could always move to such a point, keeping utility constant and decreasing his expenditure. The equation of all these "elbow" points is 3*x* = 5*y*, or *y* = 0.6*x*. Therefore the optimum point must be such that 3*x* = 5*y*.

The usual budget constraint must hold of course. That is, $5x+10y=220$. Combining these two conditions, we get (*x*, *y*) = (20, 12).



4.4 From the given information we know that $P_H = 3$, $P_M = 1$, and $MRS_{H,M} = 2$. Comparing the $MRS_{H,M}$ to the price ratio,

$$MRS_{H,M} = 2 < \frac{P_H}{P_M} = \frac{3}{1}$$

Since these are not equal Jane is not currently at an optimum. In addition, we can say that

$$\frac{P_H}{P_M} > MRS_{H,M} = \frac{MU_H}{MU_M}$$

which is equivalent to

$$\frac{MU_M}{P_M} > \frac{MU_H}{P_H}$$

That is, the "bang for the buck" from milkshakes is greater than the "bang for the buck" from hamburgers. So Jane can increase her total utility by reallocating her spending to purchase fewer hamburgers and more milkshakes.

4.5 Compare MU_H/P_H with MU_R/P_R, where the subscripts "H" and "R" refer respectively to hamburgers and root beer. We have all the information to make this comparison except for the price of a hamburger. But we can determine the price of a hamburger from Sam's budget constraint:

$P_HH + P_RR$ = Income, or $P_H(15) + 2(20) = 100$.

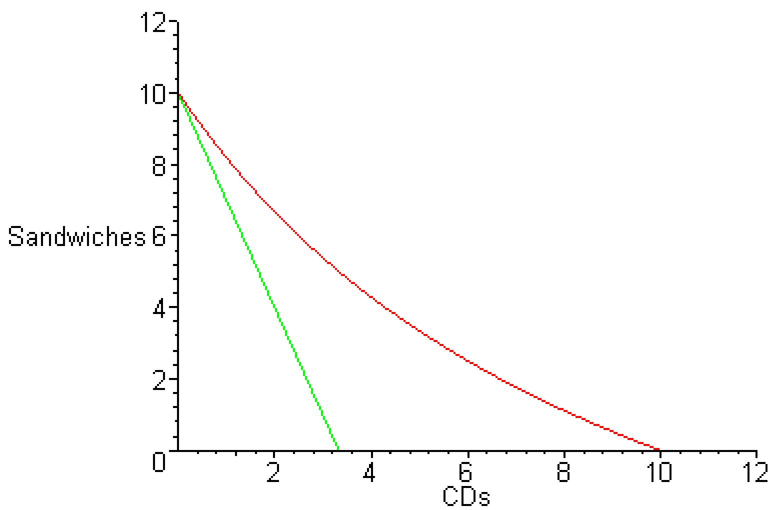
So P_H = \$4 per hamburger.
Now we can see that $MU_H/P_H = 8/4 = 2$ and $MU_R/P_R = 6/2 = 3$.
Since the "bang for the buck" is higher for root beer than for hamburgers, he should buy fewer hamburgers (and more root beer).

4.6 To determine if this situation is optimal, determine if the tangency condition holds.
Is $\frac{MU_H}{P_H} = \frac{MU_S}{P_S}$? That is, is $\frac{5}{1} = \frac{3}{0.50}$? No ($5 \neq 6$). So $\frac{MU_H}{P_H} < \frac{MU_S}{P_S}$.
Since the tangency condition does not hold, Dave is not currently maximizing his utility. To increase his utility he should purchase more soda and fewer hot dogs (since the 'bang for the buck' for sodas is higher).

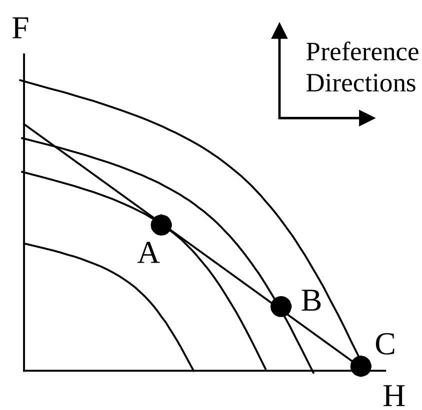
4.7 See the graph below. The fact that Helen's indifference curves touch the axes should immediately make you want to check for a corner point solution.

To see the corner point optimum algebraically, notice if there was an interior solution, the tangency condition implies $(S + 10)/(C + 10) = 3$, or $S = 3C + 20$. Combining this with the budget constraint, $9C + 3S = 30$, we find that the optimal number of CDs would be given by $18C = -30$ which implies a negative number of CDs. Since it's impossible to purchase a negative amount of something, our assumption that there was an interior solution must be false. Instead, the optimum will consist of $C = 0$ and Helen spending all her income on sandwiches: $S = 10$.

Graphically, the corner optimum is reflected in the fact that the slope of the budget line is steeper than that of the indifference curve, even when $C = 0$. Specifically, note that at $(C, S) = (0, 10)$ we have $P_C / P_S = 3 > MRS_{C,S} = 2$. Thus, even at the corner point, the marginal utility per dollar spent on CDs is lower than on sandwiches. However, since she is already at a corner point with $C = 0$, she cannot give up any more CDs. Therefore the best Helen can do is to spend all her income on sandwiches: $(C, S) = (0, 10)$. [Note: At the other corner with $S = 0$ and $C = 3.3$, $P_C / P_S = 3 > MRS_{C,S} = 0.75$. Thus, Helen would prefer to buy more sandwiches and less CDs, which is of course entirely feasible at this corner point. Thus the $S = 0$ corner cannot be an optimum.]



4.8 a)

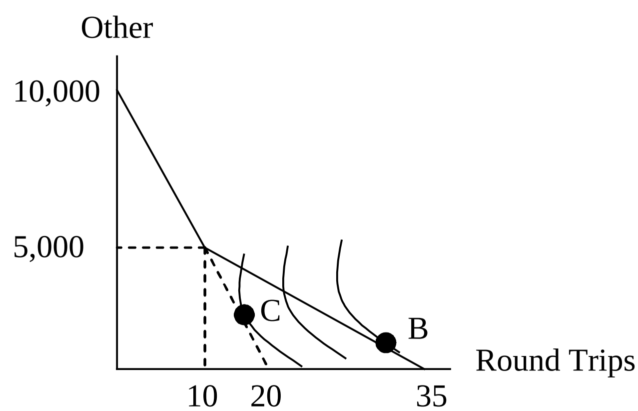


b) At point A, Dr. Strangetaste's indifference curve, which is bowed out from the origin, is tangent to his budget line. This point is not an optimum because, for example, Dr. Strangetaste could move to point B on his budget line and achieve a higher level of total utility. Point B, though, is not an optimum either because Dr.

Strangetaste could move to point C, a corner point, to achieve an even higher level of total utility. When the MRS is increasing, a corner point optimum will occur (with $F = 0$ in this picture, though it could equivalently be with $H = 0$ for another set of indifference curves).

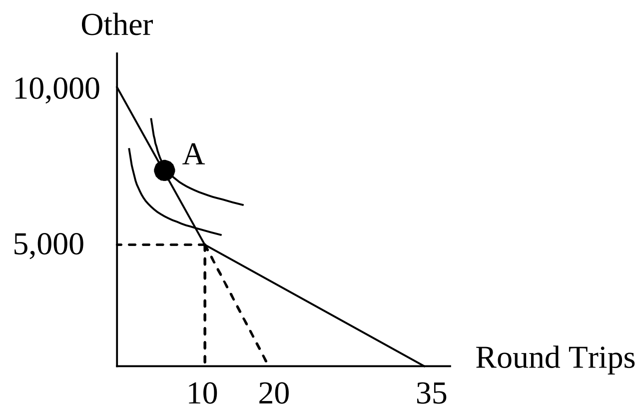
4.9 As given, Julie consumes F = 10 and C = 2 with an income of 24.
Initially (with $P_F = P_C = 2$) she spends all her income: $P_F F + P_C C = 2(10) + 2(2) = 24$.
To buy her initial basket at the new prices, she would only need to spend
$P_F F + P_C C = 1(10) + 4(2) = 18$.
Thus, her initial basket lies inside her new budget constraint (assuming her income stays at 24). With her new budget line she would be able to choose a new basket to the "northeast" of (i.e., a basket involving more food and clothing than) her initial basket, making her better off.

4.10 a) The budget line will have a kink where round trips = 10 and other goods = 5,000. Northwest of the kink, the budget line's slope will be –500 . Southeast of the kink, the slope will be –200.



b) With the indifference curves drawn on the above graph, Toni is better off with the frequent flyer program (at point B) than she would be without it (at point C). Without the frequent flyer program the best she could achieve is point C, which lies on the hypothetical budget line where the price of round trips is always $500.

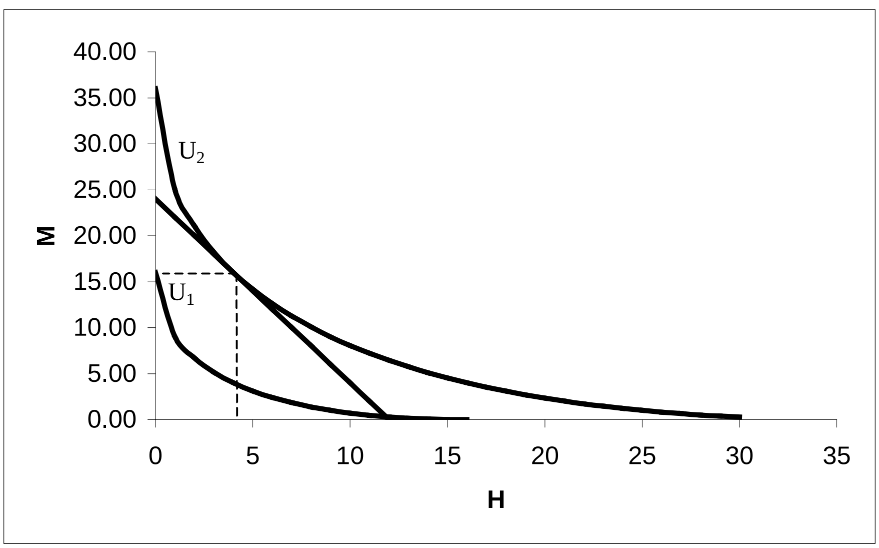
c) With the indifference curves drawn on graph below, Toni is no better off with the frequent flyer program than she would be without it (at point A). At this point, her indifference curve is tangent to a portion of the budget line where the frequent flyer program does not apply (less than 10 round trips).



4.11 a) $$MRS_{H,M} = \frac{MU_H}{MU_M} = \frac{1/(2\sqrt{H})}{1/(2\sqrt{M})} = \frac{\sqrt{M}}{\sqrt{H}}$$

This utility function has a diminishing marginal rate of substitution since $MRS_{H,M}$ declines as *H* increases and *M* decreases.

b)



Since it is possible to have $U > 0$ if either $H = 0$ (and $M > 0$) or $M = 0$ (and $H > 0$), the indifference curves will intersect both axes.

c) We know from the tangency condition that

$$\frac{\sqrt{M}}{\sqrt{H}} = \frac{2}{1}$$

$$M = 4H$$

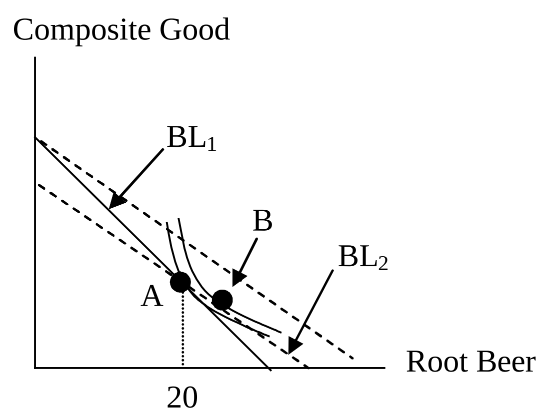
Substituting this into the budget line, $2H + M = 24$, yields

$$2H + 4H = 24$$
$$H = 4$$

Finally, plugging this back into the tangency condition implies $M = 4(4) = 16$. At the optimum the consumer will choose 4 hamburgers and 16 milkshakes. This can be seen in the graph above.

4.12 When Justin maximizes utility, his optimal consumption basket will be on the budget constraint and satisfy the tangency condition.
Any basket on the budget line will satisfy $p_x x + p_y y = I$, or $2x + 5p_y = 40$.
The tangency condition requires that $MU_x / p_x = MU_y / p_y$, or that $5 / 2 = x / p_y$. This implies that $5p_y = 2x$.
Putting these two equations together reveals that $5p_y + 5p_y = 40$; thus $p_y = 4$.

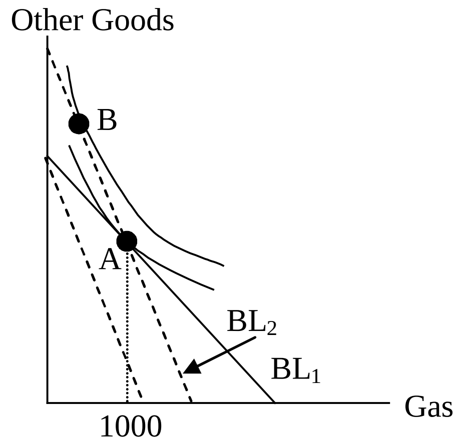
4.13



Assume the student is initially at an interior optimum, point *A*. Denote the initial price of root beer by *P* and the student's income as *M*. Point *A* then consists of $R_A = 20$ units of root beer and $Y_A = M - 20P$ units of the composite good. The effect of the proposal is to rotate the budget line outward (the price change) and then shift it inward (the lump sum tax), for a total movement from BL_1 to BL_2. Notice that BL_2 intersects BL_1 exactly at point *A*: under the proposal, (R_A, Y_A) costs the student $20(P - 0.5) + M - 20P = M - 10$, which is equal to her income under the proposal.

Because *A* was initially optimal, $MRS_{R,Y} = P$ at point *A*. Yet the price ratio along BL_2 is $(P - 0.5)$. Hence $MRS_{R,Y} > P_R / P_Y$, so the student can increase her utility by purchasing more root beer and less of the composite good, at a point such as *B* depicted in the graph above. Thus, the proposal will make the student better off.

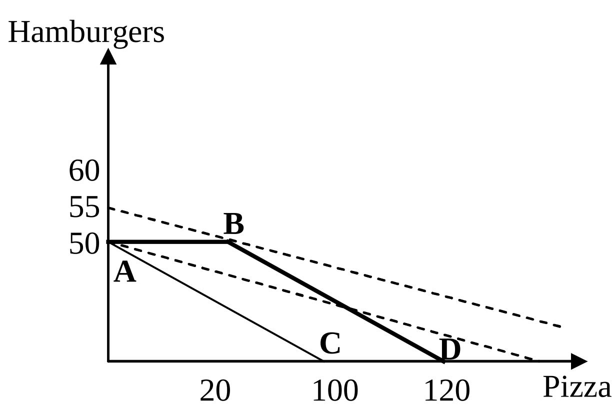
4.14



Assume Joe is initially at an interior optimum, point *A*, and that the price of other goods is $1. Let Joe's income be *M*. Point *A* then consists of G_A = 1000 units of root beer and $Y_A = M - 2000$ units of other goods. The effect of the proposal is to rotate the budget line inward (the price change) and then shift it outward (the cash transfer), for a total movement from BL_1 to BL_2. Notice that BL_2 intersects BL_1 exactly at point *A*: after the price increase, (G_A, Y_A) costs Joe 1000*2.50 + *M* – 2000 = *M* + 500, which is equal to his income after the cash transfer.

Because *A* was initially optimal, $MRS_{G,Y} = 2$ at point *A*. Yet the price ratio along BL_2 is 2.5. Hence $MRS_{G,Y} < P_G / P_Y$, so Joe can increase his utility by purchasing less gas and more of the composite good, at a point such as *B* depicted in the graph above. Thus, the proposal will make Joe better off.

4.15 Paul's initial budget constraint is the line AC, allowing him to purchase at most 50 hamburgers or at most 100 pizzas. The $60 cash certificate shifts out his budget constraint without changing the maximum number of hamburgers that he can buy. The new budget constraint is ABD and he can now buy a maximum of 120 pizzas.



Initially, Paul's optimal basket contains all hamburgers and no pizza, at point *A* where $(P, H) = (0, 50)$, because $MU_H / P_H = 4/6 > MU_P / P_P = 1/3$. His utility level at point *A* is $U(0, 50) = 200$. When he gets the gift certificate, Paul's optimal basket is at point *B*, spending all of his regular income on hamburgers and the $60 gift certificate on pizza. So point *B* is where $(P, H) = (20, 50)$ with a utility of $U(20, 50) = 220$.

However, Paul could also achieve a utility of 220 by consuming 220/4 = 55 hamburgers. To buy the extra 5 hamburgers he would require 5*6 = $30. So, if he had received a cash gift of $30 it would have made Paul exactly as well off as the $60 gift certificate for pizzas.

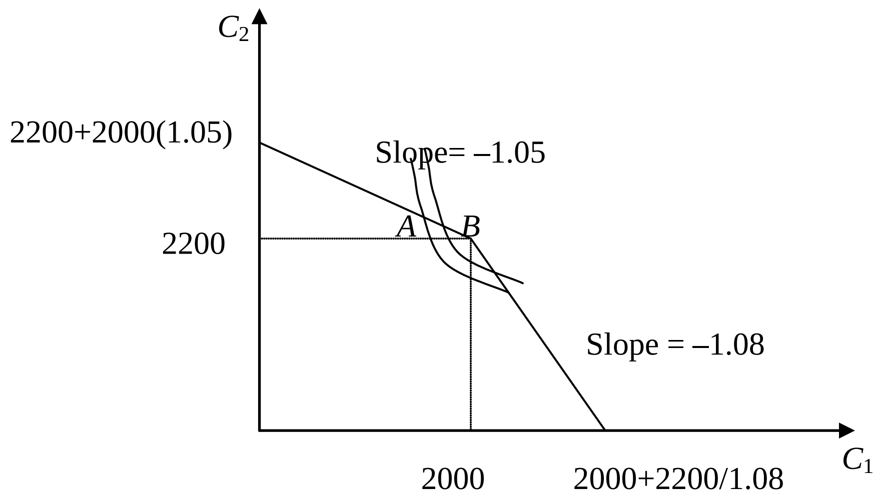
4.16 a) In this case, $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{y}{2x}$. If Jack neither borrows nor lends, then $MRS_{x,y} = 1050/(2*1000) = 0.525$. Recall that if the interest rate is r, the slope of the budget line is $-(1+r) = -1.05$. Thus, if he neither borrows nor lends it will be the case that $MRS_{x,y} < 1 + r$. That is, the "bang for the buck" for spending this month (good x) is less than that for spending next month (good y). Thus, Jack should lend some of his income this month (so $x < 1000$) in order to earn interest and have higher spending next month ($y > 1050$).

b) Now $MRS_{x,y} = 2y/x$. If Jack neither borrows nor lends, $MRS_{x,y} = 2.1 > (1 + r)$. Thus, Jack could increase his utility by borrowing in the first month (so that $x > 1000$ and $y < 1050$).

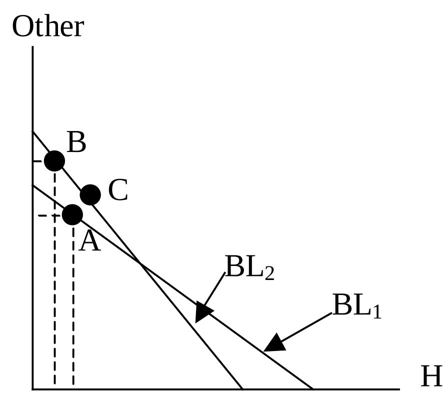
c) Now $MRS_{x,y} = y/x$. If Jack neither borrows nor lends, $MRS_{x,y} = 1.05 = (1 + r)$. Thus, Jack's utility is maximized when he neither borrows nor lends, simply spending all of his income in each month: $(x, y) = (1000, 1050)$.

4.17 The utility function implies that $MRS_{C1,C2} = C_2 / C_1$. At point *A*, $MRS_{C1,C2} = 1.10$, which lies between $(1 + r_L) = 1.05$ and $(1 + r_B) = 1.12$. Therefore Meg will neither borrow nor lend and will simply spend her entire income each month.

If the borrowing rate falls to 8%, then the lower part of the budget line pivots outward, as depicted in the graph below. Then at point *A*, $MRS_{C1,C2} > (1+ r_B) > (1 + r_L)$ since $1.10 > 1.08 > 1.05$. So Meg can increase her utility by moving away from point *A* to a point like *B*, where she borrows money, spending more than her income this month ($C_1 > 2000$) and less than her income next month ($C_2 < 2200$).



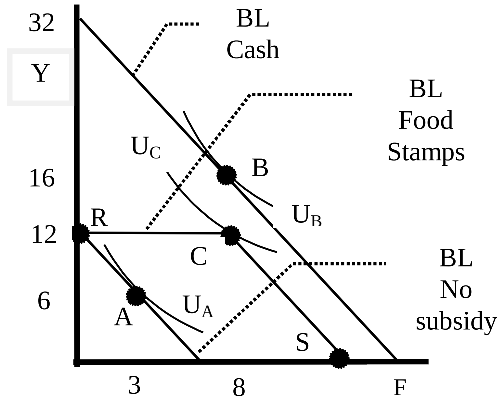
4.18



With the initial budget line, BL_1, Sally chooses point A, where $(x_A, y_A) = (2, 80)$. When her income and the price of housing increase, the budget line becomes BL_2 and she chooses point B, where $(x_B, y_B) = (1, 105)$. Importantly, because the equation for BL_1 is $10x + y = 100$ and that for BL_2 is $15x + y = 120$, we can solve to see that these lines intersect at $x = 4$, to the right of point A on BL_1.

Consider a hypothetical basket C on BL_2 but northeast of A. We can then deduce that $B \succ A$, because (i) B is at least as preferred as C (since B was chosen when C was affordable), and (ii) C is strictly preferred to A (since C lies to the northeast of A). By transitivity, B must be strictly preferred to A.

4.19

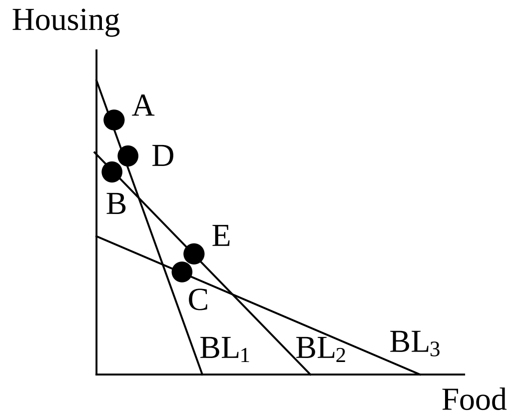


a) $MU_Y = F$ and $MU_F = Y$, so $MRS_{F,Y} = Y/F$, which diminishes as F increases along an indifference curve. Since the indifference curves do not intersect either axis, an optimal basket will be interior. At such an optimum two conditions must be satisfied: (1) tangency: $MRS_{F,Y} = P_F / P_Y$, or $Y = 2F$, and (2) budget line ("BL No subsidy" in the graph): $2F + Y = 12$. This $F = 3$ and $Y = 6$. This optimum is depicted as point A in the graph.

b) We need to find an interior optimum with $F = 8$. As income increases, the consumer chooses a basket along the Income Consumption Curve, which consists of the tangency points $Y = 2F$. So $Y = 2(8) = 16$. Total expenditure will then be $2F + Y = 2(8) + 16 = 32$. So the consumer needs an income of 32 ("BL Cash" in the graph). Since the consumer has an income of 12, she needs an additional income of 20 (=32 – 12). So the subsidy needed is 20. This optimum is shown as point B in the graph.

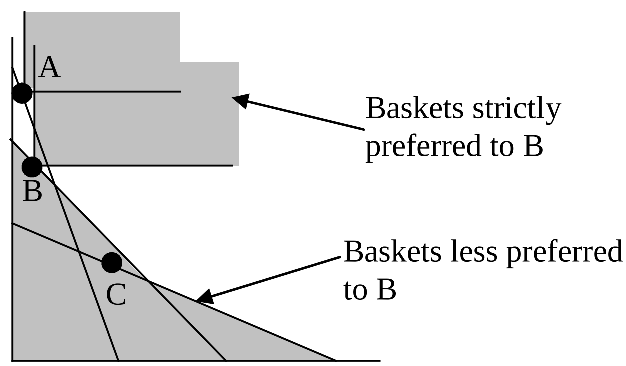
c) From part (b) we see that, with no restrictions on how the government subsidy can be spent, the consumer would like to buy 16 units of Y, more than her own budget (without subsidy) would permit. So we expect that with food stamps, she will use the voucher to purchase the required 8 units of food and spend all of her own unrestricted income (12) on Y. In other words, this consumer will be at point C on the graph, at the kink on the budget constraint RCS (labeled "BL Food Stamps"). We can verify that ($F = 8$, $Y = 12$) is her optimal choice by looking at the "bang for the buck" condition at C. MU_F/price of food = $Y/2 = 12/2 = 6$. MU_Y/price of $Y = F/1 = 8$. So the consumer would like to substitute more Y for F, but cannot do so because at basket C she has purchased all the other goods she can given her budget constraint.

4.20 a) From figure 4.21 we can infer that $A \succ B$, that $B \succ C$, and therefore by transitivity that $A \succ C$.



First, A must be strictly preferred to B since A is at least as preferred as D and D must be strictly preferred to B. Second, B must be strictly preferred to C since B is at least as preferred as E and E is strictly preferred to C. So by transitivity A must be strictly preferred to C.

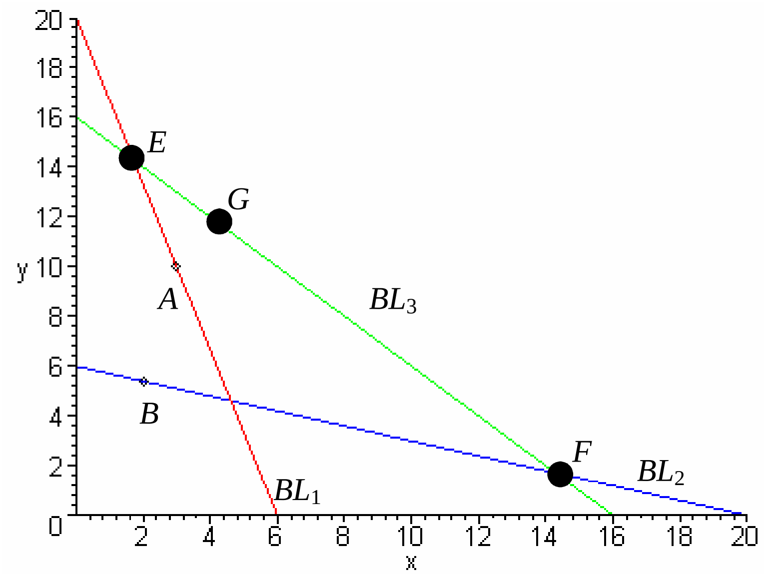
b)



B will be strictly preferred to everything inside BL_2. In addition, since B is strictly preferred to C, B will be strictly preferred to everything below BL_3, including all the points along BL_3 itself (since C is weakly preferred to everything on BL_3).

c) See the graph in part (b). Everything to the northeast of B is strictly preferred to B. In addition, since A is strictly preferred to B [see part (a)], everything to the northeast of A must also be strictly preferred to B. Notice, however, that there are points on BL_1 (both northwest and southeast of A) about which we cannot infer anything.

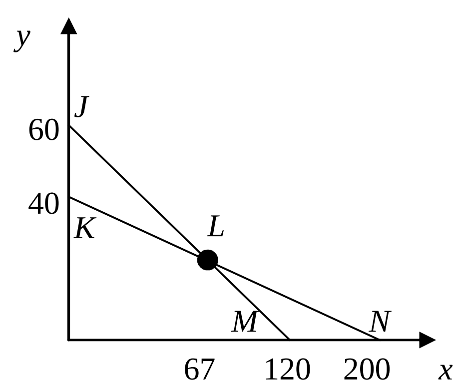
4.21 Let point *E* denote the intersection of BL_1 and BL_3, and point *F* denote the intersection of BL_2 and BL_3 (see the figure below).



First, any point to the northwest of *E*, including *E*, is in the consumer's budget set when he faces budget constraint BL_1. The fact that he chose *A* over these points implies that *A* is at least as preferred to *E* and strictly preferred to the points northwest of *E*. However, point *G* lies on BL_3 and is northeast of *A*, so $G \succ A$. Therefore, by transitivity *G* is strictly preferred to *E* and all the points on BL_3 northwest of *E*.

Similarly, *A* is northeast of *B* so $A \succ B$. Since *B* is at least as preferred as any point on BL_2, including *F*, by transitivity we know that *A* is strictly preferred to *F* and all points on BL_3 to the southeast of *F*. And since $G \succ A$, we know that *G* is strictly preferred to these points as well. Therefore the consumer could choose any point between *E* and *F* on BL_3, but neither *E* nor *F* themselves.

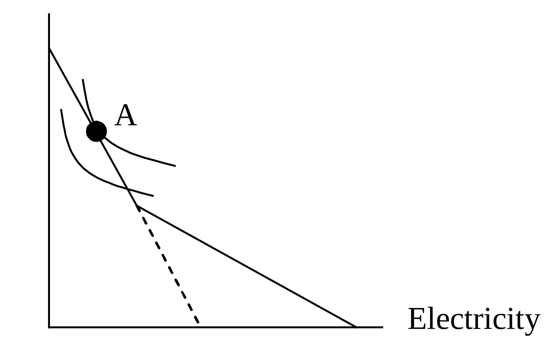
4.22 Let *x* denote the number of phone calls, and *y* denote spending on other goods. Under Plan A, Darrell's budget line is *JLM*. Under Plan B, it is *JKLN*. These budget lines intersect at point *L*, or about $x = 67$.



If we know that Darrell chooses Plan A, his optimal bundle must lie on the line segment *JL*. No point between *L* and *M* would be optimal under this plan because then Darrell could have chosen a point under Plan B, between *L* and *N*, that would have given him more minutes, and left him with more money to buy other goods. However, we cannot exclude point *L* itself (Darrell could, for instance, have perfect complements preferences with an "elbow" at point *L*). Thus, if Darrell chooses Plan A his optimal basket could be anywhere between *J* and *L*, including either of these points.

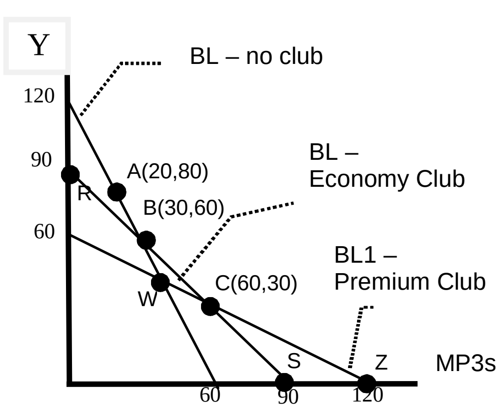
Similarly, if he chose Plan B then his optimal basket must lie between *L* and *N*. Any point between *L* and *K* (but not including point *L*) would be dominated by a point under Plan A between *L* and *J*. Thus, if Darrell chooses Plan B his optimal basket could be anywhere between *L* and *N*, including either of these points.

4.23



With this set of indifference curves, the tangency with the budget line occurs on the portion of the budget line where the quantity discount has not taken effect. Therefore, the consumer does not receive any benefit from the quantity discount.

4.24 The budget lines for the "no club," "Economy Club," and "Premium Club" opportunities available to Angela are drawn below.



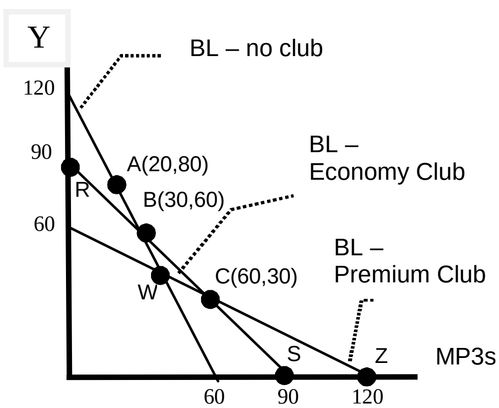
She chooses A with “no club,” and chooses not to join the “Premium Club” when given the chance. Thus, A is weakly preferred to any basket on WZ, and strongly preferred basket on the segment CS (except for C). So, if she does join the Economy Club, she would not choose a basket on CS (except possibly for C).

With “no club” she chooses A when she could have afforded B; thus A $\geq$ B. Since the rest of the segment RB lies inside the “no club” BL, these baskets are strongly inferior to A. So, if she does join the Economy Club, she would not choose a basket on RB (except possibly for B).

To summarize, if she joins the Economy Club, she might consume any basket on the segment BC, including B and C. So her consumption in that case would be $30 \leq$ MP3s ≤ 60.

But she might not join the Economy Club at all! Although we have established that A $\geq$ B and A $\geq$ C, we cannot say how she ranks A with the baskets <u>between</u> B and C.

4.25 The budget lines for the “no club,” “Economy Club,” and “Premium Club” opportunities available to Angela are drawn below.

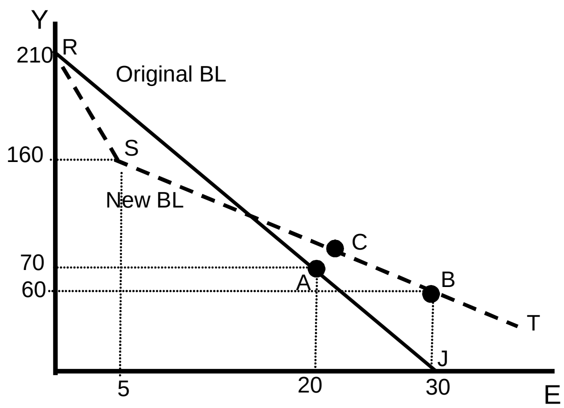


She chooses A with “no club,” and chooses not to join the “Premium Club” when given the chance. Thus, A is weakly preferred to any basket on the segment WZ, and strongly preferred basket on the segment CS (except for C). So, if she does join the Economy Club, she would not choose a basket on CS (except possibly for C).

With “no club” she chooses A when she could have afforded B; thus A is weakly preferred to B. Since the rest of the segment RB lies inside the “no club” BL, these baskets are strongly inferior to A. So, if she does join the Economy Club, she would not choose a basket on RB (except possibly for B).

To summarize, if she joins the Economy Club, she might consume any basket on the segment BC, including B and C. So her consumption in that case would be
$30 \leq$ MP3s ≤ 60. But she might not join the Economy Club at all! Although we have established that A is weakly preferred to B and A is weakly preferred to C, we cannot say how she ranks A with the baskets between B and C.

4.26

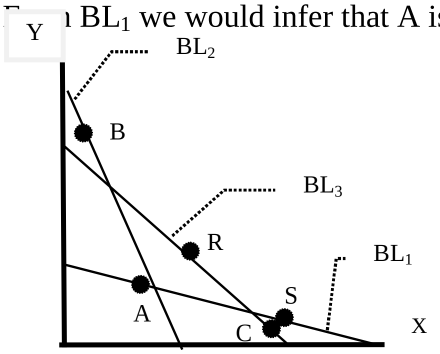


Brian's income is I = (7)(20) + 1(70) = $210.
His initial budget line is the solid curve RJ.
His initial optimal basket is A.

His new budget line is the dotted, piecewise linear curve RST. He chooses basket B, which costs 10(5) + 4(30 – 5) + 1(60) = 210.
In period 2 Basket A costs (10)(5) + (4)(20 – 5) + 1(70) = $180, so Basket A lies inside the new budget line.

In period 3 Brian will choose the plan from the second period. B is weakly preferred to a basket like C, which, in turn is strictly preferred to A (because C lies to the northeast of A). Thus he strictly prefers B to A, and he can only reach B by choosing the plan from period 2.

4.27 From BL_1 we would infer that A is weakly preferred to S, and that S is strongly preferred to C; by transit



From BL_3 we would infer that C is weakly preferred to R, which is strongly preferred to A; by transitivity we conclude that **C** is strongly preferred to **A.**

It cannot be simultaneously true that A is strongly preferred to C and C is strongly preferred to A, for this would imply that A is strongly preferred to C, which is strongly preferred to A, or that A is strongly preferred to itself. The preferences are either intransitive, or else the consumer is failing to maximize utility in each of the three time periods.

Chapter 5

The Theory of Demand

Solutions to Review Questions

1. The price consumption curve plots the set of optimal bundles for two goods, say X and Y, by changing the price of one good while holding the price of the other good and income constant.

2. The price consumption curve plots the set of optimal bundles for two goods as the price of one good changes while the price of the other good and income remain constant. The income consumption curve, on the other hand, plots the set of optimal bundles for two goods as the consumer's income changes while holding the prices of both goods constant.

3. With a normal good, when income increases, consumption of the good will increase. This implies the income elasticity for a normal good will be positive. With an inferior good, when income increases consumption of the good will decrease. This implies the income elasticity for an inferior good will be negative.

4. If indifference curves are bowed in toward the origin and the price of, say, good X falls, consumption of X will always increase; so the substitution effect will always be positive. A decrease in the price of X implies that the slope of the budget line becomes flatter. When indifference curves are bowed in, a direct consequence of this change in relative prices is that any tangency will occur "southeast" of the original bundle along the initial indifference curve. The only way for consumption to fall when price falls is for the income effect to be negative (an inferior good) and for its magnitude to more than offset the substitution effect. In this rare situation, the good is known as a Giffen good.

5. If the consumer purchases only three goods and income increases, it is possible that consumption of all three goods will increase. For example, the consumer might allocate one-third of the increase to each of the three goods. Thus, it is possible for all three goods to be normal. If the consumer purchases only three goods and income increases, it is not possible that consumption of all three goods will decrease. Recall that if consumption falls when income increases the good is inferior. If this were to occur, the consumer would be spending less income than he did prior to the income increase. Thus, it is not possible for all three goods to be inferior.

6. Generally speaking demand curves are downward sloping. Economic theory, however, suggests a special case of an inferior good whose negative income effect is greater than its positive substitution effect. In this event, consumption of the good falls when the

price of the good falls. This type of good is known as a Giffen good. While economic theory suggests that such a good could exist, in practice no such good has been confirmed for humans (although the text suggests an experiment on rats where a good was determined to be a Giffen good).

7. Consumer surplus is the difference between the maximum amount a consumer is willing to pay for a good and what he must actually pay when he purchases it in the marketplace. For example, if Joe is willing to pay $20 for a cap but purchases it at the store for only $5, Joe will receive $15 in consumer surplus. This measure indicates how much better off the consumer is after purchasing the good.

8. Compensating variation answers the question, "How much would the consumer be willing to give up *after* a price reduction to achieve the same level of satisfaction as she had *before* the price change?" Equivalent variation, on the other hand, answers the question, "How much money would we have to give the consumer *before* a price reduction to leave her level of satisfaction the same as it would be *after* the price reduction?" In essence, both of these are measures of the "distance" between the initial and final indifference curves after a price change.

   Typically the compensating and equivalent variation measures will not be the same. In the case of quasi-linear utility functions, however, the compensating and equivalent variation measures will always be the same (they will be equal to the change in consumer surplus). In general, these two measures will be identical when there is no income effect associated with a price change.

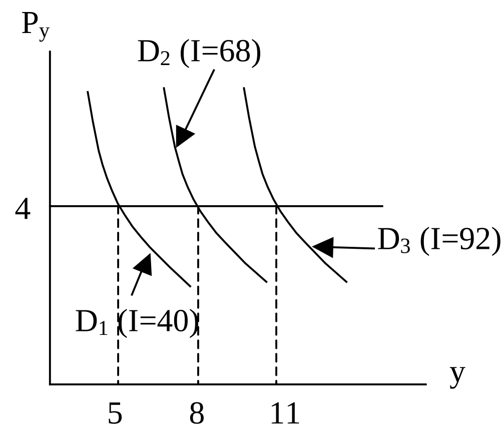
9.

   (i.) No network externality
   (ii.) Negative network externality
   (iii.) Positive network externality
   (iv.) Since sales fall when income increases, this might be a negative network externality if some consumers stopped buying hot dogs not only because of a lower income, but also because other consumers bought fewer hot dogs.

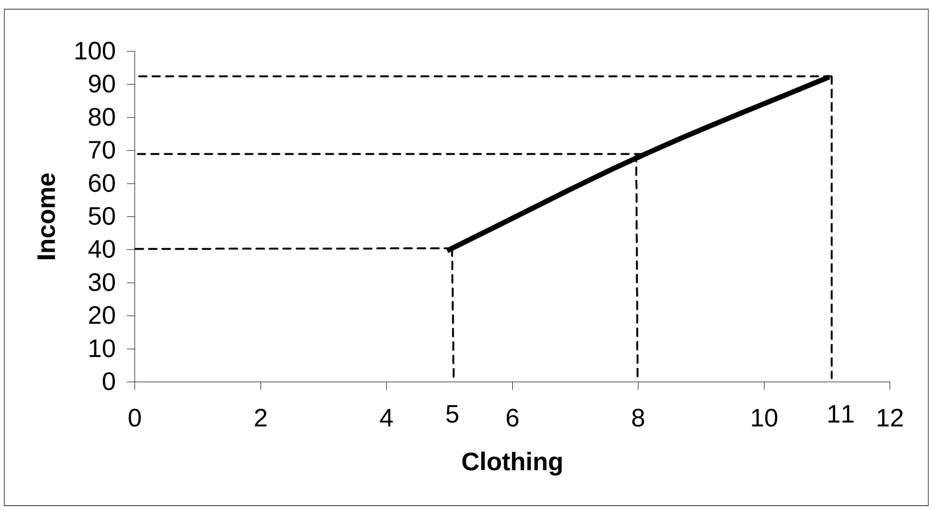
10. When the wage rate rises, the substitution effect will induce a worker to supply more hours of labor. The income effect, on the other hand, may induce the worker to increase the amount of leisure and decrease the amount of labor. If the income effect reduces the amount of labor supplied more than the substitution effect increases it, the worker will ultimately supply less hours of labor.

Solutions to Problems

5.1



5.2



5.3 a) $\varepsilon_{Q,I} = \frac{\%\Delta Q}{\%\Delta I} = \frac{\Delta Q / Q}{\Delta I / I} = \left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{I}{Q}\right)$

I and Q must be greater than zero. In addition, assume income increases, *i.e.*, $\Delta I > 0$. If the good is inferior, then $\Delta Q < 0$. Thus, the first term $(\Delta Q / \Delta I) < 0$ and the second term $(I / Q) > 0$. Multiplying these two terms together implies $\varepsilon_{Q,I} < 0$. Inferior goods have a negative income elasticity of demand.

b) If income elasticity of demand is negative then

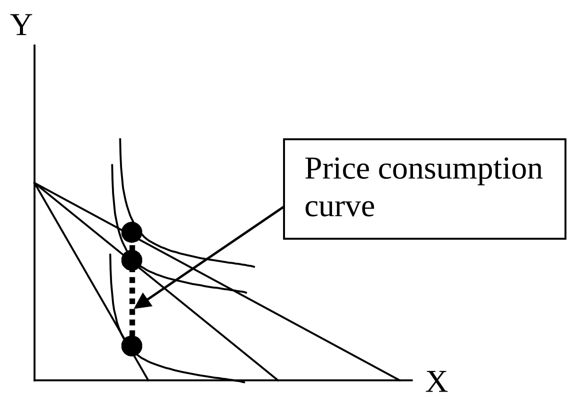
$$\varepsilon_{Q,I} = \left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{I}{Q}\right) < 0 .$$

Since I and Q must be greater than zero, for $\varepsilon_{Q,I}$ to be negative, we must have

$$\frac{\Delta Q}{\Delta I} < 0 .$$

This can only happen if either a) $\Delta Q < 0$ and $\Delta I > 0$ or b) $\Delta Q > 0$ and $\Delta I < 0$. In both instances, the change in quantity demanded moves in the opposite direction as the change in income implying the good is inferior.

5.4 If demand for good X is perfectly price inelastic then the demand curve is a vertical line and quantity remains constant as price changes. Graphing the price consumption curve for good X on an optimal choice diagram would appear as



The price consumption curve is a straight line because the level of consumption of of X is constant.

5.5 a) At the consumer's optimum we must have

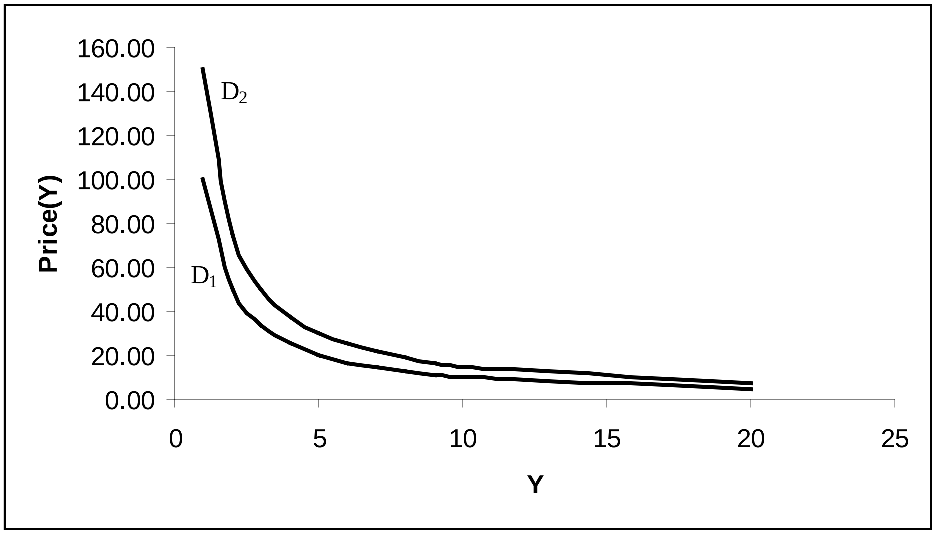
$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

$$\frac{y}{P_x} = \frac{x}{P_y}$$

Substituting into the budget line, $P_x x + P_y y = I$, gives

$$P_x\left(y\left(\frac{P_y}{P_x}\right)\right)+P_y y=I$$
$$2P_y y=I$$
$$y=\frac{I}{2P_y}$$

b) Yes, clothing is a normal good. Holding P_y constant, if I increases y will also increase.



c) The cross-price elasticity of demand of food with respect to the price of clothing must be zero. Note in part a) that with this utility function the demand for *y* does not depend on the price of *x*. Similarly, you can show that the demand for food is $x = I / (2P_x)$, which does not depend on the price of *y*. In fact, the consumer divides her income equally between the two goods regardless of the price of either. Since the demands do not depend on the prices of the other goods, the cross-price elasticities must be zero.

5.6 a) Karl's optimal bundle will always be such that $2H = 3B$. If this were not true then he could decrease the consumption of one of the two goods, staying at the same level of utility and reducing expenditure. Also, at the optimal bundle, it must be true that $P_H H + P_B B = I$. Substituting the first condition into the second we get $B(1.5P_H + P_B) = I$ which implies that the demand curve for beer is given by, $B = \dfrac{I}{(1.5P_H + P_B)}$

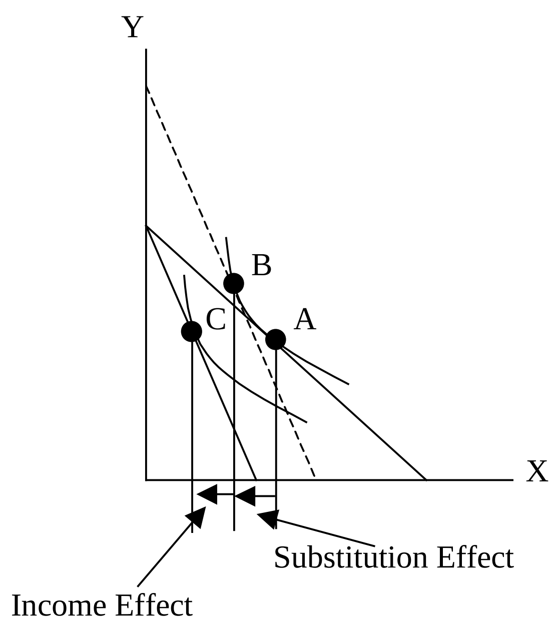
b) You can answer this just by looking at the demand curve. Because it has a larger coefficient, the price of hamburgers affects the demand for beer more than the price of beer. A one dollar increase in P_H decreases demand for beer more than a one dollar increase in P_B.

5.7 a) Denoting the level of income by *I*, the budget constraint implies that $p_x x + p_y y = I$ and the tangency condition is $\frac{1}{2\sqrt{x}} = \frac{p_x}{p_y}$, which means that $x = \frac{{p_y}^2}{4{p_x}^2}$. The demand for *x* does not depend on the level of income.

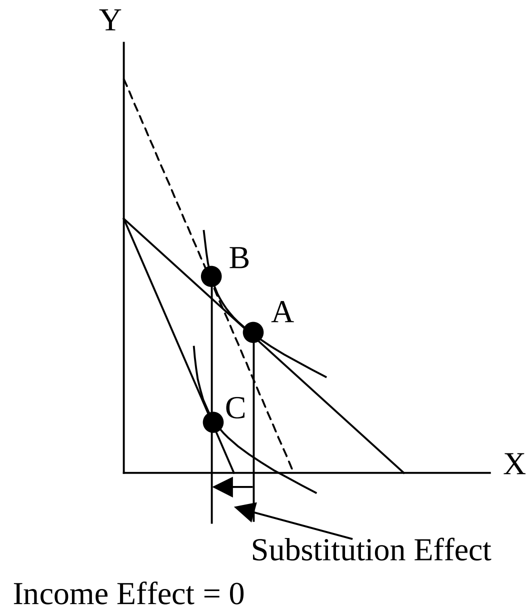
b) From the budget constraint, the demand curve for *y* is, $y = \frac{I - p_x x}{p_y} = \frac{I}{p_y} - \frac{p_y}{4p_x}$.

You can see that the demand for *y* increases with an increase in the level of income, indicating that *y* is a normal good. Moreover, when the price of *x* goes up, the demand for *y* increases as well.

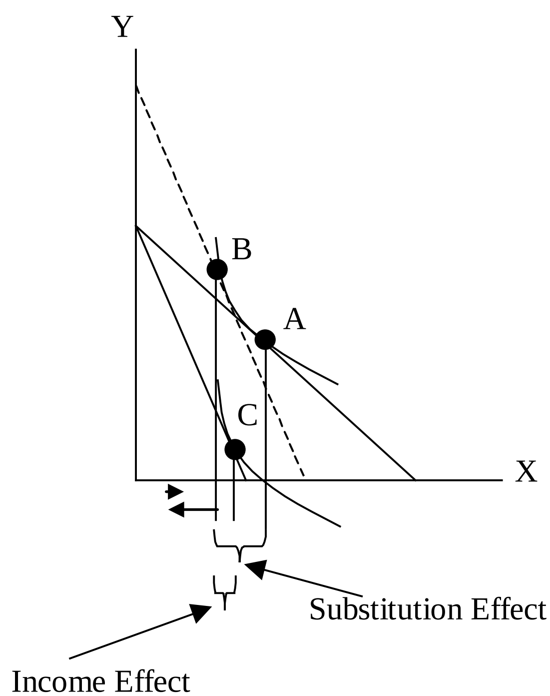
5.8 a)



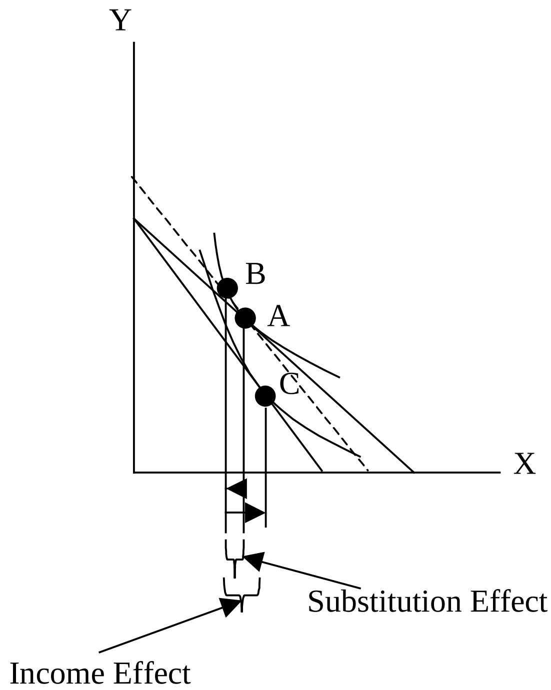
b)


Y
B
A
C
X
Substitution Effect
Income Effect = 0

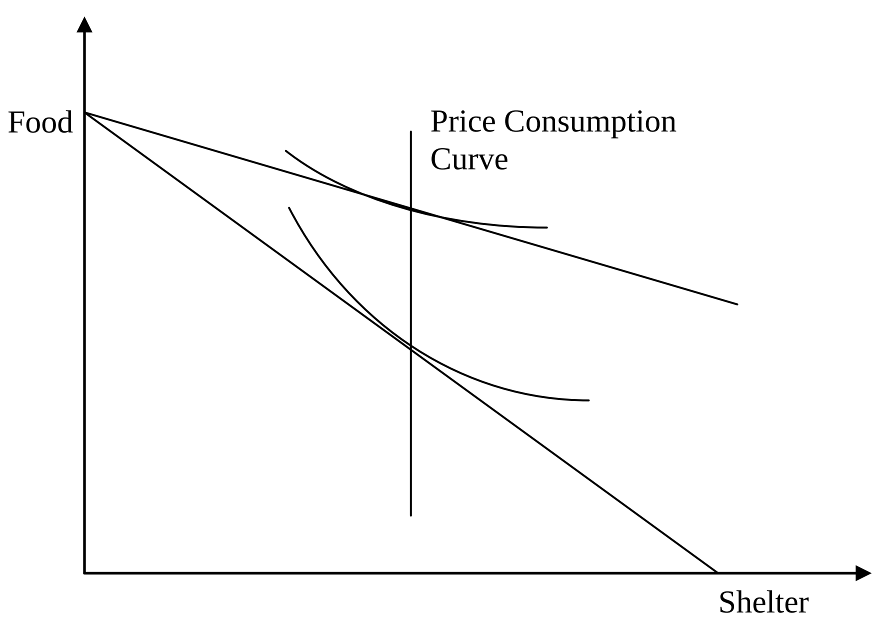
c)


Y
B
A
C
X
Substitution Effect
Income Effect

d)


Y
B
A
C
X
Substitution Effect
Income Effect

5.9


Food
Price Consumption
Curve
Shelter

A pair of possible indifference curves and budget lines are shown above. For the Price consumption curve to be a vertical line, it must be that Reggie's demand for shelter does not change even when the price of shelter changes and the budget line rotates.

The fact that his optimal bundle stays the same, despite a price change, means that Reggie's income and substitution effects as a result of a change in the price of shelter must cancel each other out so as to leave a net zero effect. For example, if the price of shelter were to decrease, the substitution effect would be positive and this would imply a *negative* income effect, just large enough to cancel out the substitution effect. In other words, the two effects have the same magnitude but opposite signs. This also implies that Reggie views shelter as an inferior good.

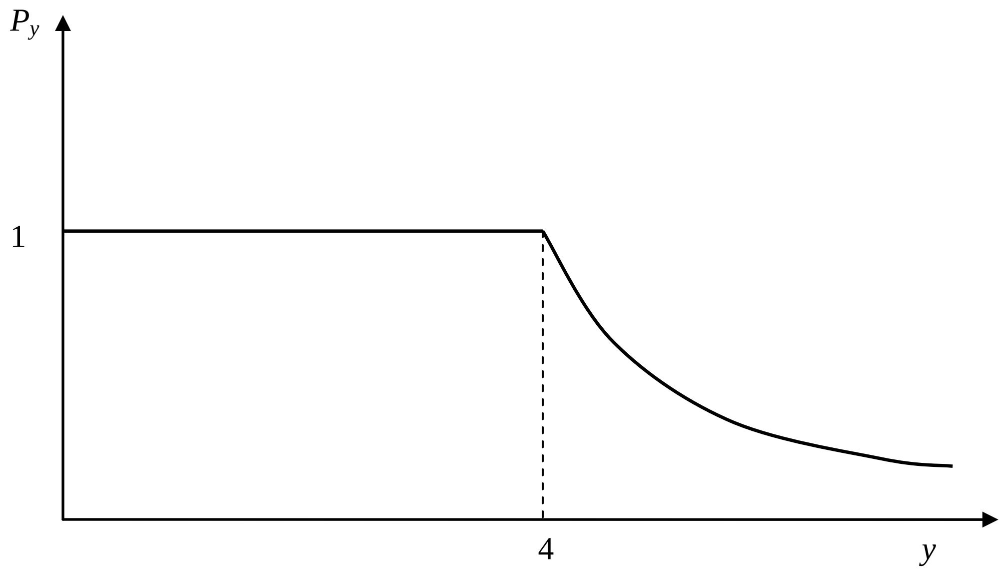
5.10 a) The budget constraint is $8x + 2y = 240$ and the tangency condition is $\frac{2y}{x} = \frac{8}{2} = 4$. Solving, the optimal bundle is (x, y)=(20, 40) with a utility of $20^2(40)$=16,000.

b) Now p_y=8. We need to calculate p_x such that, with the new prices, Ginger reaches exactly the same indifference curve as before. The new optimal bundle (x,y) must be such that: $p_x x + 8y = 240$, and $x^2 y = 16000$. The tangency condition now implies that $\frac{2y}{x} = \frac{p_x}{8}$ that is, $p_x x = 16y$. Substituting this into the budget constraint we find that y=10. Using the condition $x^2 y = 16000$, we find that x = 40. Finally, substituting the values of x and y back into the budget constraint, we can see that $p_x 40 + 8(10) = 240$, or p_x=4. Therefore, if the price of y were to increase to \$8, Ginger would need the price of x to decrease to \$4 in order to be exactly as well off as before.

5.11 a) Notice that MU_x / MU_y = 1 for all x and y. In this case indifference curves are straight lines with slope 1. Therefore, when P_x = 1 and P_y = 1 all pairs of x and y such that $x + y = 4$ are optimal baskets.

b) Optimal consumption in this case is at a corner point. Since the price of x is smaller than the price of y and marginal utility of reach good is the same, consumer is better off purchasing only x. (Another way to see this is to note that $MU_X/P_X = 1/1 > MU_Y/P_Y = ½$.) Hence, the optimal basket consists of 4 units of x and zero units of y.

c) When price of y is lower than 1 there are zero units of x in the optimal basket. Hence, for P_x = 1 and P_y < 1 the demand for y equals to I / P_y.



d) By the same argument Ann purchases only x when $P_x = 1$ and $P_y = 1$. Marginal utility per dollar from consumption of x is higher than marginal utility per dollar of y. When $P_x = 1$ and $P_y = 2$ marginal utilities per dollar are the same for both goods. Hence, all baskets such that $2x + y = 4$ are optimal. Construction of the demand curve is similar as in part c).

5.12 Consider any change in income ΔI. For the budget constraint to hold, it must be true that

$$\Delta I = P_x \Delta x + P_y \Delta y.$$

(For example, if income increases then some of it may be spent on x and some on y, but the total new expenditures must be equal to the change in income.) Since we are interested in income elasticities, it helps to rewrite the previous equation as

$$1 = P_x \frac{\Delta x}{\Delta I} + P_y \frac{\Delta y}{\Delta I}$$

Since $\varepsilon_{x,I} = (\Delta x/\Delta I)(I/x)$ and $\varepsilon_{y,I} = (\Delta y/\Delta I)(I/y)$, we can write this as

$$1 = P_x \frac{x}{I}\varepsilon_{x,I} + P_y \frac{y}{I}\varepsilon_{y,I}$$

Or

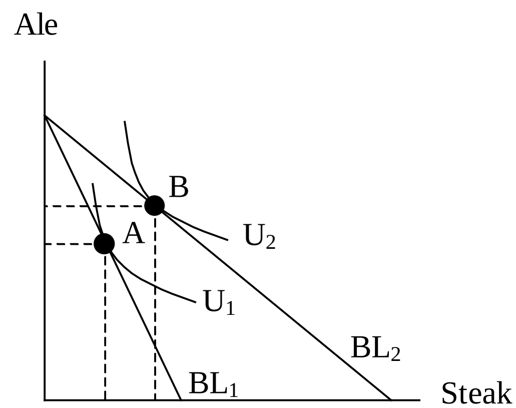
$$I = (P_x x)\varepsilon_{x,I} + (P_y y)\varepsilon_{y,I}$$

But if both goods are luxury goods, then $\varepsilon_{x,I} > 1$ and $\varepsilon_{y,I} > 1$ so that the previous equation implies

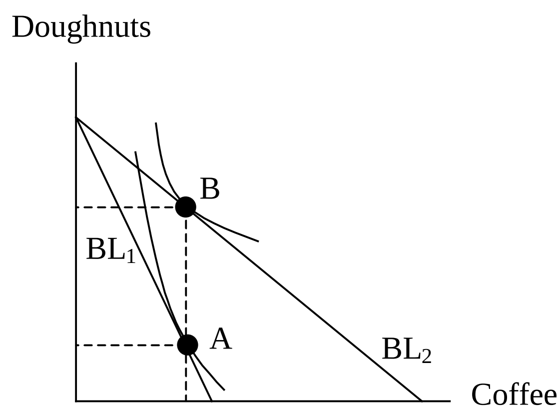
$$I > (P_x x)(1) + (P_y y)(1)$$

Thus, if both *x* and *y* are luxury goods then *I* > *I*, which obviously is untrue! Therefore, both goods cannot simultaneously be luxury goods.

5.13 When the price of steak falls, the budget line rotates from BL_1 to BL_2. The consumer now maximizes utility on U_2 at point B on BL_2. The amounts of steak and ale consumed at point B are greater than the initial amounts consumed at point A. This is shown in the following figure.



5.14 a)



In the diagram above, the consumer purchases the same amount of coffee and more doughnuts after the price of coffee falls.

b) No, this behavior is not consistent with a quasi-linear utility function. While it is true that there is no income effect with a quasi-linear utility function, the substitution effect would still induce the consumer to purchase more coffee when the price of coffee falls.

5.15 a) If we are at an interior optimum the tangency condition must hold:

$$\frac{y}{x+10} = \frac{P_x}{P_y}$$
$$P_y y = P_x(x+10)$$

Substituting into the budget line, $P_x x + P_y y = I$, yields

$$P_x x + P_x(x+10) = I$$
$$2P_x x + 10P_x = I$$
$$2P_x x = I - 10P_x$$
$$x = \frac{I}{2P_x} - 5$$

b) If $I = 100$, then

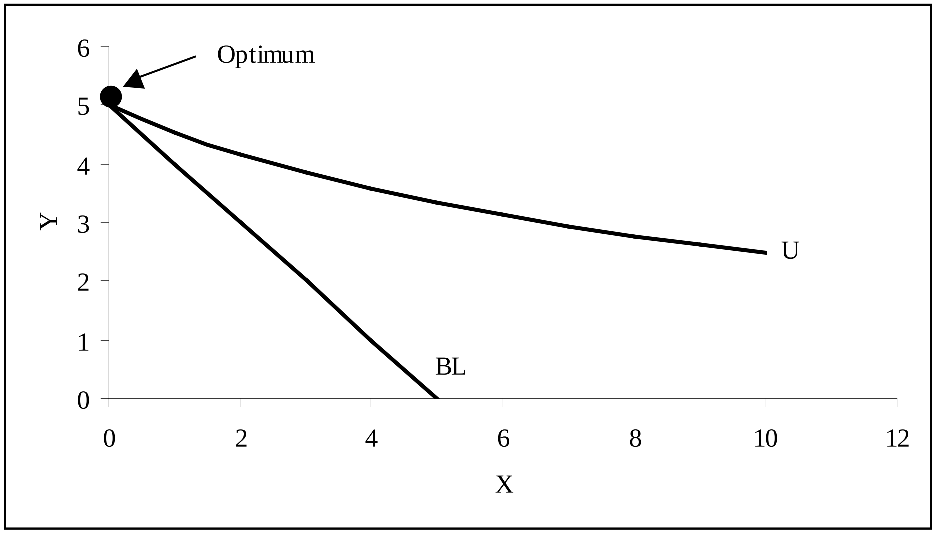
$$x = \frac{100}{2P_x} - 5$$
$$x = \frac{50}{P_x} - 5$$

Since we must have $x \geq 0$, we must have

$$\frac{50}{P_x} - 5 \geq 0$$
$$\frac{50}{P_x} \geq 5$$
$$50 \geq 5P_x$$
$$P_x \leq 10$$

So the consumer would only purchase x for prices less than 10.

c)



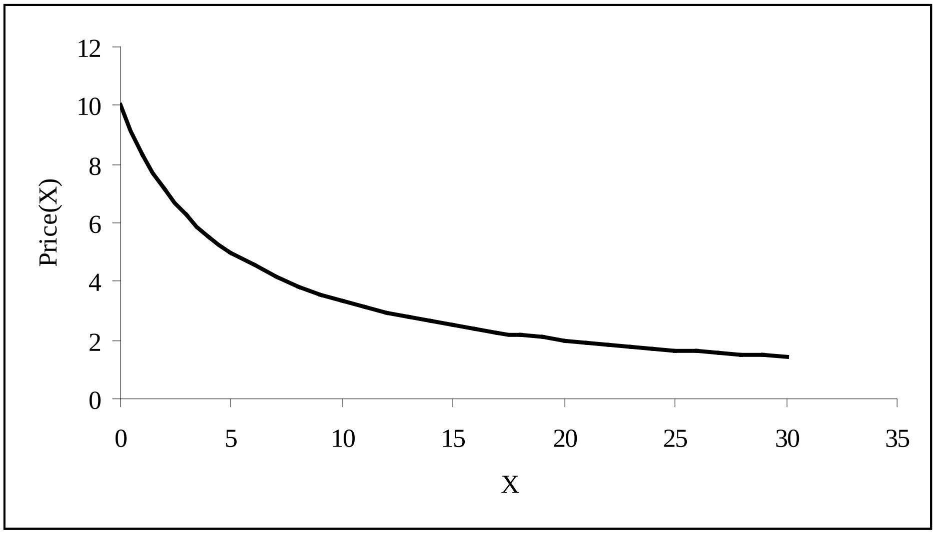
Given $P_x = P_y = 20$, the slope of the budget line is –1. At the corner point optimum, the slope of the indifference curve is

$$-\frac{MU_x}{MU_y} = -\frac{y}{x+10} = -\frac{5}{10} = -\frac{1}{2}$$

Because the indifference curve has a flatter slope than the budget line, the consumer would like to substitute more y for x, but has no more x to give up at the corner point.

d) $\frac{MU_x}{P_x} = \frac{5}{20} < \frac{10}{20} = \frac{MU_y}{P_y}$. If the consumer were to purchase any x, since the "bang for the buck" for x is less than the "bang for the buck" for y, the consumer would reduce total utility by increasing x above zero.

e)



As shown in part a), the demand for x depends only on I and P_x. Therefore, the location of the demand curve does not depend on P_y.

5.16 As the figure shows, a decrease in the price from p_1 to p_2 induces an increase in quantity from q_1 to q_2. The resulting change in consumer surplus is due to two things:

First, the consumer is paying a lower price, per unit, on all the units of the good that he was consuming before the price change. That is, for the q_1 units he was earlier consuming, he now pays a lower price and therefore enjoys a higher consumer surplus, denoted by the area of the rectangle ABCD. Another way of putting this is that if he continued to consume q_1 even after the price change his consumer surplus would increase by only area ABCD.

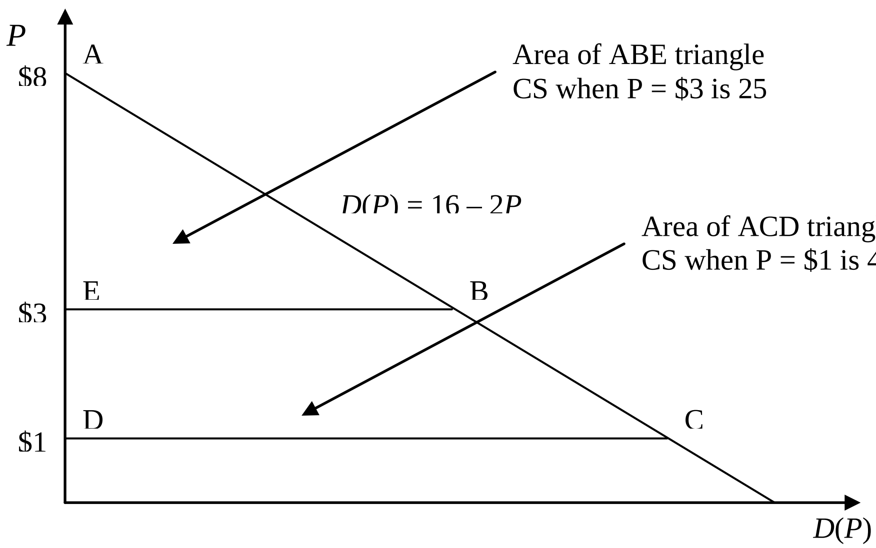
Second, the lower price induces him to consume more of the good in question. In fact he consumes $(q_2 – q_1)$ *more* units. The additional benefit he gets from this is the area of triangle BDE.

5.17 For price of a widget equal to $1 consumer surplus is

$$CS_{\$1} = ½ \cdot (8 – 1) \cdot D(1) = ½ \cdot 7 \cdot 14 = 49.$$

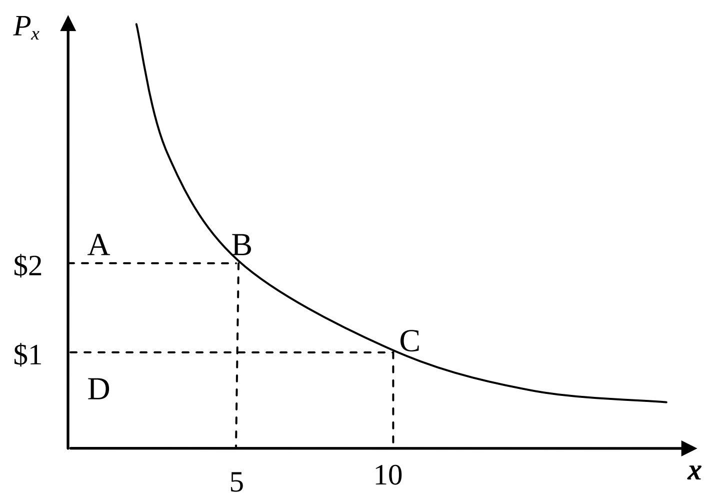
When price is equal to $3 consumer surplus is

$$CS_{\$3} = ½ \cdot (8 – 3) \cdot D(3) = ½ \cdot 5 \cdot 10 = 25.$$



5.18 a) Jim's optimal basket is a solution to equations $MU_x / MU_y = P_x / P_y$ and $P_x x + P_y y = I$. Hence, we have $y / x = P_x$ and $P_x x + y = 20$ with solution $x = 10 / P_x$ and $y = 10$. Demand schedule for x is $D(P_x) = 10 / P_x$.

b)



The change in consumer surplus is area of region ABCD under the demand curve. The are of this region can be computed by simple integration: $-\int_{[1,2]} 10/p \, dp = -10 \ln(2)$.

5.19 a) Using the tangency condition, $\frac{y}{x} = 4$, and the budget constraint, $4x + y = 120$, Lou's initial optimum is the basket (x, y) = (15, 60) with a utility of 900.

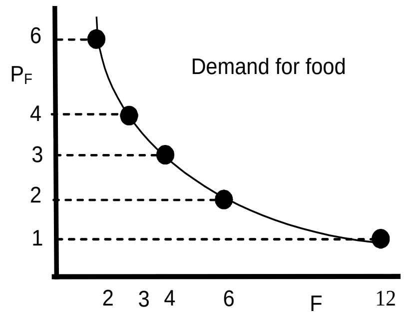
b) First we need the decomposition basket. This would satisfy the new tangency condition, $\frac{y}{x} = 3$ and would give him as much utility as before, i.e. $xy = 900$. This gives $(x, y) = (10\sqrt{3}, 30\sqrt{3})$ or approximately (17.3,51.9). Now we need the final basket, which satisfies the same tangency condition as the decomposition basket and also the new budget constraint: $3x + y = 120$. Together, these conditions imply that (x, y) = (20, 60). The substitution effect is therefore 17.3 – 15 = 2.3, and the income effect is 20 – 17.3 = 2.7.

c) The compensating variation is the amount of income Lou would be willing to give up after the price change to maintain the level of utility he had before the price change. This equals the difference between the consumer's actual income, $120, and the income needed to buy the decomposition basket at the new prices. This latter income equals: 3*17.3 + 1*51.9 = 103.8. The compensating variation thus equals 120 – 103.8 = $16.2.

d) The equivalent variation is the amount of income that Lou would need to be given *before* the price change in order to leave him as well off as he would be after the price change. After the price change his utility level is 20(60)=1200. Therefore the additional income should be such that it allows Lou to purchase a bundle (x, y) satisfying the initial tangency condition, $\frac{y}{x} = 4$, and also such that $xy = 1200$. This implies that $(x, y) = (10\sqrt{3}, 40\sqrt{3})$ or approximately (17.3, 69.2). How much income would Lou need to purchase this bundle under the original prices? He would need 4(17.3) + 69.2 = 138.4. That is he would need to increase his income by (138.4 – 120) dollars in order to be as well off as if the price of pizza were to decrease instead. Therefore his equivalent variation is $18.4.

5.20 a) $MU_F = C + 1$ $MU_C = F$
Tangency: $MU_F/MU_C = P_F / P_C$. $(C + 1)/ F = P_F/4$ => $4C + 4 = P_F F$. (Eq 1)
Budget Line: $P_F F + P_C C = I$. $P_F F + 4C = 20$. (Eq 2)
Substituting (Eq 1) into (Eq 2): 4C + 4 + 4C = 20. Thus C = 2, independent of P_F.

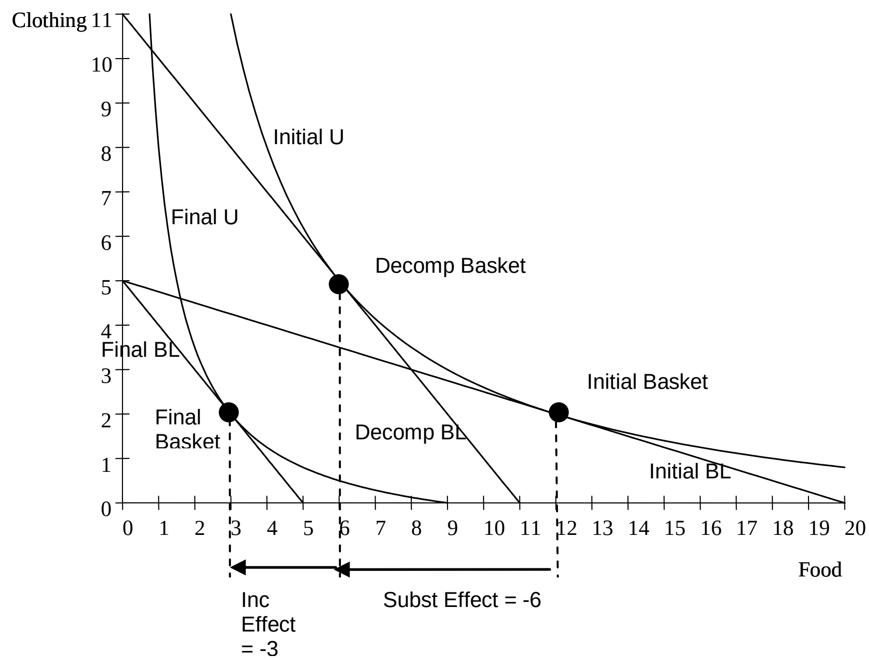
From the budget line, we see that $P_F F + 4(2) = 20$, so **the demand for F is F = 12/P_F .**



b) Initial Basket: From the demand for food in (a), F = 12/1 = 12, and C = 2.
Also, the initial level of utility is U = FC + F = 12(2) + 12 = 36.
Final Basket: From the demand for food in (a), we know that F = 12/4 = 3, and C = 2. (Also, U = 3(2) + 3 = 9.)
Decomposition Basket: Must be on initial indifference curve, with U = FC + F = 36 (Eq 5)
Tangency condition satisfied with final price: $MU_F/MU_C = P_F / P_C$. (C + 1)/ F = 4/4 => C + 1 = F. (Eq 3)
Eq 5 can be written as F(C + 1) = 36. Using Eq 3, $(C + 1)^2 = 36$, and thus, C = 5.
Also, by Eq 3, F = 6.
So the decomposition basket is F = 6, C = 5.

Income effect on F: $F_{\text{final basket}} - F_{\text{decomposition basket}} = 3 - 6 = -3$.
Substitution effect on F: $F_{\text{decomposition basket}} - F_{\text{initial basket}} = 6 - 12 = -6$.

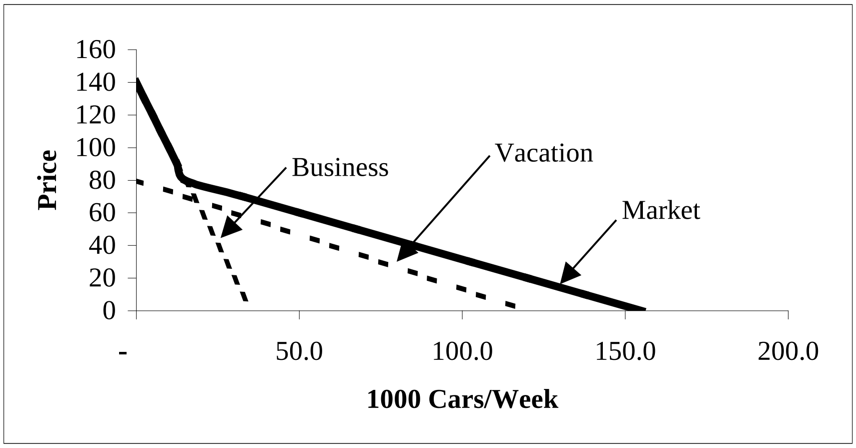
c) $P_F F + P_C C = 4(6) + 4(5) = 44$. So she would need an additional income of 24 (plus her actual income of 20).
The compensating variation associated with the increase in the price of food is -24.



5.21 a)

Price ($/day)	Business (000 cars/Week)	Vacation (000 cars/Week)	Market Demand (000 cars/Week)
100	10.0	-	10.0
90	12.5	-	12.5
80	15.0	-	15.0
70	17.5	15.0	32.5
60	20.0	30.0	50.0
50	22.5	45.0	67.5

b)



c) For price greater than $80, vacation traveler's demand will be zero. So above $P = 80$, market demand is $Q_b = 35 - 0.25P$.

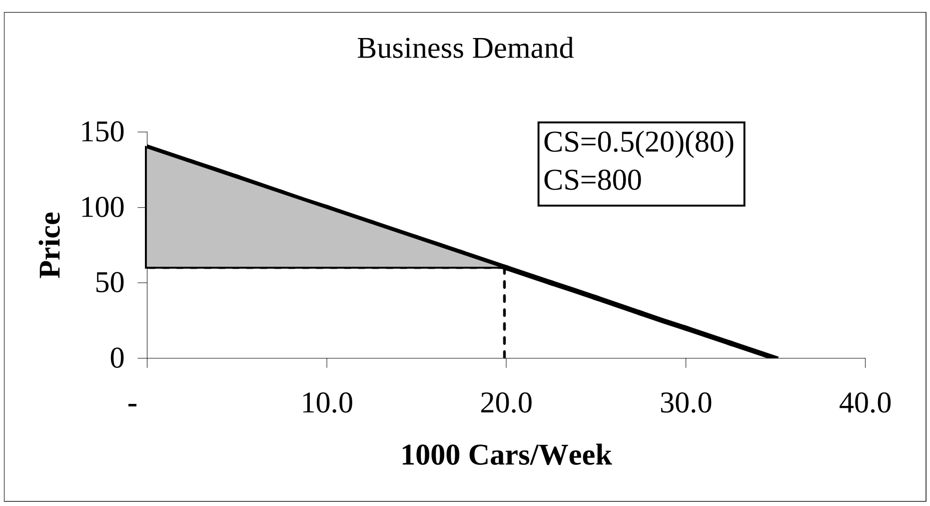
For price between $0 and $80, market demand is the sum of the vacation and business demand, $Q_m = Q_b + Q_v$, or

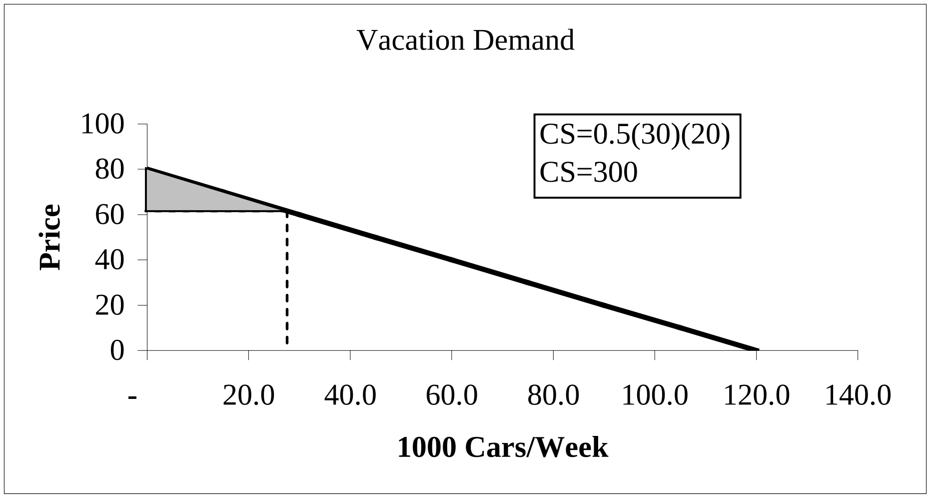
$$Q_m = 35 - 0.25P + 120 - 1.5P$$
$$Q_m = 155 - 1.75P$$

Above a price of $140, no purchases will be made so market demand is zero. In summary,

$$Q_m = \begin{cases} 0, & \text{when } P \geq 140 \\ 35 - 0.25P, & \text{when } 80 \leq P < 140 \\ 155 - 1.75P, & \text{when } P < 80 \end{cases}$$

d)





5.22 a) Jim's optimal basket is a solution to equations
$MU_x / MU_y = P / P_y$ and $P\, x + P_y\, y = I_J$.
Hence, we have $2xy / x^2 = P$ and $P\, x + y = 100$
with solution $x = 200 / (3P)$ and $y = 100 / 3$.
Analogous system of equations for Donna is
$y / x = P$ and $P\, x + y = 150$ with solution $x = 75 / P$ and $y = 75$.

b) Approximate shape of the demand curve for Jim and Donna is depicted below.

c) Aggregate demand is

$$D_x(P) = 200 / (3P) + 75 / P = 445 / (3P).$$

d) When there is one more consumer that has preferences identical to Donna's then her demand is also 75 / P and hence aggregate demand is

$$D_x(P) = 200 / (3P) + 75 / P + 75 / P = 650 / (3P).$$

Shape of the demand curve in this case is the same as in part b).

5.23 The market demand and individual demand will have the same price elasticity given any particular price. Denote an individual's demand curve by $Q_i(P)$. With 1,000,000 identical individuals the market demand curve will be $Q_m(P) = 1{,}000{,}000Q_i(P)$. At a given price P, an individual's demand curve will have elasticity $\varepsilon_{Q_i,P} = (\Delta Q_i / \Delta P)(P / Q_i)$. Since $Q_m(P) = 1{,}000{,}000Q_i(P)$, it must also be true that

$$\frac{\Delta Q_m}{\Delta P} = 1{,}000{,}000\frac{\Delta Q_i}{\Delta P}$$

The elasticity for the market demand curve will be

$$\varepsilon_{Q_m,P} = \frac{\Delta Q_m}{\Delta P}\frac{P}{Q_m} = 1{,}000{,}000\frac{\Delta Q_i}{\Delta P}\frac{P}{1{,}000{,}000Q_i} = \frac{\Delta Q_i}{\Delta P}\frac{P}{Q_i} = \varepsilon_{Q_i,P}$$

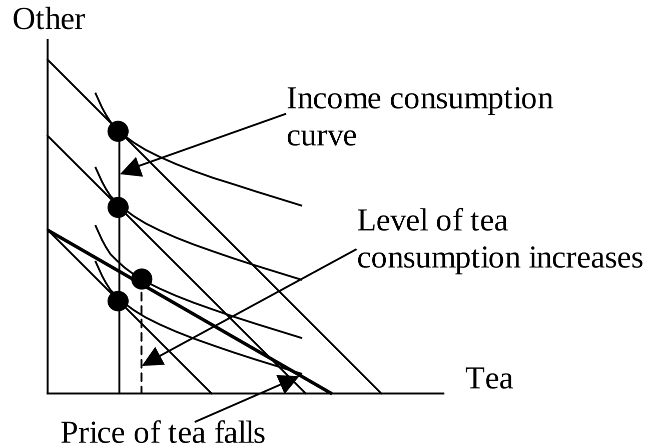
In other words, with identical consumers the elasticity of the market demand curve will equal the elasticity of the individual demand curve at any price P.

5.24 Bart will only consume when the price is less than 10. Therefore his demand curve for 7-UP is $Q_B = \frac{10-P}{4}$, when P<10 and zero otherwise. Homer will only consume if the price is less than 25 so his demand curve is $Q_H = \frac{25-P}{2}$, when $P < 25$ and zero otherwise.

Therefore the market demand curve for 7-UP as a function of all possible values of price is:

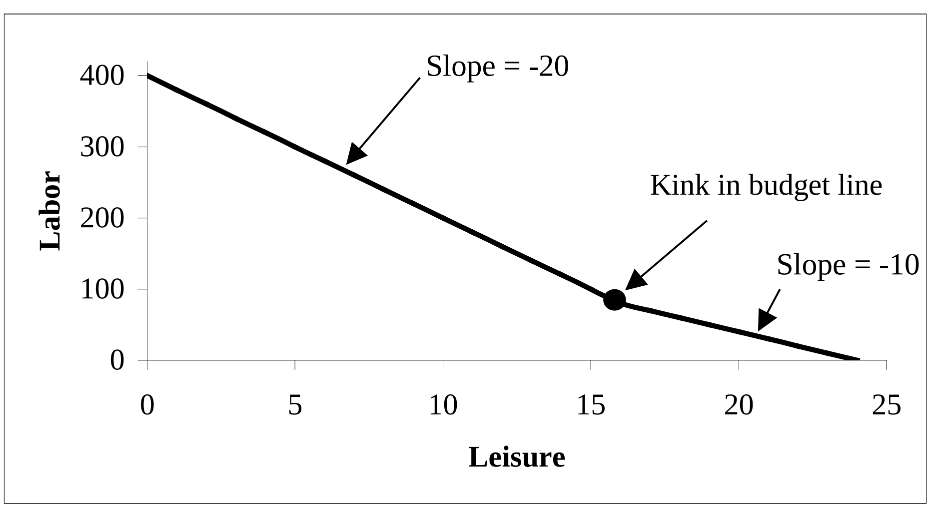
$$Q^M = 0, \text{if } P > 25$$
$$Q^M = \frac{25-P}{2}, \text{if } 10 < P < 25$$
$$Q^M = \frac{60-3P}{4}, \text{if } P < 10$$

5.25 a) If the income consumption curve is vertical the utility function has no income effect. This will occur, for example, with a quasi-linear utility function. This utility function will have the same marginal rate of substitution for any particular value of tea regardless of the level of total utility. If the price of tea falls, flattening the budget line, the consumer will reach a new optimum where the marginal rate of substitution is equal to the slope of the new budget line. Since the budget line has flattened, this cannot occur at the previous optimum amount of tea. The substitution effect implies that this new optimum level of tea will be greater than the previous level. Thus, when the price of tea falls, the quantity of tea demanded increases, implying a downward sloping demand curve. This can be seen in the following figure.



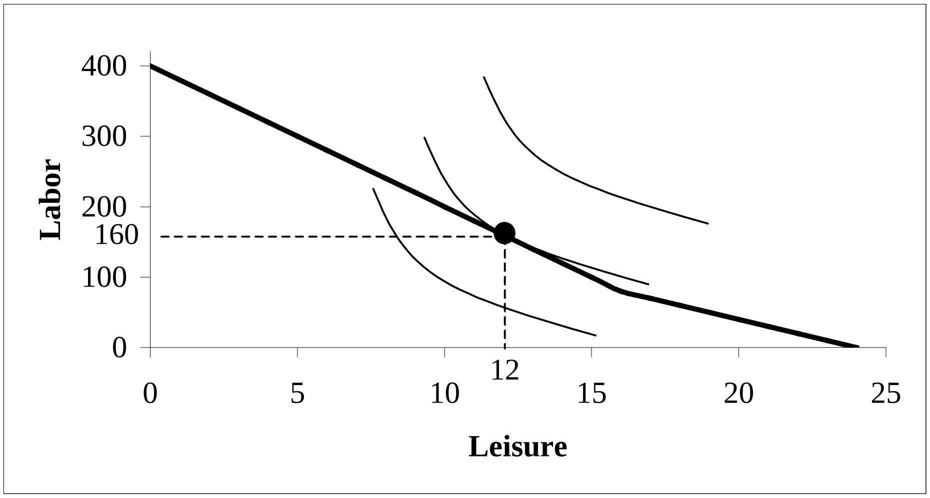
b) Yes, the values will be exactly $30. When the income consumption curve is vertical, the consumer's utility function has no income effect. As stated in the text, when there is no income effect, compensating and equivalent variation will be identical and these will also equal the change in consumer surplus measured as the change in the area under the demand curve.

5.26 a)



Because the wage rate changes for any hours worked over eight (leisure less than sixteen) the budget line has a kink at sixteen hours of leisure.

b)



With this set of indifference curves, the consumer reaches an optimum at 12 hours of leisure and 12 hours of labor, or \$160 of income.

5.27 If Terry's wage rate is w, then the income he earns from working is $(24 – L)w$. Since P_Y = 1, the number of units of other goods he purchases is $Y = (24 – L)w$.

Now at an optimal bundle, Terry's $MRS_{L,Y}$ must equal the price ratio $w/P_Y = w$.

Therefore, the tangency condition tells us that $\frac{Y}{1+L} = w$. The two conditions imply $w(1+L) = (24-L)w$. This means that the optimal amount of leisure is $L = 11.5$. You can see that this does not depend on the wage rate.

5.28 If Noah's wage rate is w, then the income he earns from working is $(24 – L)w$. Since P_Y = 1, the number of units of other goods he purchases is $Y = (24 – L)w$. Also, the tangency condition gives us $\sqrt{\frac{Y}{L}} = w$. Combining the two conditions, $w^2 L = (24-L)w$, or $L = \frac{24}{w+1}$. Clearly, the amount of leisure that Noah consumes decreases with an increase in the wage rate, and this is true no matter what the wage rate is. Since the amount of labor that Noah supplies equals $(24 – L)$, we see that his supply of labor always increases with an increase in the wage rate. So, his labor supply curve is always positively sloped – that is, it is not backward bending.

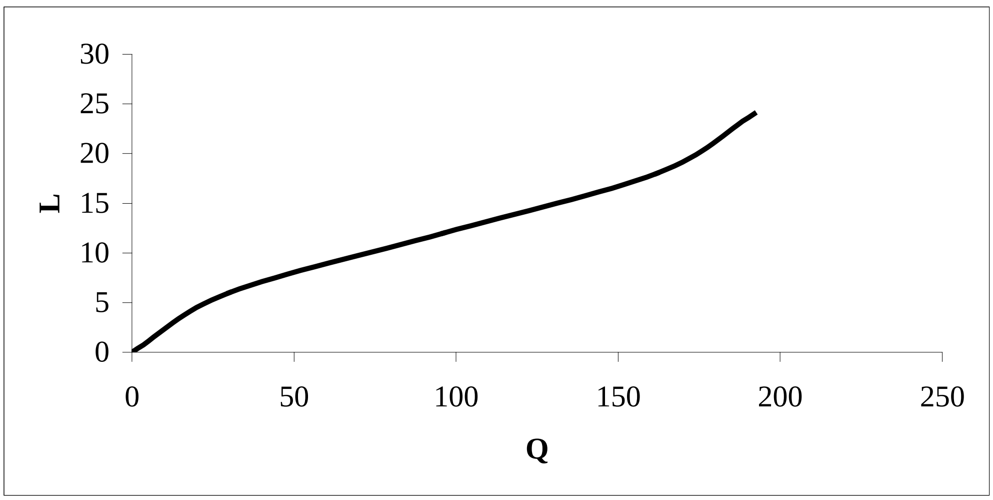
Chapter 6

Inputs and Production Functions

Solutions to Review Questions

1. The production function tells us the maximum volume of output that may be produced given a combination of inputs. It is possible that the firm might produce less than this amount of output due to inefficient management of resources. While it is possible to produce many levels of output with the same level of inputs, some of which are less technically efficient than others, the production function gives us the upper bound on (the *maximum* of) the level of output.

2. The labor requirements function, which is the inverse of the production function, tells us the minimum amount of labor that is required to produce a given amount of output.



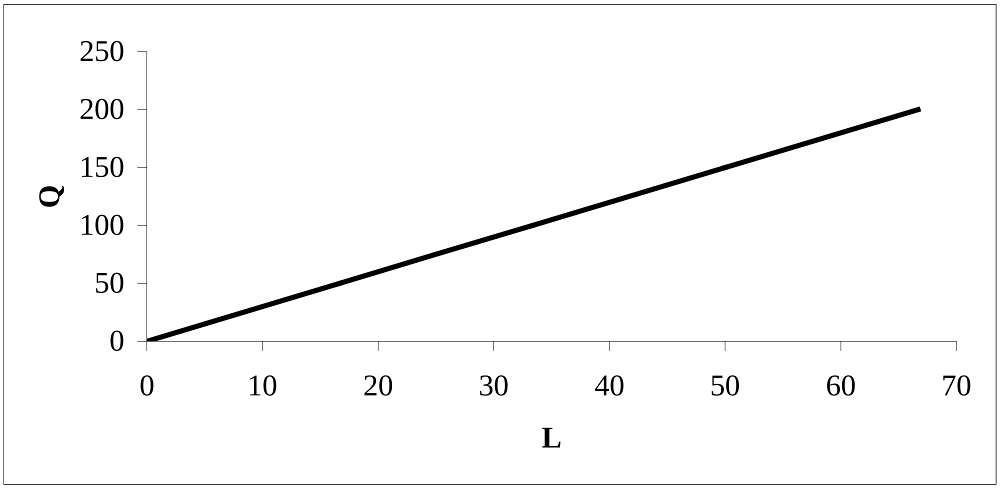
3. The average product of labor is the average amount of output per unit of labor.

$$AP_L = \frac{\text{Total Product}}{\text{Quantity of Labor}} = \frac{Q}{L}$$

The marginal product of labor is the rate at which total output changes as the firm changes its quantity of labor.

$$MP_L = \frac{\text{Change in Total Product}}{\text{Change in Quantity of Labor}} = \frac{\Delta Q}{\Delta L}$$

The total product function in the graph below $Q = 3L$, which is linear, would have the average and marginal products coincide. In particular, for all values of Q we would have $AP_L = MP_L = 3$.

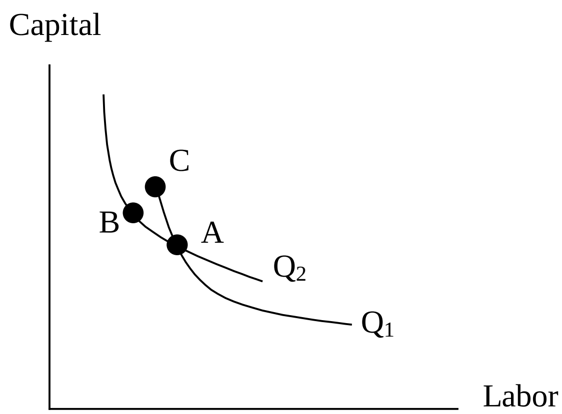


4. With diminishing total returns to an input, increasing the level of the input will decrease the level of total output holding the other inputs fixed. Diminishing marginal returns to an input means that as the use of that input increases holding the quantities of the other inputs fixed, the marginal product of that input will become less and less. Essentially, diminishing total returns implies that output is decreasing while with diminishing marginal returns we could have output increasing, but at a decreasing rate as the amount of the input increases.

   It is entirely plausible to have a total product function exhibit diminishing marginal returns but not diminishing total returns. This would occur when each additional unit of an input increased the total level of output, but increased the level of output less than the previous unit of the input did. Essentially, this occurs when output is increasing at a decreasing rate as the level of the input increases.

5. If the marginal product of labor is positive, then when we increase the level of labor holding everything else constant this will increase total output. To keep the level of output at the original level, we need to stay on the same isoquant. To do so, since the marginal product of capital is positive we would then need to *reduce* the amount of capital being used. So, to keep output constant, when the level of one input increases the level of the other input must decrease. This negative relationship between the inputs implies the isoquant will have a negative slope, *i.e.,* be downward sloping.

6. No, as with indifference curves, isoquants can never cross. For example, suppose we draw isoquants for two levels of output Q_1 and Q_2 with $Q_2 > Q_1$. In addition, suppose that these isoquants crossed at some point A as in the following diagram.

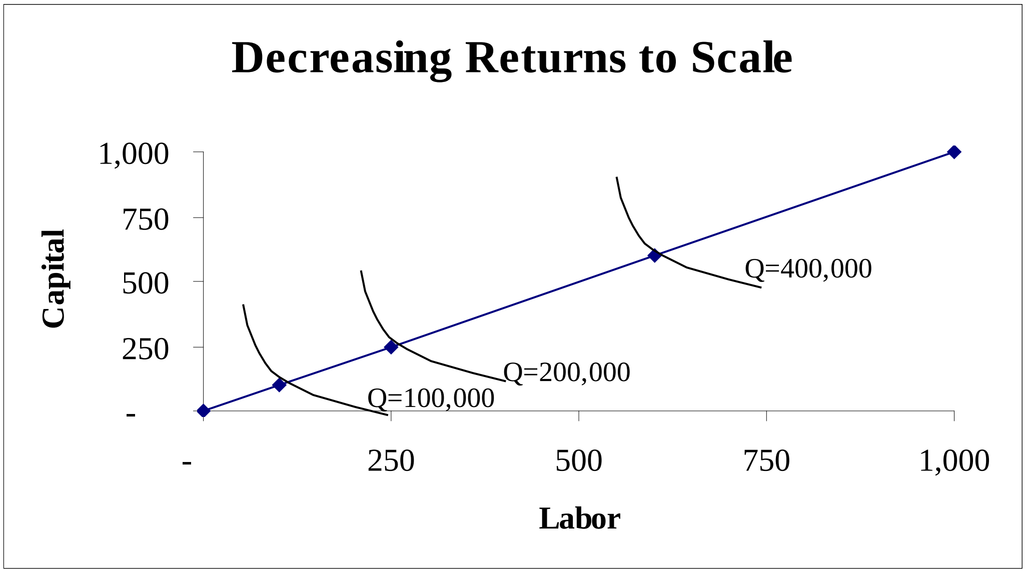


Because A and B are on Q_2, both achieve the same level of output. Since A and C are on Q_1, both achieve the same level of output. This would imply that B and C achieve the same level of output. However, this is not possible since point C contains more of both inputs which would achieve a higher level of output. Therefore, isoquants cannot cross.

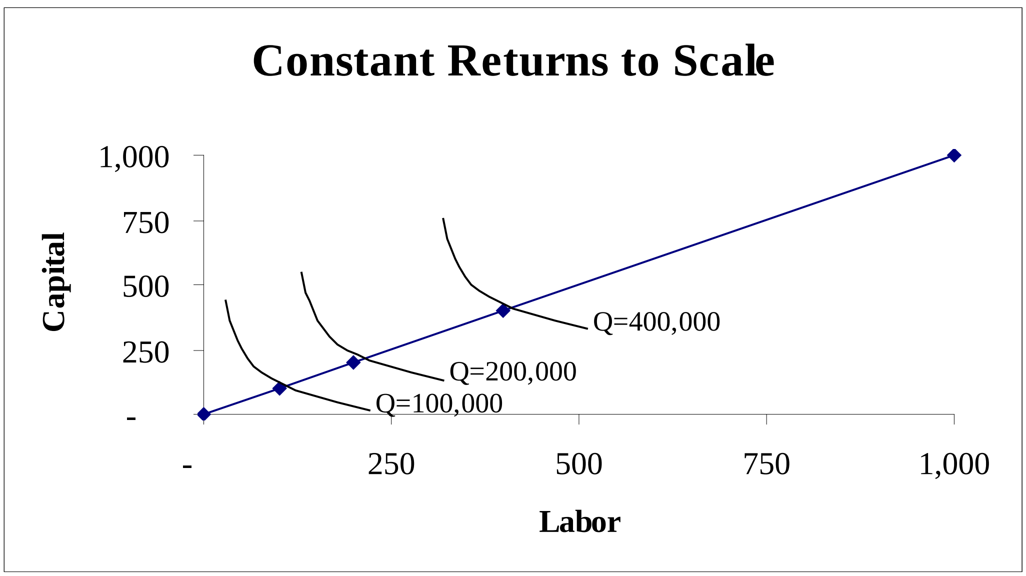
7. By operating on the uneconomic portion of an isoquant, the firm would be using a combination of inputs in which one of the inputs has a negative marginal product, *i.e.*, increasing the input decreases the level of total output. At a point such as this, the firm could increase output by decreasing the level of the input. By decreasing the level of the input, the firm could decrease total cost. Thus, if a firm were operating on the uneconomic region of an isoquant it could simultaneously increase output and decrease total cost. Thus, a cost-minimizing firm would never operate on this portion of an isoquant because it would always take advantage of this opportunity.

8. The elasticity of substitution measures how the marginal rate of technical substitution of labor for capital changes as we move along an isoquant. Essentially this value tells us the level of substitutability between capital and labor, *i.e.*, how easily the firm can substitute capital for labor to maintain the same level of total output.

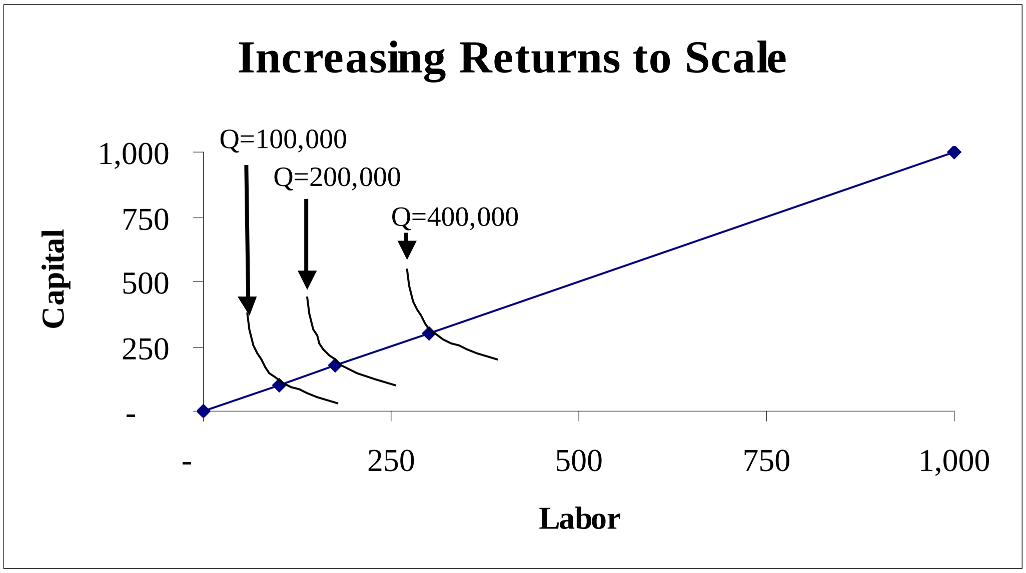
9.



With decreasing returns to scale the firm needs to more than double inputs to double output. Equivalently, doubling inputs less than doubles output.



With constant returns to scale the firms needs to double inputs to double output.



With increasing returns to scale the firm needs to less than double the inputs to double the output. Equivalently, doubling inputs more than doubles output.

Solutions to Problems

6.1 a) $AP_F = \frac{Q}{F} = \frac{2200}{4} = 550$.

b) $MP_F = \frac{\Delta Q}{\Delta F} = \frac{2600-2500}{6-5} = 100$.

c) Diminishing marginal returns set in when MP_F for some unit is lower than MP_F for the previous unit. This occurs for $F > 3$.

d) Diminishing total returns set in at the point where total output begins to fall. This occurs for $F > 6$.

6.2 a) The input combination gives *Q* = 97 so it is infeasible.

b) *Q* = 129.6 which is greater than 100, so feasible, but inefficient.

c) *Q* = 112 so again feasible but inefficient.

d) *Q* = 100 therefore the required output is feasible and the input combination is efficient.

6.3 The completed table is shown below:

L	Q
0	0
1	5
2	16
3	27
4	32
5	25
6	0

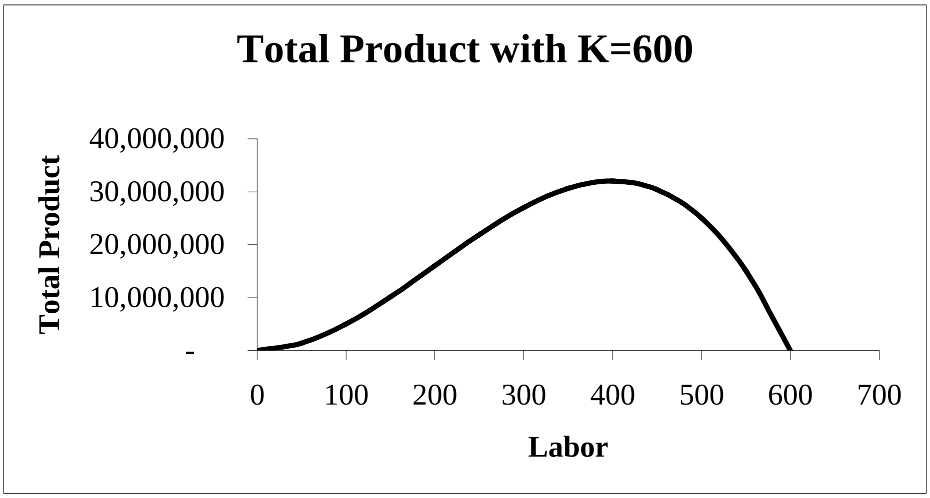
a) You can calculate the average product at each point by just dividing total output by L. The values obtained are 0,5,8,9,8,5,0. Therefore Average Product is maximized when L = 3.

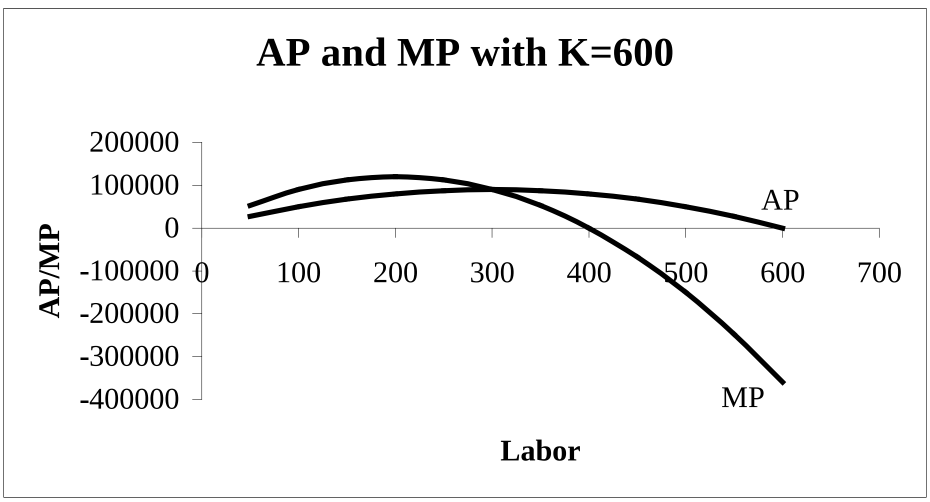
b) The marginal product at values 1 through 6 are respectively: 5,11,11,5,–7, –25. Therefore both the second and the third unit of L give the greatest marginal increase in output [if you use calculus techniques it can be seen that marginal product is maximized when L = 2].

c) From the Table it is clear that total product is maximized when L = 4.

d) Average Product will be zero only when Total Product is zero. This happens when L = 6.

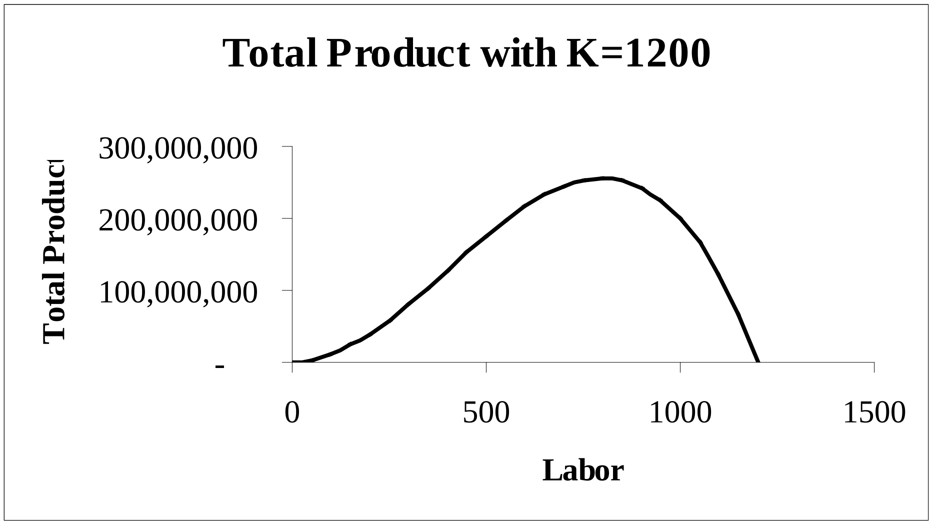
6.4 a)

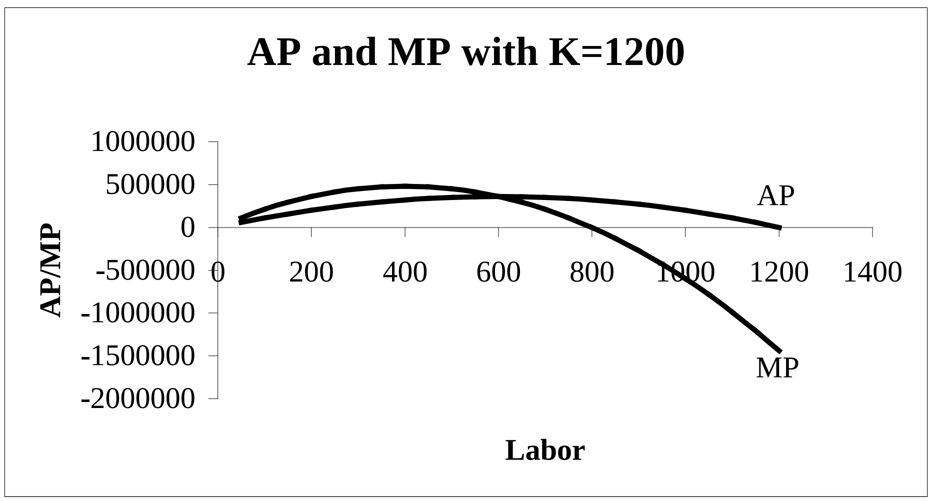




Based on the figure above it appears that the average product reaches its maximum at L = 300. The marginal product curve appears to reach its maximum at L = 200.

b)





Based on the figure above it appears that the average product curve reaches its maximum at $L = 600$. The marginal product curve appears to reach its maximum at $L = 400$.

c) In both instances, for low values of L the total product curve increases at an increasing rate. For $K = 600$, this happens for $L < 200$. For $K = 1200$, it happens for $L < 400$.

6.5 a) Incorrect. When $MP > AP$ we know that AP is increasing. When $MP < AP$ we know that AP is decreasing.

b) Incorrect. If MP is negative, $MP < 0$. But $AP = Q / L$ can never be negative since total product Q and the level of input L can never be negative. Thus, $MP < 0 < AP$, which only implies that AP is falling.

c) Incorrect. Average product is always positive, so this tells us nothing about the change in total product. Whether or not total product is rising depends on whether or not marginal product is positive.

d) Incorrect. If total product is increasing we know that $MP > 0$. If diminishing marginal returns have set in, however, marginal product will be positive but decreasing, but that does not preclude $MP > 0$.

6.6 To develop the answer, suppose that we were initially producing 64 units of steel. According to the table, we could do this with 8 units of labor and 100 units of capital.

Now, since we have constant returns to scale, if we double the amount of labor *and* capital, i.e., L = 16 and K = 200, we can double output, i.e., produce Q = 128 units of steel.

But notice from the table that the input combination L = 16 and K = 100 results in an even greater output of Q = 256 units of steel. Thus, by *reducing* the amount of capital it uses (from K = 200 to K = 100), holding the quantity of labor fixed, the firm can produce more output! That is, the marginal product of capital is negative over this range.

We can see the same thing if we start with any other input combination. For example, suppose the firm is initially producing 4 units of steel using 2 units of labor and 100 units of capital. Because of constant returns to scale, if we double the amount of labor and capital, i.e., L = 4 and K = 200, we can double output, i.e., produce Q = 8 units of steel.

But notice from the table that the input combination L = 4 and K = 100 results in an even greater output of Q = 16 units of steel. Again, by reducing the amount of capital it uses (holding the quantity of labor fixed), the firm can produce more output. Again, we see that the marginal product of capital is negative.

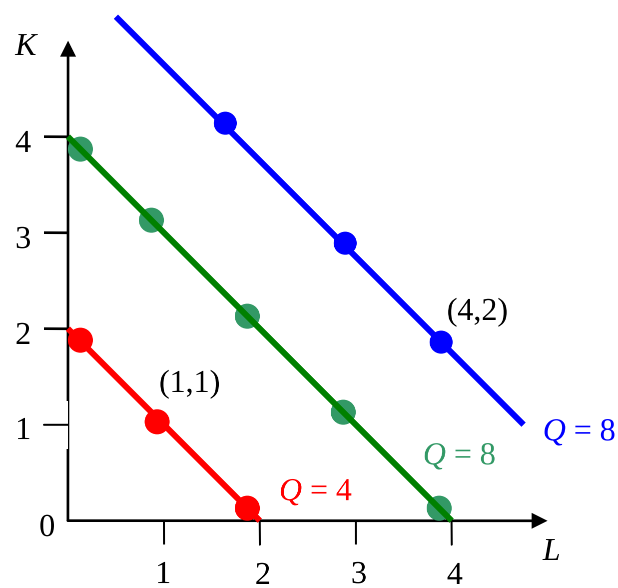
The above calculations illustrate that a two-input production function with (a) constant returns to scale and (b) increasing marginal returns to labor must necessarily imply that the marginal product of capital is negative. And, of course, if the marginal product of capital is negative, the firm can expand output by reducing the amount of capital it uses. It could, theoretically, produce an enormous amount of steel in a backyard blast furnace.

Because this conclusion is absurd, the point of the illustration is that with constant returns to scale, marginal returns to labor cannot be everywhere increasing. Eventually the law of diminishing marginal returns must set in.

6.7 The correct answers are shown in bold face red type.

Labor, *L*	Total product, *Q*	AP_L	MP_L
0	0	0	----
1	19	**19**	19
2	**72**	36	**53**
3	**153**	**51**	**81**
4	256	64	103
5	375	**75**	**119**
6	**504**	**84**	129
7	637	91	133
8	**768**	96	**131**
9	891	**99**	**123**
10	**1000**	100	**109**
11	1089	**98**	89
12	**1152**	96	**63**
13	**1183**	**91**	**31**
14	**1176**	**84**	-7
15	**1125**	75	-51

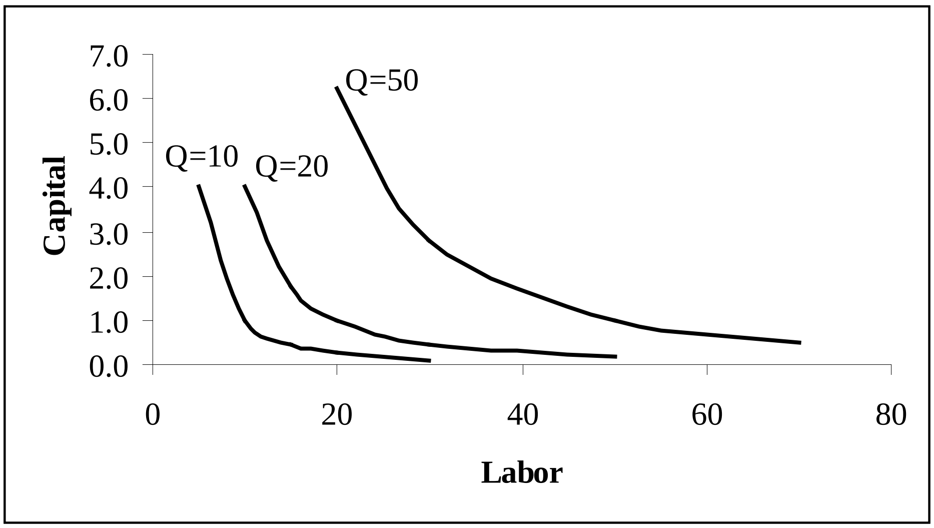
6.8 a) Three sample isoquants: red for production of 4 widgets (Q = 4), green for production of 8 widgets (Q = 8), and blue for production of 12 widgets (Q = 12). The dots represent particular combinations of inputs.



b) Recall that marginal product of an input, say labor, is given by $\Delta Q/\Delta L$. If we compute the marginal product of labor and capital at any point in the table, we find that it always equals 2. For example, in moving from input combination (2,2) to (3,2), we increase output from 8 to 10. Hence, MP_L = (10 – 8)/(3 – 2) = 2.

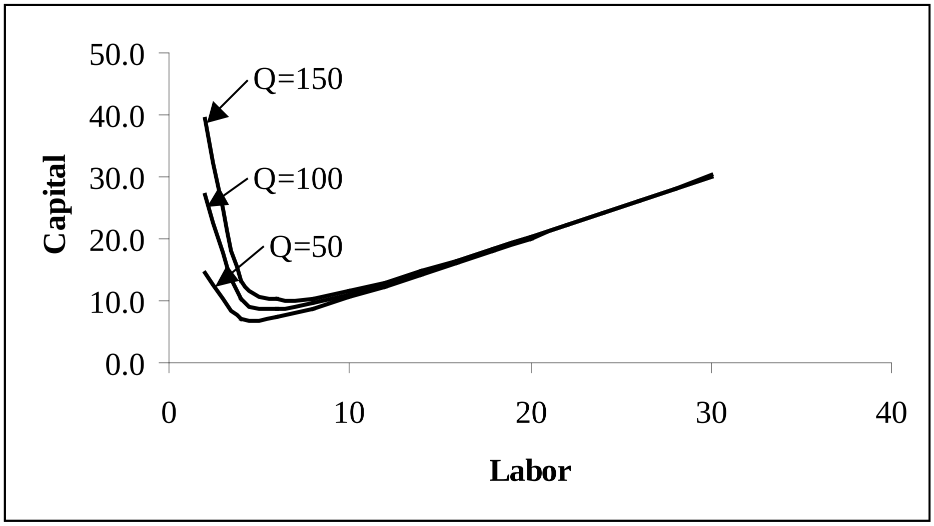
c) From the table, we see that as we increase the quantity of each input by a given proportion, the quantity produced increases by the same proportion. Hence, in moving from input combination (1,1) to (3,3), we are tripling the quantity of labor and capital used. As a result, the quantity of output produced triples as well.

6.9



Because these isoquants are convex to the origin they do exhibit diminishing marginal rate of technical substitution.

6.10 a)



b) Because each of these isoquants has an upward-sloping portion beyond some level of labor, each one does indeed have an uneconomic region.

6.11 For this production function $MP_L = a$ and $MP_K = b$. The $MRTS_{L,K}$ is therefore

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{a}{b}$$

6.12 First, note that $MRTS_{L,K} = L/K$, which diminishes as L increases and K falls as we move along an isoquant. So $MRTS_{L,K}$ is diminishing. However, the marginal product of capital MP_K is *increasing* (not diminishing) as K increases (remember, the amount of labor is held fixed when we measure MP_K.) Similarly, the marginal product of labor is *increasing* as L grows (again, because the amount of capital is held fixed when we measure MP_L). This exercise demonstrates that it is possible to have a diminishing marginal rate of technical substitution even though both of the marginal products are increasing.

6.13 a) At point *A*, the firm produces 18 units of output. Therefore, since *B* is on the same isoquant, it must be that $L = 17$ at *B*.

b) The capital-to-labor ratio at *A* is 3/5 and $MRTS_{L,K} = ½$. At *B*, the capital-to-labor ratio is 1/17, and $MRTS_{L,K} = 1/18$.

Therefore the elasticity of substitution is

$$\frac{(1/17 - 3/5)/(3/5)}{(1/18 - 1/2)/(1/2)} = \frac{69}{68}.$$

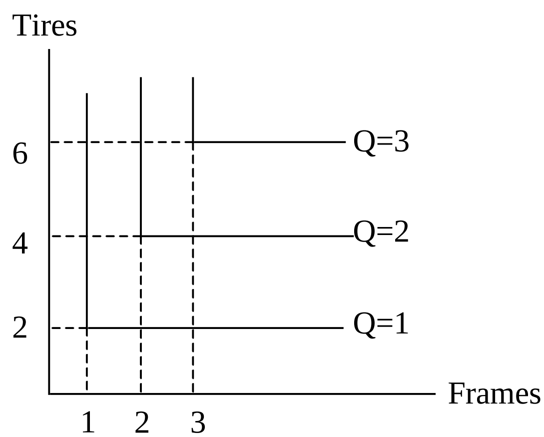
A Cobb-Douglas production function has an elasticity of substitution of 1. Therefore this production function has a slightly higher elasticity of substitution, indicating a slightly greater ease of substitutability of inputs.

6.14 Since the capital-labor ratio at B is twice that at A, it implies that the percent change in this ratio as we move from A to B is 100%. If we denote the percent change in the MRTS over these two points as x then using the definition of elasticity of substitution,

$$\frac{100}{\%\Delta MRTS_{L,K}} = 2, \text{ which means that } \%\Delta MRTS_{L,K} = 50.$$

Equivalently, $\frac{MRTS_B - MRTS_A}{MRTS_A} \times 100 = 50.$ Solving, $\frac{MRTS_A}{MRTS_B} = \frac{2}{3}$.

6.15 a) This isoquants for this situation will be L-shaped as in the following diagram



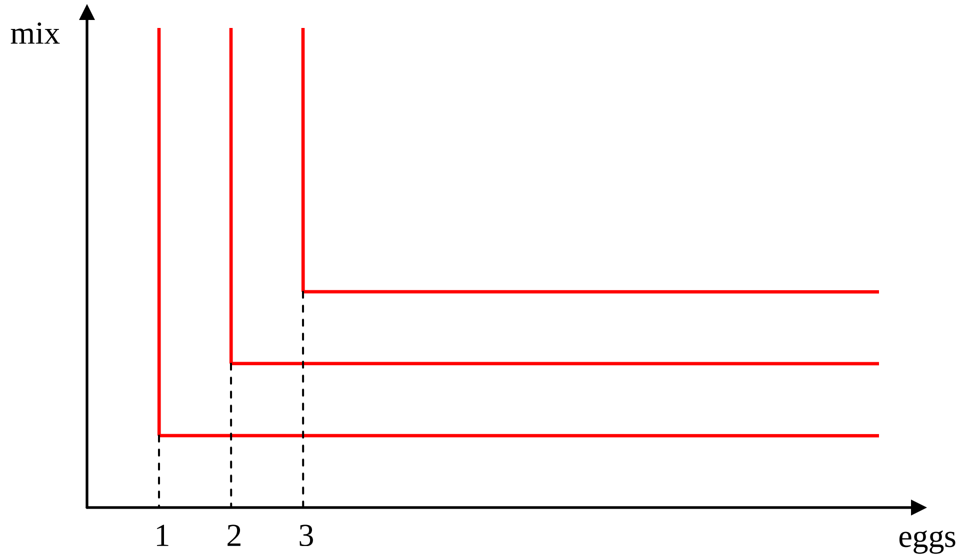
These L-shaped isoquants imply that once you have the correct combination of inputs, say 2 frames and 4 tires, additional units of one resource without more units of the other resource will not result in any additional output.

b) Mathematically this production function can be written

$$Q = \min(F, \frac{1}{2}T)$$

where F and T represent the number of frames and tires.

6.16 a)



b) The formula is # of cakes = Min{# of eggs, # of ingredients' packages}. This production function has constant returns to scale. To see why, let x and y denoted the quantities of eggs and mix, respectively, and let Q denote the number of cakes produced. The equation of our production function is: Q = Min(x,y). If we increase each input by a factor of a, we have the following quantity of cake: $\min\{ax, ay\} = a \min\{x, y\} = aQ$. Hence, increasing the quantities of inputs by a given proportion results in the same proportionate increase in output, and the production function thus exhibits constant returns to scale.

6.17 If we were to scale up all inputs by a factor λ (that is, replace K by λK, and L by λL), the resulting output would equal λQ. Therefore a linear production function has constant returns to scale.

6.18 A general fixed proportions production function is of the form $Q = \min(aK, bL)$. If we were to scale up all inputs by a factor λ (that is, replace K by λK, and L by λL), the resulting output would be $\min(a\lambda K, b\lambda L) = \lambda \min(aK, bL) = \lambda Q$. Therefore the production function has constant returns to scale.

6.19 a) To determine the nature of returns to scale, increase all inputs by some factor λ and determine if output goes up by a factor more than, less than, or the same as λ.

$$Q_\lambda = 50\sqrt{\lambda M \lambda L} + \lambda M + \lambda L$$
$$Q_\lambda = 50\lambda\sqrt{ML} + \lambda M + \lambda L$$
$$Q_\lambda = \lambda\left[50\sqrt{ML} + M + L\right]$$
$$Q_\lambda = \lambda Q$$

By increasing the inputs by a factor of λ output goes up by a factor of λ. Since output goes up by the same factor as the inputs, this production function exhibits constant returns to scale.

b) The marginal product of labor is

$$MP_L = 25\sqrt{\frac{M}{L}} + 1$$

Suppose $M > 0$. Holding M fixed, increasing L will have the effect of decreasing MP_L. The marginal product of labor is decreasing for all levels of L. The MP_L, however, will never be negative since both components of the equation above will always be greater than or equal to zero. In fact, for this production function, $MP_L \geq 1$.

6.20 a) Notice that $(aK)^{1/3}(aL)^{2/3} = a^{1/3}a^{2/3}\ K^{1/3}\ L^{2/3} = a\ K^{1/3}\ L^{2/3} = aQ$. This production function exhibits constant returns to scale.

b) $MRTS_{L,K} = MP_L\ /\ MP_K = 2\ K/L$.

c) Because this is a Cobb-Douglas production function, its elasticity of substitution equals 1.

6.21 a) For a CES production function of the form

$$Q = \left[aL^{\frac{\sigma-1}{\sigma}} + bK^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

the elasticity of substitution is σ. In this example we have a CES production function of the form

$$Q = \left[K^{0.5} + L^{0.5}\right]^2.$$

To determine the elasticity of substitution, either set $(\sigma-1)/\sigma = 0.5$ or $\sigma/(\sigma-1) = 2$ and solve for σ.

$$\begin{aligned}\frac{\sigma-1}{\sigma} &= 0.5\\ \sigma-1 &= 0.5\sigma\\ 0.5\sigma &= 1\\ \sigma &= 2.\end{aligned}$$

In either case, the elasticity of substitution is 2.

b)

$$\begin{aligned}Q_\lambda &= \left[(\lambda K)^{0.5} + (\lambda L)^{0.5}\right]^2\\ Q_\lambda &= \left[(\lambda^{0.5})(K^{0.5} + L^{0.5})\right]^2\\ Q_\lambda &= \lambda\left[K^{0.5} + L^{0.5}\right]^2\\ Q_\lambda &= \lambda Q.\end{aligned}$$

Since output goes up by the same factor as the inputs, this production function exhibits constant returns to scale.

c)

$$\begin{aligned}Q_\lambda &= \left[100 + (\lambda K)^{0.5} + (\lambda L)^{0.5}\right]^2\\ Q_\lambda &= \left[100 + \lambda^{0.5}(K^{0.5} + L^{0.5})\right]^2\\ Q_\lambda &= \lambda\left[\frac{100}{\lambda^{0.5}} + K^{0.5} + L^{0.5}\right]^2 < \lambda Q.\end{aligned}$$

When the inputs are increased by a factor of λ, where $\lambda > 1$ output goes up by a factor less than λ implying decreasing returns to scale.

Intuitively, in this production function, while you can increase the K and L inputs, you cannot increase the constant portion. So output cannot go up by as much as the inputs.

6.22 a) For each pair of inputs, except those where there are no eggs or no other ingredients, new recipe produces more cookies. Hence, the new recipe represents technological progress.

b) $MP_E = \Delta Q/\Delta E$, where E denotes the quantity of eggs. With mixed ingredients held fixed at 8, we have:

MP_E = (8 – 0)/(1 – 0) = 8, when E goes from 0 to 1.
MP_E = (16 – 8)/(2 – 1) = 8, when E goes from 1 to 2.
MP_E = (16 – 16)/(3 – 2) = 0, when E goes from 2 to 3.
MP_E is zero for all subsequent changes in E.

After the technological progress we have:

MP_E = (10 – 0)/(1 – 0) = 10, when E goes from 0 to 1.
MP_E = (19 – 10)/(2 – 1) = 9, when E goes from 1 to 2.
MP_E = (22 – 19)/(3 – 2) = 3, when E goes from 2 to 3.
MP_E = (23 – 22)/(4 – 3) = 1, when E goes from 3 to 4.
Comparing the marginal products, we see that MP_E (when mix equals 8 is higher after the technological progress.

6.23 a) It is possible to write the two production functions as

$$Q_1 = 500L + 1,500K$$
$$Q_2 = 500L + 10,000K$$

Since $Q_2 > Q_1$ for given quantities of K and L, the firm can achieve more output for a given combination of inputs. This innovation has therefore resulted in technological progress as defined in the text.

b) Initially $MP_K = 1,500$ and $MP_L = 500$ implying the $MRTS_{L,K} = 0.33$. After the innovation the $MP_K = 10,000$ and $MP_L = 500$ implying the $MRTS_{L,K} = 0.05$. Since the marginal rate of technical substitution of labor for capital has decreased after the innovation this is labor-saving technological progress.

6.24 a) With any positive amounts of K and L, $\sqrt{KL} < KL$ so more Q can be produced with the final production function. So there is indeed technological progress.

b) With the initial production function $MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L}$.

With the final production function $MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L}$.

For any ratio of capital to labor, $MRTS_{L,K}$ is the same for the two production functions. Thus, the technological progress is neutral.

6.25 a) With any positive amounts of K and L, $\sqrt{KL} < K\sqrt{L}$ so more Q can be produced with the final production function. So there is indeed technological progress.

b) With the initial production function $MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L}$.

With the final production function $MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{0.5K}{L}$.

For any ratio of capital to labor, $MRTS_{L,K}$ is lower with the second production function. Thus, the technological progress is labor-saving.

Chapter 7

Costs and Cost Minimization

Solutions to Review Questions

1. Acquisition cost and opportunity cost are not necessarily the same. As the text points out, opportunity costs are forward looking. The opportunity cost is the payoff associated with the best of the alternatives that are not chosen. Once the test tubes are purchased, the decision is to use the tubes to clone snake cells or something else. It is possible that someone values the tubes for some purpose at higher (or lower) than $0.50 so that selling the tubes would earn the firm something more (or less) than $0.50 per tube. The opportunity cost then is different than the acquisition cost.

2. Since the business is computer consulting, an explicit cost, a cost involving a direct monetary outlay, might be the cost of paper and ink used to advertise your service. An implicit cost, a cost not involving a direct monetary outlay, might be the opportunity cost of your time, *e.g.*, to earn money working at the student fitness center or to study for your own classes.

3. Whether or not a particular cost is sunk or not depends on the decision being made. If the cost does not change as a result of the decision the cost is sunk, while if the cost does change the cost is not sunk.

4. A firm's total costs are $TC = rK + wL$, so the equation for a typical isocost line is

$$K = \frac{TC}{r} - \frac{w}{r}L.$$

Since the slope of the isocost line is given by $-w/r$, if the price of labor increases the isocost line will become steeper and if the price of capital increases the isocost line will become flatter.

5. The solution to the firm's cost minimization problem must lie on an isoquant. While the firm *could* produce a given output with a combination of inputs not on the isoquant, say by using more labor and more capital than necessary, a combination such as this would not be efficient and therefore not cost minimizing.

6. To understand why at an interior optimum the additional output the firm gets from a dollar spent on labor must equal the additional output the firm gets from a dollar spent on capital, assume these were not equal. For example, suppose the firm could get more

output from a dollar spent on labor than on a dollar spent on capital. Then the firm could take one dollar away from capital and reallocate it to labor. Since the firm gets more output from a dollar of labor than from a dollar of capital, it will require the firm to spend less than one dollar on labor to offset the decline in output from taking one dollar away from capital. This implies the firm can keep output at the same level but do so at a lower cost. Therefore, if these amounts are not equal the firm is not minimizing cost.

This requirement does not necessarily hold at a corner solution. While the firm could potentially reduce cost by reallocating spending to the more productive input, at a corner solution, by definition, the firm is not using one of the inputs. There is no further opportunity to reallocate spending if the firm is spending nothing on one of the inputs, *i.e.*, the firm cannot move to a point where one of the inputs is negative.

7. The expansion path traces out the cost minimizing combinations of all inputs as the level of output is increased (expanded) holding the prices of the inputs fixed. An input demand curve traces out a firm's cost minimizing quantity of one input as the price of that input varies holding the level of output and the prices of the other inputs fixed.

8. Giffen goods arise when the income effect is so severely negative that it offsets the substitution effect. This can happen because in consumer choice, income was an exogenous variable – therefore, changes in price affect both the relative substitutability of goods (via the tangency condition) as well as the consumer's purchasing power (via the budget constraint). By contrast, in the cost minimization problem output is exogenous while the expenditure is the objective function. Thus, a change in an input price affects only the relative substitutability of inputs (via the tangency condition) – there is no corresponding effect on the production constraint, since prices do not appear there. So while there is a "substitution effect" in cost minimization, there is no corresponding "income effect" as in consumer choice. Therefore, increases in input prices will always lead to decreases in the use of that input (except at corner solutions, where there might be no change). So there cannot be a Giffen input.

9. Assuming quantity is fixed, the short-run demand for a variable input would equal its long-run demand if the level of the fixed input in the short run was cost minimizing for the quantity of output being produced in the long run.

Solutions to Problems

7.1 a) $500

b) 30% of $500, or $150

c) By not lowering the price and assuming the firm cannot sell any more printers, the best the firm can hope for is the $150 the firm can receive from the manufacturer. If the firm drops the price to $200 and sells the printers on their own they can actually "profit" an additional $50 over their best available alternative.

7.2 The accounting costs are simply the sum: 25,000 + 75,000 + 80,000 + 6,000 = $186,000 and the shop's accounting profit is $64,000 which means that Mr. Moore's total gain from this venture is 80,000 + 64,000 = $144,000.

The economic costs also include the opportunity cost of the land rental ($100,000) and of Mr. Moore's next best alternative, which in this case is $95,000. That is, Mr. Moore loses $15,000 by not choosing his next best alternative. Therefore Mr. Moore's total economic costs are 186,000 + 100,000 + 15,000 = $301,000, which exceeds his revenues by $51,000.

If he were to shut down the shop, Mr. Moore would earn 100,000 + 95,000 = $195,000 which is more than the $144,000 he currently earns (by precisely the $51,000 figure from above). Therefore he should shut down the shop.

7.3 At the optimum we must have

$$\frac{MP_K}{r} = \frac{MP_L}{w}$$

In this problem we have

$$\frac{200}{0.25} > \frac{1000}{10}$$
$$800 > 100$$

This implies that the firm receives more output per dollar spent on an additional machine hour of fermentation capacity than for an additional hour spent on labor. Therefore, the firm could lower cost while achieving the same level of output by using fewer hours of labor and more hours of fermentation capacity.

7.4 a) If the price of both inputs change by the same percentage amount, the slope of the isocost line will not change. Since we are holding the level of output fixed, the isocost line will be tangent to the isoquant at the same point as prior to the price increase. Therefore, the cost-minimizing quantities of the inputs will not change.

b) If the price of capital increases by a larger percentage than the price of labor, then, relatively speaking, the price of labor has become cheaper. The firm will substitute away from capital and add labor until either the tangency condition holds or a corner solution is reached.

7.5 a) The amount of land used in production is fixed in the short-run. Hence, in the short-run the farmer chooses amount of capital and labor. It follows that cost-minimizing quantities of labor and capital have to satisfy equation MP_L / MP_K = w/r where w and r denote prices of labor and capital. Notice that w/r = (1.05 w)/(1.05 r). The cost-minimizing quantities of inputs, for each level of output, do not change when prices of both inputs go up by 5% and quantity of land is fixed.

b) For a given output level, the cost-minimizing farmer uses more capital and less labor.

7.6 Imagine that two expansion paths did cross at some point. Recall that the expansion path traces out the cost- minimizing combinations of inputs as output increases. Essentially the expansion path traces out all of the tangencies between the isocost lines and isoquants. These tangencies occur at the point where

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

If the expansion paths cross at some point then the cost minimizing combination of inputs must be identical with both sets of prices. This would require that

$$\frac{MP_K}{r_1} = \frac{MP_L}{w_1} \text{ and } \frac{MP_K}{r_2} = \frac{MP_L}{w_2}$$

Unless the input prices are proportional, i.e. unless $w_1 / r_1 = w_2 / r_2$, it is not possible for both of these equations to hold. Therefore, it is not possible for the expansion paths to cross unless the prices are proportional, in which case the two expansion paths will be identical.

7.7 The tangency condition implies

$$\frac{\left[L^{1/2}+K^{1/2}\right]K^{-1/2}}{r}=\frac{\left[L^{1/2}+K^{1/2}\right]L^{-1/2}}{w}$$
$$\frac{1}{r\sqrt{K}}=\frac{1}{w\sqrt{L}}$$
$$w\sqrt{L}=r\sqrt{K}$$
$$\frac{K}{L}=\frac{w^2}{r^2}$$

Given that $w=10$ and $r=1$, this implies

$$100=\frac{K}{L}$$
$$100L=K$$

Returning to the production function and assuming $Q=121,000$ yields

$$121,000=\left[L^{1/2}+K^{1/2}\right]^2$$
$$121,000=\left[L^{1/2}+(100L)^{1/2}\right]^2$$
$$121,000=\left[L^{1/2}+10L^{1/2}\right]^2$$
$$121,000=\left[11L^{1/2}\right]^2$$
$$121,000=121L$$
$$1,000=L$$

Since $K=100L$, $K=100(1000)=100,000$. The cost minimizing quantities of capital and labor to produce 121,000 airframes is $K=100,000$ and $L=1,000$.

7.8 The tangency condition implies

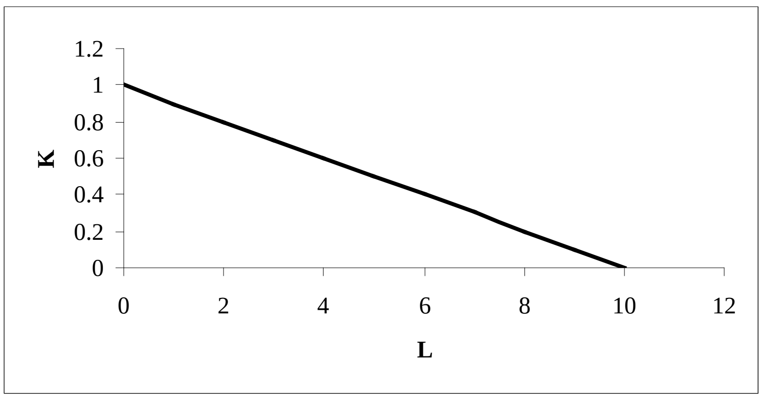
$$10=\frac{K}{L}$$
$$10L=K$$

Substituting into the production function yields

$$121,000 = LK$$
$$121,000 = L(10L)$$
$$121,000 = 10L^2$$
$$12,100 = L^2$$
$$110 = L$$

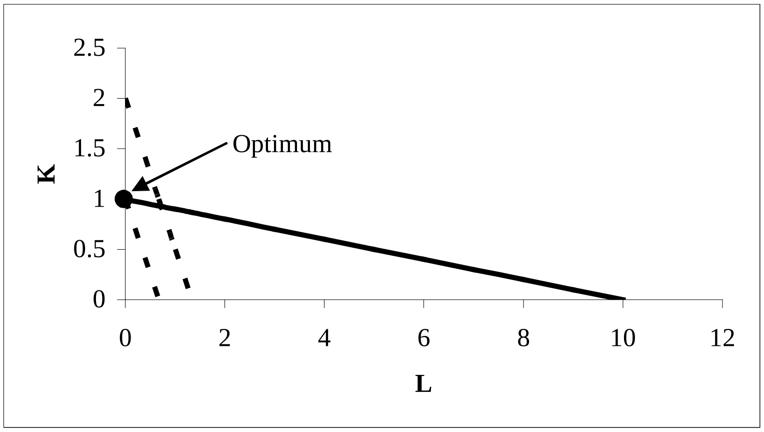
Since $K = 10L$, $K = 1,100$. The cost-minimizing quantities of labor and capital to produce 121,000 airframes are $K = 1,100$ and $L = 110$.

7.9 a)



K and *L* are perfect substitutes, meaning that the production function is linear and the isoquants are straight lines. We can write the production function as *Q* = 10,000*K* + 1000*L*, where *Q* is the number of workers for whom payroll is processed.

b) If $r = 5$ and $w = 7.50$, the slope of a typical isocost line will be $-7.5/5.0 = -1.5$. This is steeper than the isoquant implying that the firm will employ only computer time (K) to minimize cost. The cost minimizing combination is $K = 1$ and $L = 0$. This outcome can be seen in the graph below. The isocost lines are the dashed lines.



The total cost to process the payroll for 10,000 workers will be
$TC = 5(1) + 7.5(0) = 5$.

c) The firm will employ clerical time only if $MP_L / w > MP_K / r$. Thus we need 0.1 / 7.5 > 1/r or r > 75.

7.10 Currently the firm must be using $L = Q/K = 32/16 = 2$ units of labor. Let the factor prices of capital and labor be, respectively, r and w.
Its total expenditure is $C = wL + rK = 2(2) + 4(16) = 68$.
If it were to minimize cost, it would hire L and K so that
(1) $MP_K/r = MP_L/w$, or $L/4 = K/2$, or $L = 2K$ and (2) $Q = LK$.
(1) and (2) imply that $Q = 2K^2$, or $32 = 2K^2$, and thus $K = 4$ and $L = 8$.
So $Q = 32$ can be produced efficiently with a cost of $C = wL + rK = 2(8) + 4(4) = 32$.
The firm could save 68 – 32 = 36 by producing efficiently.

7.11 From the tangency condition, we get

$$\frac{K}{L} = \frac{w}{r}$$
$$K = \left(\frac{w}{r}\right)L$$

Substituting into the production function yields

$$Q = LK$$
$$Q = L\left(\frac{w}{r}\right)L$$
$$Q = \left(\frac{w}{r}\right)L^2$$
$$L = \left(\frac{rQ}{w}\right)^{1/2}$$

This represents the input demand curve for L. Since

$$K = \left(\frac{w}{r}\right)L$$

we have

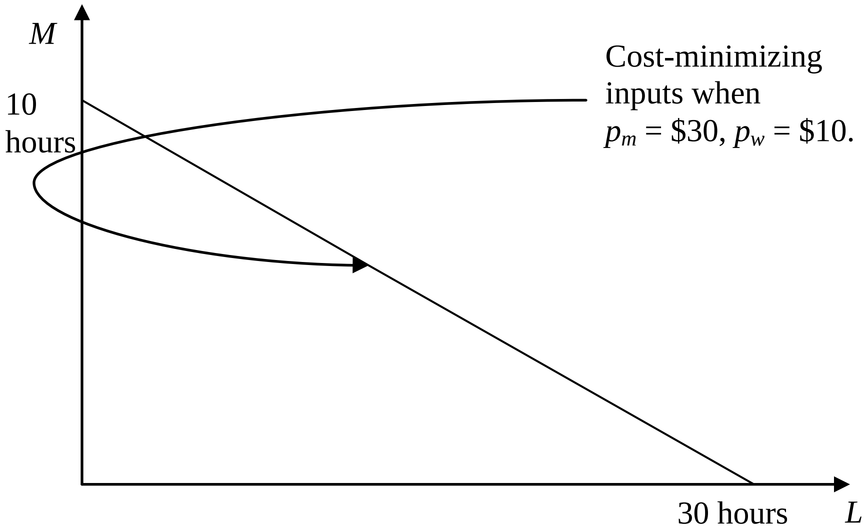
$$K = \left(\frac{w}{r}\right)\left(\frac{rQ}{w}\right)^{1/2}$$
$$K = \left(\frac{wQ}{r}\right)^{1/2}$$

This represents the input demand curve for K.

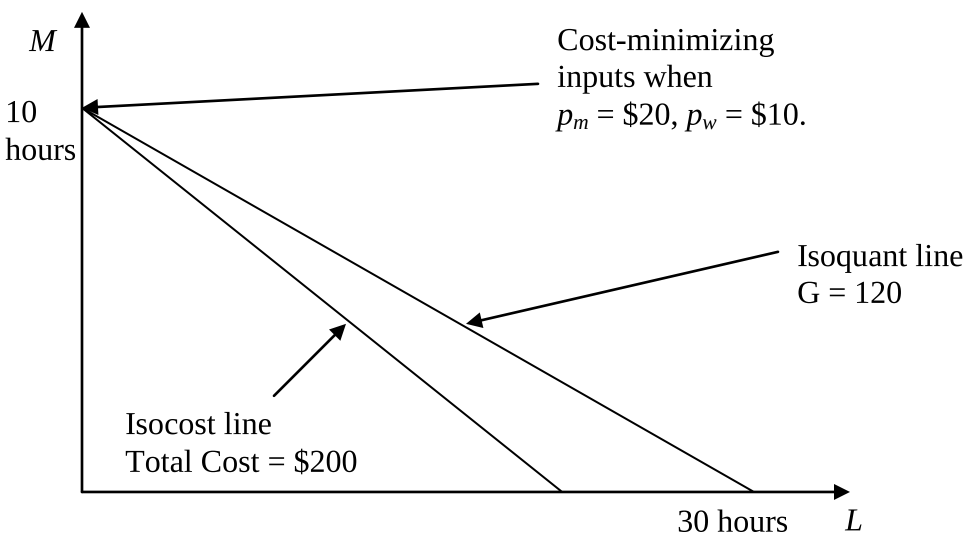
7.12 Using the tangency condition, with the original input prices: $\frac{K}{L} = \frac{w}{r} = 2$. So, $K = 2L$. Also, using the information on total costs, $4L + 2K = 160$. Combining these two equations, we get (L, K) = (20, 40). Therefore the firm produces 20*40 = 800 units of output.

After the prices change, even though we don't know the numerical values of the input prices, we can still answer the question using the fact that we're told $w = 8r$. The tangency condition implies that $\frac{K}{L} = 8$, so $K = 8L$. Also, we have $KL = 800$. This implies that the optimal input combination is (L, K) = (10, 80).

7.13 a) Isoquants for the production function are straight lines. At the given input prices slope of an isoquant is equal to the ratio of the input prices. Hence, all positive input quantities (measured in work hours) such that $4L + 12M = 120$ are cost-minimizing.



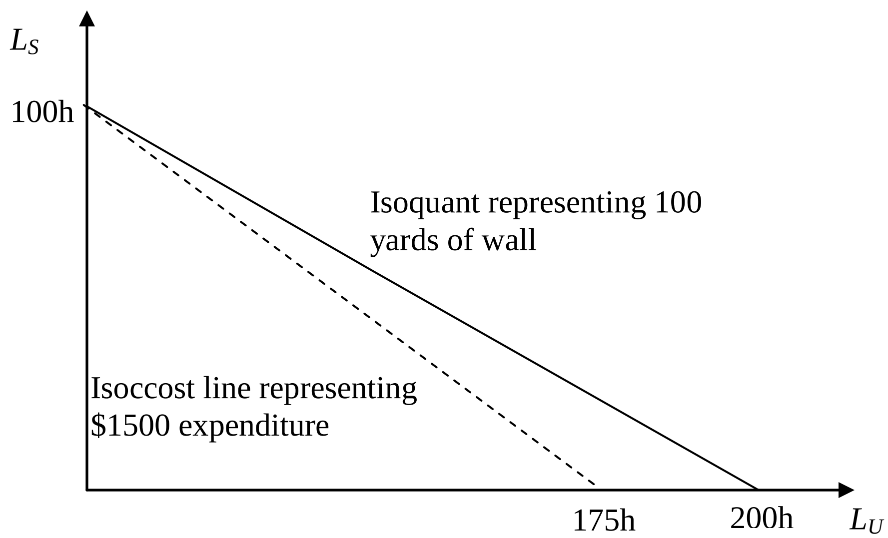
b) When one hour of the machine's work costs \$20 cost-minimizing firm does not use manual work at all. The cost-minimizing quantity of the machine's work necessary to produce 120 widgets is equal to $M = 120/12 = 10$ hours. The firm spends \$200. (Note that if the firm were to use only manual labor, the cost would be \$300 (= 30 hours x \$10 per hour).



c) $G = 4L + 12M$

7.14 a) The production function is $Q = L_S + ½ L_U$ where L_S denotes hours worked by skilled workers and L_U denotes hours worked by unskilled workers. Both types of labor are perfect substitutes.

b) The isoquant is a straight line.



c) $MP_{Ls}/w_s = 1/15$; $MP_{Lu}/w_u = 0.5/8 = 1/16$. Thus, the "bang for the buck" is higher for skiled labor, and the firm will use only skilled labor.
Note that the total cost of building 100 yards with skilled labor is (100 hours)($15/ hour) = $1500.
The total cost of building 100 yards with unskilled labor is (200 hours)($8/ hour) = $1600.

The isocost line representing a $1500 expenditure is drawn as a dotted line in the graph in (b). The isocost line is more steeply sloped than the isoquant in the graph because the marginal rate of technical substitution of unskilled labor for unskilled labor is equal to ½, while the ratio of input prices is equal to 8/15.

7.15 a) First, note that this production function has diminishing $MRS_{L,K}$. The tangency condition would imply that $1/2\sqrt{L} = 1/50$ or $L = 625$. Substituting this back into the production function we see that $K = 10 – 25 = –15$. Since the firm cannot use a negative amount of capital, the tangency condition is not valid in this case.

Looking at the corner with K = 0, since Q = 10 the firm requires $L = Q^2 = 100$ units of labor. At this point, $MP_L / w = (1/20)/1 = 0.05 > MP_K / r = 1/50 = 0.02$. Since the marginal product per dollar is higher for labor, the firm will use only labor and no capital.

b) The firm will use a positive amount of capital when $\frac{MP_L}{w} = \frac{MP_K}{r}$, or $2\sqrt{L} = r$. Thus $L = 0.25r^2$. From the production constraint $K = Q - \sqrt{L} = 10 - 0.5r$. So if $K > 0$ then we must have $10 - 0.5r > 0$, or $r < 20$.

c) Again, using the tangency condition we must have $2\sqrt{L} = r$. Therefore, since r = 50, L = 625. From the production constraint, the input demand for capital is $K = Q - \sqrt{L} = Q - 25$. So if $K > 0$ then we must have $Q > 25$.

7.16 No, these are not valid input demand curves. In both cases the quantity of the input is positively related to the input's price. Such upward-sloping input demand curves cannot exist.

7.17 If K = 0, then the firm must hire L = 5 units of labor. For this to be optimal, it must be that $MP_L / w > MP_K / r$, or $1/w > 6$. In other words, $w < 1/6$.

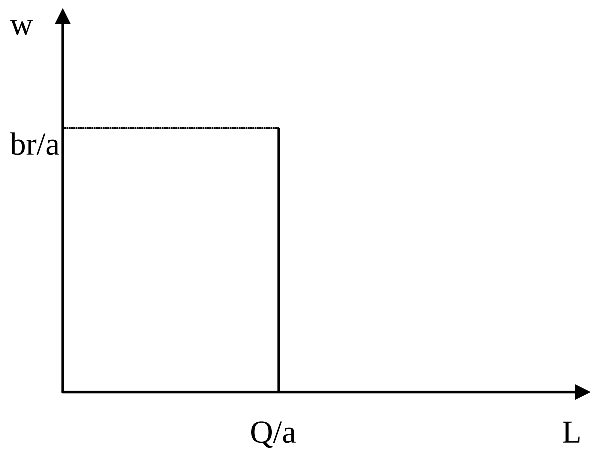
If L = 0, then the firm must hire K = 5 units of capital. For this to be optimal, it must be that $MP_L / w < MP_K / r$, or $6/w > 1$. In other words, $w > 6$.

For the firm to use both capital and labor, it must be that $1/6 < w < 6$. To see why, notice that the indifference curves will have diminishing $MRTS_{L,K}$. In particular, $MRTS_{L,K} = 6$ where the Q = 5 indifference curve intersects the K-axis (where L = 0). Diminishing $MRTS_{L,K}$ implies that the Q = 5 indifference curve will gradually flatten out until it intersects the L-axis (where K = 0), at which point $MRTS_{L,K} = 1/6$.

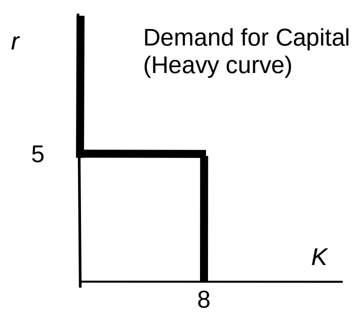
7.18 The input demand curves will be vertical lines, representing the fact that the demand by firms for such inputs is inelastic. If the firm's production function is $Q = \min(L, K)$ then, holding fixed the quantity of production and the price of capital, if the wage rate were to increase it would not change the firm's requirement for labor. Therefore, the demand for each input is independent of price and the demand curves are vertical lines.

7.19 Recall that with a linear production function we are usually going to get corner point solutions. In this case, the firm will employ only labor and no capital if labor is cheap enough or, $\frac{MP_L}{w} > \frac{MP_K}{r}$ i.e. if $w < \frac{br}{a}$. Similarly it will use just capital if the rental rate is low enough i.e. $r < \frac{aw}{b}$. If the firm uses only labor, it will use $L = \frac{Q}{a}$ units regardless of the price, and similarly it will use $K = \frac{Q}{b}$ units of capital if it uses any capital at all.

The input demand curve for labor for a given price, r, of capital, is shown below.



7.20 With this production function the firm views K and L as perfect substitutes.
The firm will be at a corner point with $K = 0$ when $MP_K/r < MP_L/w$, or when $10/r < 2/1$, or when $r > 5$.
The firm will be at a corner point with $L = 0$ when $MP_K/r > MP_L/w$, or when $10/r > 2/1$, or when $r < 5$. When the firm needs to produce $Q = 80$, how much capital will it need? The production function shows that $80 = 10K$, or $K = 8$ units.
When r = 5, the firm might use any combination of K and L along the isoquant $80 = 10K + 2L$. The firm might therefore use any K such that $0 \leq K \leq 8$.
The graph of the demand for labor is as shown.



7.21 The tangency condition implies that $\frac{1}{2\sqrt{L}} = \frac{w}{r}$, or $\sqrt{L} = \frac{r}{2w}$. Clearly the demand curve for L is not a function of the level of output, Q. Therefore, as the level of output changes, the amount of labor is constant. Therefore, if we were to graph isoquants with labor on the horizontal axis, the expansion path for labor would just be a straight, vertical line.

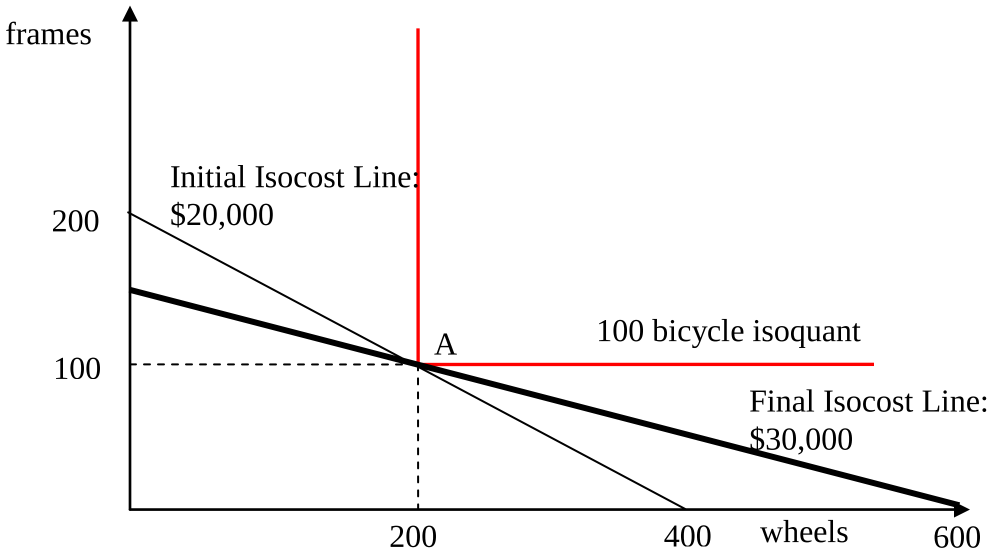
The demand curve for capital can be derived by substituting the demand curve for labor into the production function. That is, $K + \frac{r}{2w} = Q$, so $K = Q - \frac{r}{2w}$.

7.22 Using the tangency condition, initially $\frac{K}{L} = 1$, implying that $K = L$. Since $KL = 100$, we get $K = L = 10$.

Under the new prices, the tangency condition implies that K=4L. This means that the optimal input combination is (L, K) = (5, 20).

The percent change in price is (4 – 1)*100 = 300%. While the percent change in the demand for labor is [(5 – 10)/10]*100 = –50%. Therefore the price elasticity of demand over this range of prices is –50/300 = –1/6.

7.23 a) The production function is $Q = \min(F, ½\,W)$, where F denotes the number of frames and W denotes the number of wheels.



b) To produce 100 bicycles in the least costly manner, the firm always needs to choose basket A, with 200 wheels and 100 frames.
Initially, when the price of a frame is $100 and the price of a wheel is $50, the isocost line is the lighter one shown in the graph; all points on the isocost line indicate an expenditure of $20,000.
Later, when the price of a frame is $200 and the price of a wheel is $50, the isocost line is the lighter one shown in the graph; all points on the isocost line indicate an expenditure of $30,000.

7.24 With just two inputs, there is no tangency condition to worry about in the short run. To find the short-run cost-minimizing quantity of labor, we need only solve the production function for L in terms of Q and $\overline{K}$:

$$Q = 10\overline{K}L^{\frac{1}{3}}$$

This gives us:

$$L = \frac{Q^3}{1000\overline{K}^3}$$

This is the cost-minimizing quantity of labor in the short run.

7.25

a) Since $\overline{K} = 9$, we get $18L + 9 = 45$ which implies that L = 36/18 = 2. Therefore the firm's total cost with this input combination is 4(2) + 5(9) = $53.

b) If the firm could operate optimally, it would choose labor and capital to satisfy the tangency condition: $\frac{2K}{2L+1} = \frac{4}{5}$, implying that $10K = 8L + 4$. Also, $2KL + K = 45$. Combining these two conditions, $K = \sqrt{18}$ = 4.24 and L = 4.8. Now the firm's expenditure would be 4(4.24) + 5(4.8) = $41 approximately. Therefore the firm loses about $12 because of its constraint on capital.

7.26

a) Here we have two tangency conditions and the requirement that L, K, and M produce Q units of output.

$$\frac{MP_L}{MP_M} = \frac{w}{m} \Rightarrow \frac{\frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}}M^{\frac{1}{3}}}{\frac{1}{3}K^{\frac{1}{3}}L^{\frac{1}{3}}M^{-\frac{2}{3}}} = \frac{1}{1} \Rightarrow M = L$$

$$\frac{MP_L}{MP_K} = \frac{w}{r} \Rightarrow \frac{\frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}}M^{\frac{1}{3}}}{\frac{1}{3}K^{-\frac{2}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}}} = \frac{1}{1} \Rightarrow K = L$$

$$Q = K^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}}$$

This is a system of three equations in three unknowns. The solution to this system gives us the long-run cost-minimizing input combination:

$$L = Q$$
$$M = Q$$
$$K = Q$$

b) The tangency condition $\frac{MP_L}{MP_M} = \frac{w}{m}$ is

$$\frac{\frac{1}{3}\overline{K}^{\frac{1}{3}}L^{-\frac{2}{3}}M^{\frac{1}{3}}}{\frac{1}{3}\overline{K}^{\frac{1}{3}}L^{\frac{1}{3}}M^{-\frac{2}{3}}} = \frac{1}{1},$$

which implies

$$M = L$$

To find the short-run cost-minimizing quantity of labor, we plug this back into the production function and solve for L in terms of Q and $\overline{K}$.

$$Q = \overline{K}^{\frac{1}{3}}L^{\frac{1}{3}}L^{\frac{1}{3}}$$

which when we solve for L gives us the short-run cost-minimizing quantity of labor

$$L = \frac{Q^{\frac{3}{2}}}{\overline{K}^{\frac{1}{2}}}$$

Since $M = L$, the short-run cost-minimizing quantity of materials is

$$M = \frac{Q^{\frac{3}{2}}}{\overline{K}^{\frac{1}{2}}}$$

c) Plugging $Q = 4$ into the expressions for the long-run cost-minimizing quantities of labor and materials gives us

$$L = 4$$
$$M = 4$$

Plugging $Q = 4$ and $\overline{K} = 4$ into the expressions for the short-run cost-minimizing quantities of labor and materials gives us

$$L = \frac{4^{\frac{3}{2}}}{4^{\frac{1}{2}}} = 4^{(\frac{3}{2}-\frac{1}{2})} = 4$$
$$M = \frac{4^{\frac{3}{2}}}{4^{\frac{1}{2}}} = 4^{(\frac{3}{2}-\frac{1}{2})} = 4$$

7.27 a) With three inputs, we need two tangency conditions to ensure that the marginal product per dollar spent is equal across all inputs. (We could write down a third tangency condition, but it would be redundant.) Equating the "bang for the buck" between labor and capital implies $1/2\sqrt{L} = 1/2\sqrt{K}$ or $L = K$. Similarly, equating the "bang for the buck" between labor and materials implies $1/2\sqrt{L} = 1/2\sqrt{M}$ or $L = M$. Then using the production constraint to find the input demand for labor yields $Q = \sqrt{L} + \sqrt{L} + \sqrt{L}$ or $L = (1/3)Q^2$. Since $L = M = K$ from the tangency conditions, we also have $K = (1/3)Q^2$ and $M = (1/3)Q^2$.

b) First, note that with $K = 4$, the firm can produce up to $Q = \sqrt{0} + \sqrt{4} + \sqrt{0} = 2$ units of output without hiring *any* labor or materials. To produce more than $Q = 2$, the firm still balances the marginal product per dollar spent on labor and materials; in part (a), we saw this implied $L = M$. Substituting this and $K = 4$ into the production constraint, we have $Q = \sqrt{L} + \sqrt{4} + \sqrt{L}$ which yields $L = (1/4)(Q - 2)^2$ as the input demand for labor. Then $L = M$ implies that the input demand for materials is $M = (1/4)(Q - 2)^2$. Therefore, the input demand functions are

$$L(Q) = M(Q) = \begin{cases} 0 & Q \le 2 \\ \frac{1}{4}(Q-2)^2 & Q > 2 \end{cases}$$

c) Again, with $K = 4$ and $L = 9$, the firm can produce up to $Q = 5$ units of output without hiring any materials. Should it desire to produce greater levels of output, it can hire materials according to $Q = \sqrt{9} + \sqrt{4} + \sqrt{M}$, or $M = (Q - 5)^2$. Therefore, the input demand for materials is

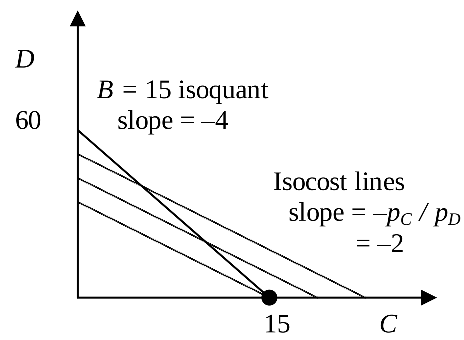
$$M(Q) = \begin{cases} 0 & Q \le 5 \\ (Q-5)^2 & Q > 5 \end{cases}$$

7.28 The information in the problem tells us that $MP_L = 200$ and $MP_K = 150$ while $w = 25$ and $r = 10$. So $MP_L/w = 8 < MP_K/r = 15$. Thus Acme could maintain its current level of output while reducing costs by employing more capital and less labor. So it is not employing the optimal input bundle.

7.29 We have $MP_L/w = 4/1 = 4 > MP_K/r = 2/2 = 1$. Thus the firm cannot be minimizing its long-run total cost. By employing more labor and less capital, it could maintain 32 units of output while lowering total costs.

7.30 a) Computers are four times as productive as draftsmen; an alternative way of saying this is that $MP_C = 4MP_D$. Since C and D are perfect substitutes, we know the production function has the form $B = aC + bD$, where a and b are positive constants. Thus we can write the production function as $B = C + (1/4)D$. Note that this is consistent with generating one blueprint ($B = 1$) from the following combinations of inputs: $(C, D) = (1, 0)$, $(C, D) = (0, 4)$, and $(C, D) = (0.5, 2)$.

b) Notice that $MP_C / p_C = 1/10 > MP_D / p_D = 0.25/5 = 1/20$. That is, the marginal product per dollar spent on computer time is always higher than the marginal product per dollar spent on draftsman time. So the optimal input combination involves D = 0 and C = 15. The graph below illustrates the (dotted) isocost lines with slope = $-p_C / p_D$ = –2, along with the (solid) B = 15 isoquant with slope = $-MP_C / MP_D$ = –4.



Chapter 8

Cost Curves

Solutions to Review Questions

1. The long-run total cost curve plots the minimized total cost for each level of output holding input prices fixed. In other words, for a given set of input prices, the long-run total cost curve represents the total cost associated with the solution to the long-run cost minimization problem for each level of output.

2. When the price of one input increases, the isocost line for a particular level of total cost will rotate in toward the origin. Assuming the isocost line was tangent to the isoquant for the firm's selected level of output, when the isocost line rotates it will no longer touch the original isoquant. In order for an isocost line to reach a tangency with the original isoquant, the firm would need to move to an isocost line associated with a higher level of cost, *i.e.* an isocost line further to the northeast.

3. If the price of a single input goes up leaving all other input prices the same and the level of output constant, total cost will rise but by a smaller percentage than the increase in the input price. This occurs because the firm will substitute away from the now relatively more expensive labor to the now relatively less expensive other inputs. So, if the price of labor rises by 20% holding all other input prices constant, total cost will rise by less than 20%.

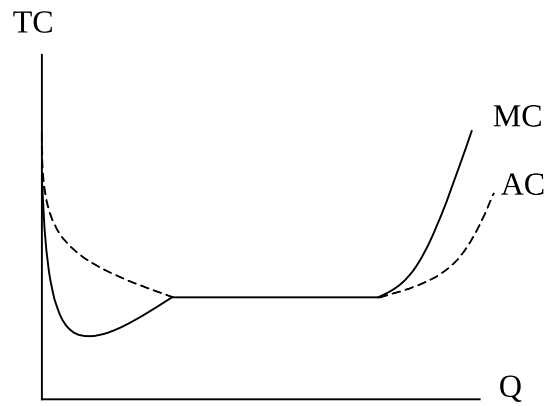
   If the prices of all inputs go up by the same percentage, total cost will rise by exactly that same percentage. So, if input prices rise by 20%, total cost will also rise by 20%.

4. An increase in the price of labor would result in a long-run total cost curve that lies above the initial long-run total cost curve at every quantity except $Q = 0$. Since $AC = TC / Q$, increasing total cost will raise average cost at every quantity except $Q = 0$. Therefore, the long-run average cost curve will shift up.

5. a) When $MC > AC$, average cost is increasing, and when $MC < AC$, average cost is decreasing. So, if the average cost curve is increasing it must lie *below* the marginal cost curve.

   b) If the marginal cost curve is increasing, it may lie above or below the average cost curve. The only determining factor here is whether or not marginal cost lies above or below average cost. If it lies above, average cost will be increasing and if it lies below, average cost will be decreasing. Knowing that marginal cost is increasing or decreasing tells us nothing about average cost.

6.



When average cost is falling, marginal cost will lie below average cost, and when average cost is increasing, marginal cost will lie above average cost. Over the flat-bottomed portion where average cost is neither increasing nor decreasing, marginal cost and average cost will be equal.

7. The output elasticity of total cost, when simplified can be written as

$$\varepsilon_{TC,Q} = \frac{MC}{AC}$$

Since $AC = TC/Q$, and since TC and Q must always be positive, AC will always be positive. Marginal cost, *MC*, represents the change in total cost associated with an increase in output. When output increases, total cost must always rise for a given set of input prices, implying that MC is also always positive. Therefore, the output elasticity of total cost must always be positive.

8. Because fixed cost does not change, marginal costs reflect the change in variable costs. Thus, as with the relationship between any average and marginal, if average variable cost is decreasing, marginal cost must be below average variable cost, and if average variable cost is increasing, marginal cost must lie above average variable cost. This implies marginal cost will intersect average variable cost at the minimum of average variable cost.

9. If the average variable cost curve is flat, average variable cost is neither increasing nor decreasing. Marginal cost will therefore be equal to average variable cost and the marginal cost curve will therefore also be flat. Since average fixed cost is always declining, and since average total cost is the vertical sum of average variable and average

fixed costs, average total cost must also be declining at all levels of Q if average variable cost is constant. Graphically, average total cost will be declining and asymptotic to the average variable cost curve.

10. The long-run average cost curve is the envelope to the short-run average cost curves associated with each level of output. If each of these short-run average cost curves has the same minimum point, the long-run average cost curve will be a horizontal line tangent to all of these minimum points. Because the long-run average cost curve will be flat, long-run average cost is neither increasing nor decreasing, and the long-run marginal cost curve will also be flat and equal to long-run average cost.

11. Economies of scale refer to a situation when average total cost for a single product declines as the level of output for that product increases. These economies of scale might occur, for example, because workers can specialize in tasks as the level of output increases and the workers' productivity may increase. Economies of scope refer to efficiencies that arise when a firm produces more than one product. In particular, economies of scope exist if one firm producing *N* products does so at a lower total cost than *N* separate firms producing the same quantities of each product individually.

The notion of economies of scale can actually be applied to a multi-product firm as well. We can use this extension to further refine the distinction between economies of scale and scope. Suppose a firm is producing N products, with output levels measured by $Q_1, Q_2, \ldots, Q_N$. If it operates with economies of scale, the total cost of production will rise by less than 1% when production of all outputs increases by 1%. If it operates with diseconomies of scale, the total cost of production will rise by more than 1% when production of all outputs increases by 1%. By contrast, economies of scope exist if it is less costly to have the outputs produced by one firm instead of by N firms, each specializing in the production of one of the outputs.

Note that information about economies of scope does not tell us whether the firm has economies of scale. If a production process has economies of scope, there may not be economies of scale. Further, information about economies of scale does not tell us whether the firm has economies of scope. If a production process has economies of scale, there may not be economies of scope.

12. The experience curve represents the relationship between average variable cost and cumulative production volume over time. One would expect that as cumulative production volume increased, average variable cost would fall. Economies of scale refer to a situation when average cost declines as the level of output for that product increases within a given time frame.

In general, economies of scale would occur if the average cost curve declined as the level of output increased. Economies of experience would occur if, as cumulative production volume increased, the average cost curve *shifted* downward for all levels of output. So, economies of scale refer to lower average costs that occur as output increases and economies of experience refer to lower average costs for all levels of output as cumulative production volume increases.

Solutions to Problems

8.1 The table is reproduced below. First, since fixed costs are independent of quantity, the entire TFC column can be easily filled in. Proceeding through the table row by row, for Q = 1 it is easy to see that TVC = TC – TFC = 80, and the rest of the row is similarly straightforward. For Q = 2, TC = TVC + TFC = 180, and the rest of the follows easily. For Q = 3, all we have is TFC = 20; thus, we cannot infer anything else. For Q = 4, TC = Q*AC = 380. It's then possible to get TVC and AVC; however, we cannot find MC since we don't know TC or TVC for Q = 3. For Q = 5, the important step is to use MC(5) = TC(5) – TC(4) to find TC(5) = 550. For Q = 6, the important step is TVC = AVC*Q = 720.

Q	TC	TVC	TFC	AC	MC	AVC
1	**100**	80	20	100	80	80
2	180	**160**	20	90	80	80
3	-	-	**20**	-	-	-
4	380	360	20	**95**	-	90
5	550	530	20	110	**170**	106
6	740	720	20	123.3	190	**120**

8.2 It helps to rewrite this table adding an extra column for Total Fixed Costs at each level of output. The TFC for Q = 2 is just 2*30 = 60, and this is also the TFC value for every other output level. Then for Q = 1, we know TC = AC*Q = 100, TVC = TC – TFC = 40 and the rest is straightforward. Similarly we can fill in the rows for Q = 2, 3, 4, and 6. For Q = 5, we need to use the fact that MC(6) = TC(6) – TC(5) to infer TC(5) = 250. The rest is straightforward.

Q	TC	TVC	AFC	AC	MC	AVC	TFC
1	100	40	60	**100**	40	40	60
2	110	**50**	**30**	55	10	25	60
3	120	60	20	40	**10**	20	60
4	180	120	15	45	60	**30**	60
5	250	190	12	50	70	38	60
6	**330**	270	10	55	**80**	45	60

8.3

Q	TC	TVC	TFC	AC	MC	AVC
1	18	8	10	18	8	8
2	30	20	10	15	12	10
3	46	36	10	46/3	16	12
4	66	56	10	66/4	20	16
5	90	80	10	18	24	16
6	118	108	10	118/6	28	18

8.4

Q	TC	TVC	TFC	AC	MC	AVC
1	20	10	10	20	10	10
2	36	26	10	18	16	13
3	55	45	10	18.33	19	15
4	82	72	10	20.5	28	18
5	112	102	10	22.4	30	20.4
6	144	134	10	24	32	22.33

8.5 Starting with the tangency condition, we have

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\frac{K}{L} = \frac{2}{1}$$

$$K = 2L$$

Substituting into the production function yields

$$Q = LK$$

$$Q = L(2L)$$

$$L = \sqrt{\frac{Q}{2}}$$

Plugging this into the expression for K above gives

$$K = 2\sqrt{\frac{Q}{2}}$$

Finally, substituting these into the total cost equation results in

$$TC = 2\left(\sqrt{\frac{Q}{2}}\right) + 2\left(\sqrt{\frac{Q}{2}}\right)$$
$$TC = 4\left(\sqrt{\frac{Q}{2}}\right)$$
$$TC = \sqrt{8Q}$$

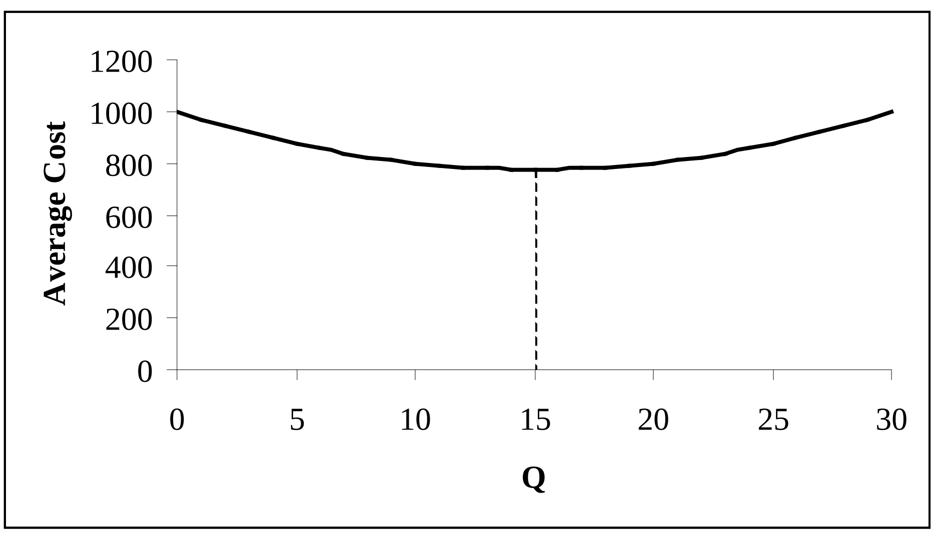
and average cost is given by

$$AC = \frac{TC}{Q} = \frac{\sqrt{8Q}}{Q}$$
$$AC = \sqrt{\frac{8}{Q}}$$

8.6

$$AC = \frac{TC}{Q} = \frac{1000Q - 30Q^2 + Q^3}{Q}$$
$$AC = 1000 - 30Q + Q^2$$

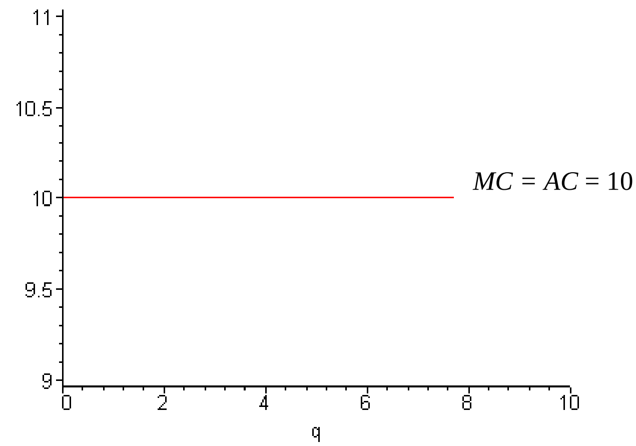
Graphically, average cost is



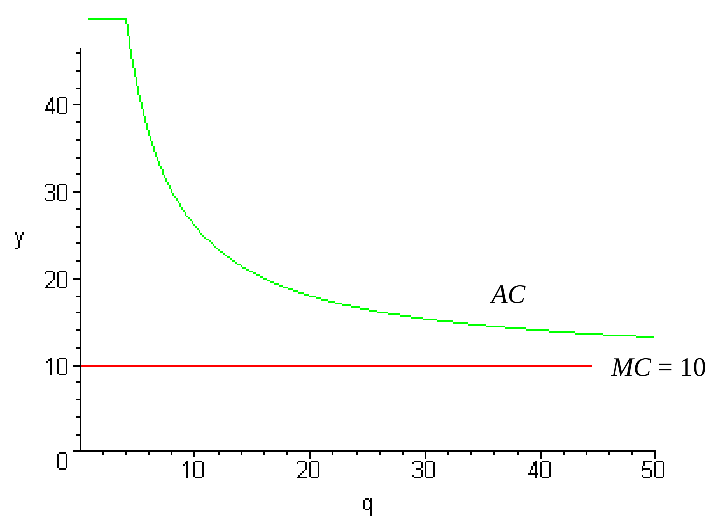
Minimum efficient scale occurs where the average cost curve reaches a minimum, $Q = 15$ for this cost function.

8.7 From the total cost curve, we can derive the average cost curve, $AC(Q) = 40 - 10Q + Q^2$. The minimum point of the AC curve will be the point at which it intersects the marginal cost curve, i.e. $40 - 10Q + Q^2 = 40 - 20Q + 3Q^2$. This implies that *AC* is minimized when *Q* = 5. By definition, there are economies of scale when the *AC* curve is decreasing (i.e. *Q* < 5) and diseconomies when it is rising (*Q* > 5).

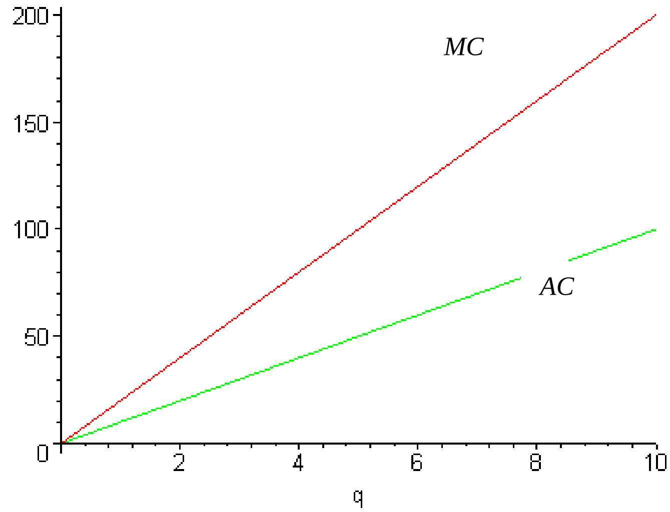
8.8 a) *TFC* = 0, *AVC* = 10, *MC* = 10.



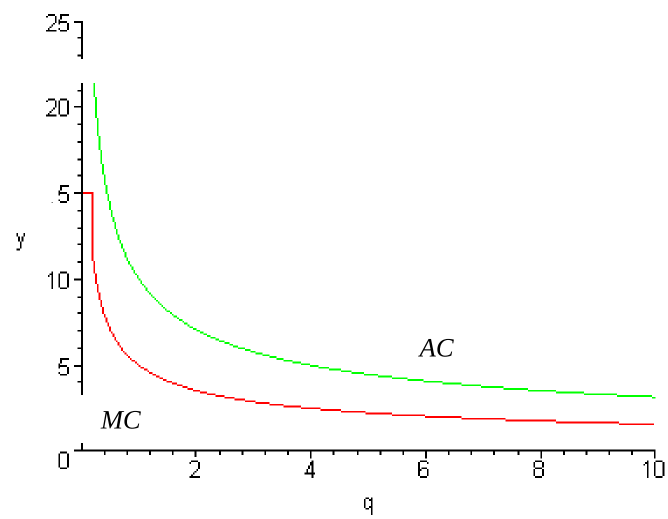
b) *TFC* = 160, *AVC* = 10, *MC* = 10.



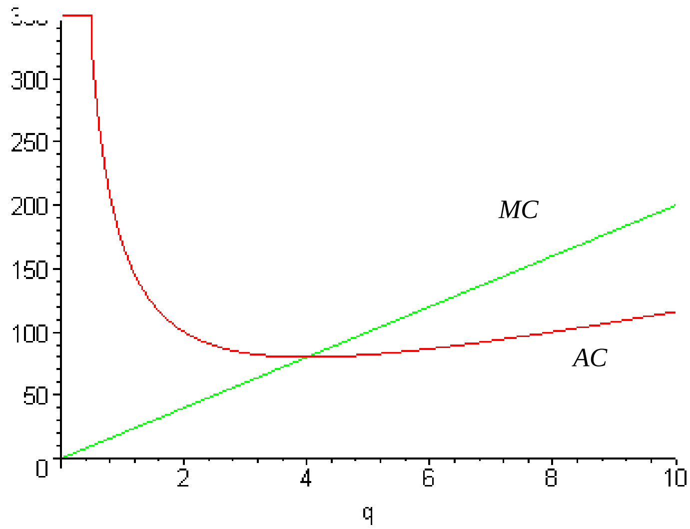
c) $TFC = 0$, $AVC = 10Q$.



d) $TFC = 0$, $AVC = 10\big/\sqrt{Q}$.

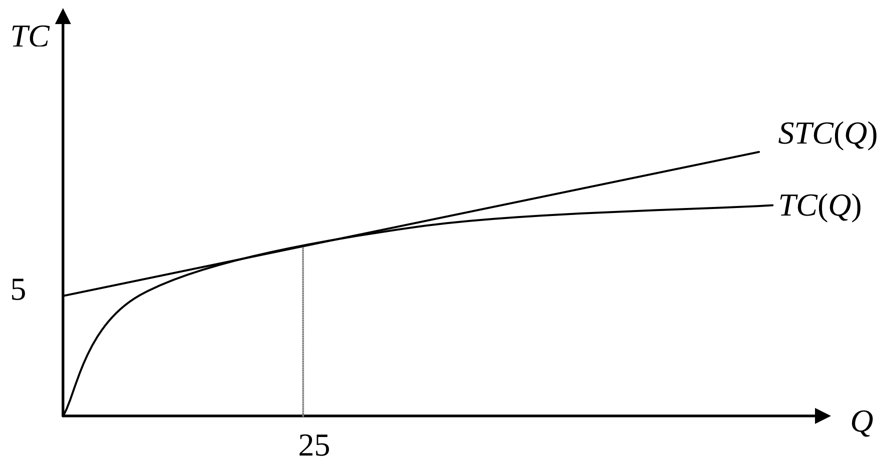


e) $TFC = 160$, $AVC = 10Q$.

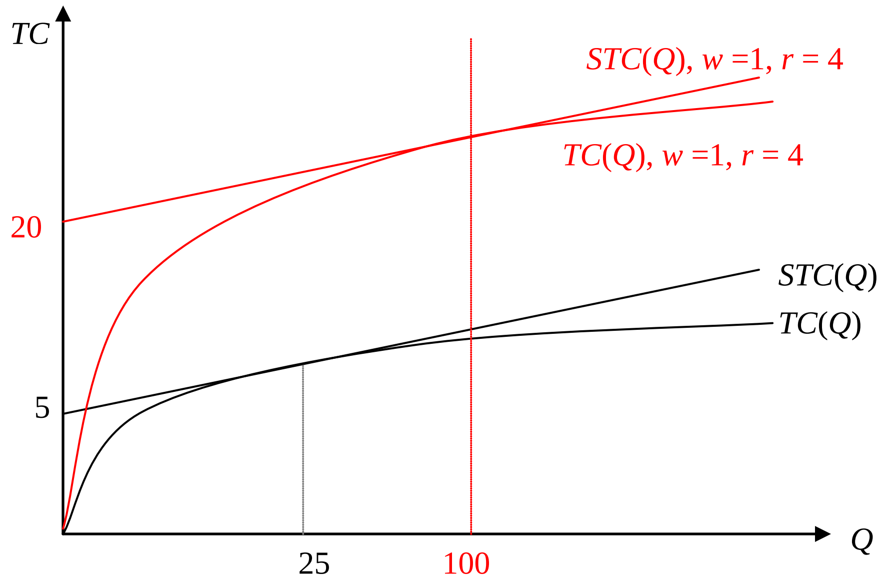


8.9 a) Cost-minimizing quantities of inputs are equal to $L = \sqrt{Q}\sqrt{(r/w)}$ and $K = \sqrt{Q} / \sqrt{(r/w)}$. Hence, in the long-run the total cost of producing Q units of output is equal to $TC(Q) = 10 + 2\sqrt{(Qrw)}$. For $w = 1$ and $r = 1$ we have $TC(Q) = 2\sqrt{Q}$.

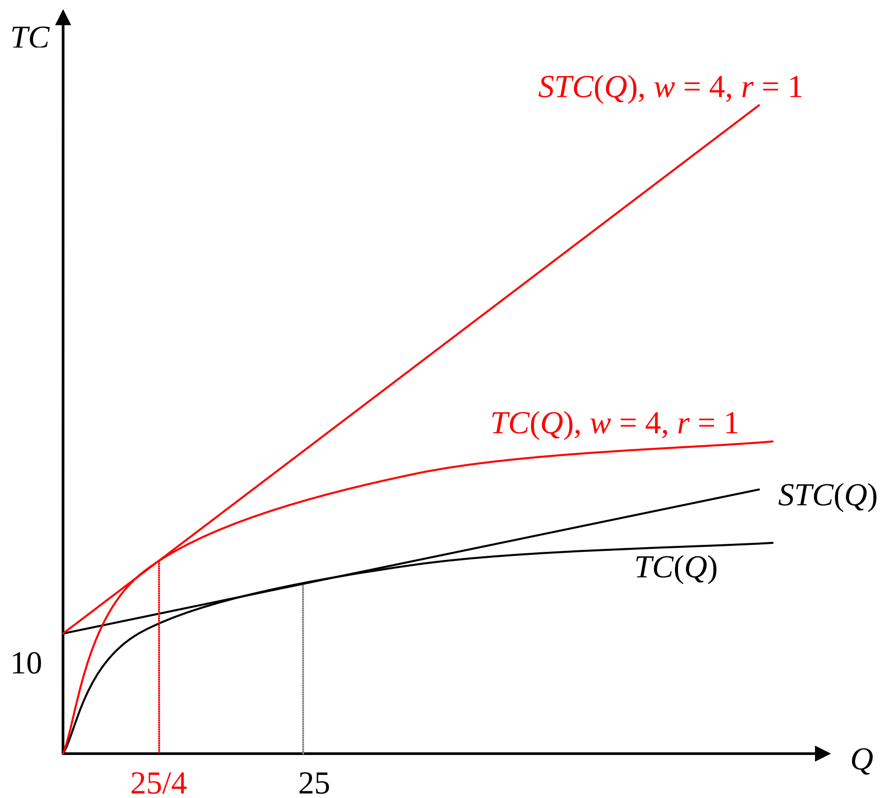
b) When capital is fixed at a quantity of 5 units (i.e., $K^* = 5$) we have $Q = K^*L = 5\,L$. Hence, in the short-run the total cost of producing Q units of output is equal to $STC(Q) = 5 + Q/5$.



c) We have $L = \sqrt{Q}\sqrt{(r/w)}$ and $K = \sqrt{Q} / \sqrt{(r/w)}$. Hence, $TC(Q) = 2\sqrt{(Qrw)}$ and $STC(Q) = 5r + wQ/5$. When $w = 1$ and $r = 4$ we have $TC(Q) = 4\sqrt{Q}$ and $STC(Q) = 20 + Q/5$.

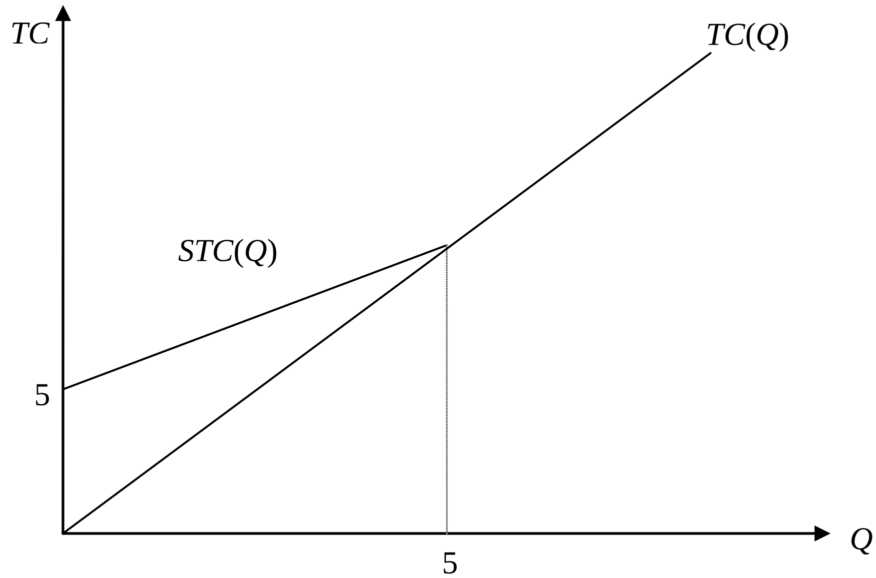


d) When $w = 4$ and $r = 1$ we have $TC(Q) = 4\sqrt{Q}$ and $STC(Q) = 4Q/5$.

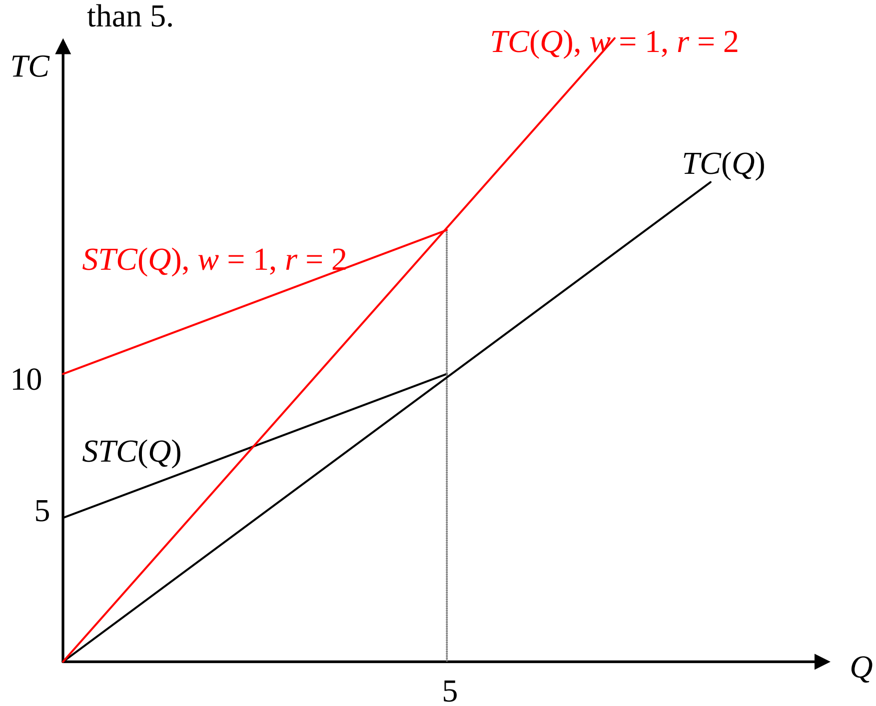


8.10 a) The inputs are complementary and the cost-minimizing firm uses them in proportions 1:1. Hence, we have $TC(Q) = Q(w + r)$.

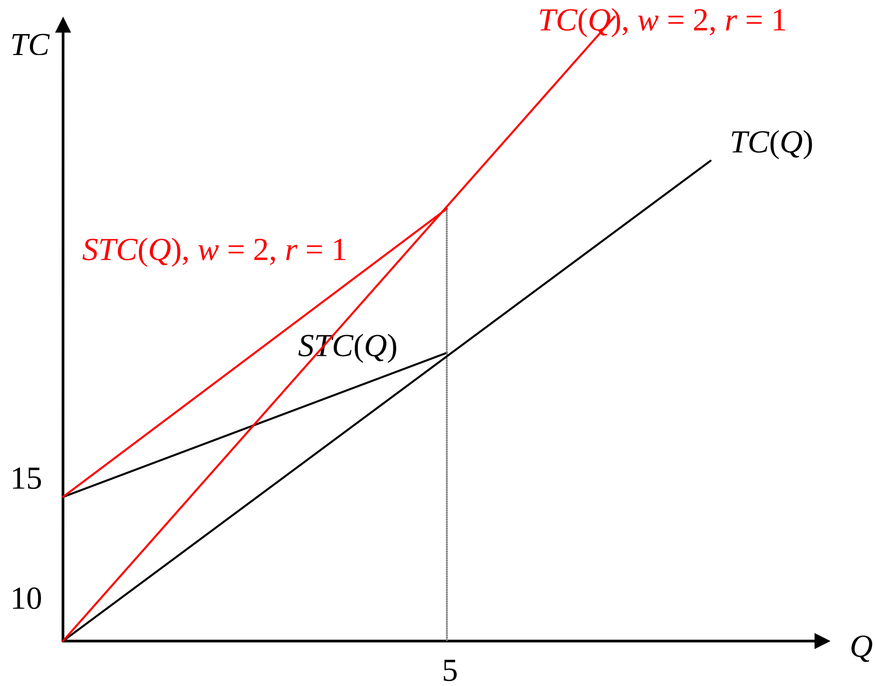
b) If $w = r = 1$, then $TC(Q) = 2Q$. In the short run, it is impossible to produce more than 5 units. This is because min(L,5) cannot be any greater than 5. To produce $Q \leq 5$ units, we set $L = Q$. With $w = r = 1$, this implies $STC(Q) = 15 + Q$. (10 is the fixed cost of the indivisible input, 5 is the fixed cost of labor, and Q is the variable cost of labor.) The diagram below shows $TC(Q)$ and $STC(Q)$ for $Q \leq 5$ when $w = r = 1$.



c) For $w = 1$ and $r = 2$ we have $TC(Q) = 3Q$ and $STC(Q) = Q + 10$ for Q not larger than 5.

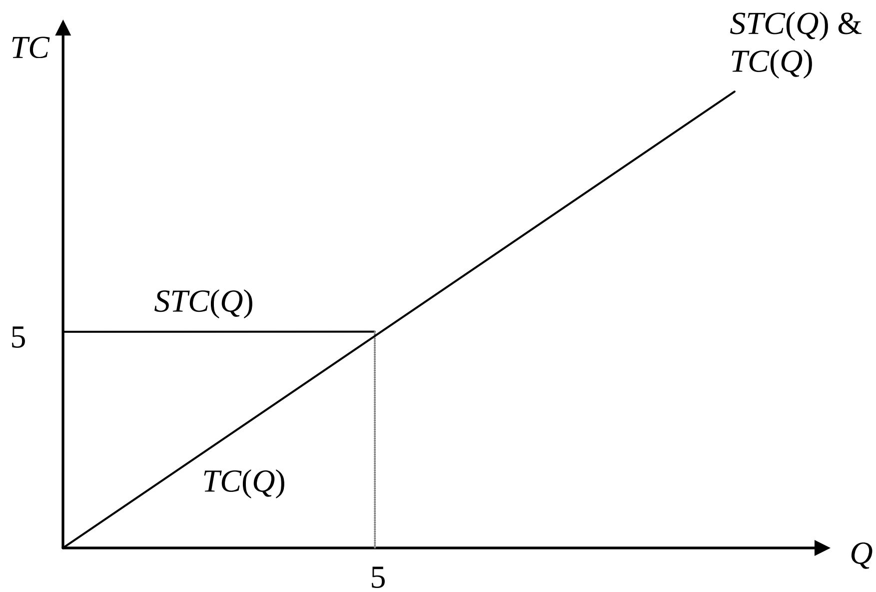


d) For $w = 2$ and $r = 1$ we have $TC(Q) = 3Q$ and $STC(Q) = 2Q + 5$ for Q not larger than 5.



8.11 a) With a linear production function, the firm operates at a corner point depending on whether $w < r$ or $w > r$. If $w < r$, the firm uses only labor and thus sets $L = Q$. In this case, the total cost (including the fixed cost) is wQ. If $w > r$, the firm uses only capital and thus sets $K = Q$. in this case, the total cost is rQ. When $w = r = 1$, the firm is indifferent among combination of L and K that make $L + K = 10$. Thus, we have $TC(Q) = Q$.

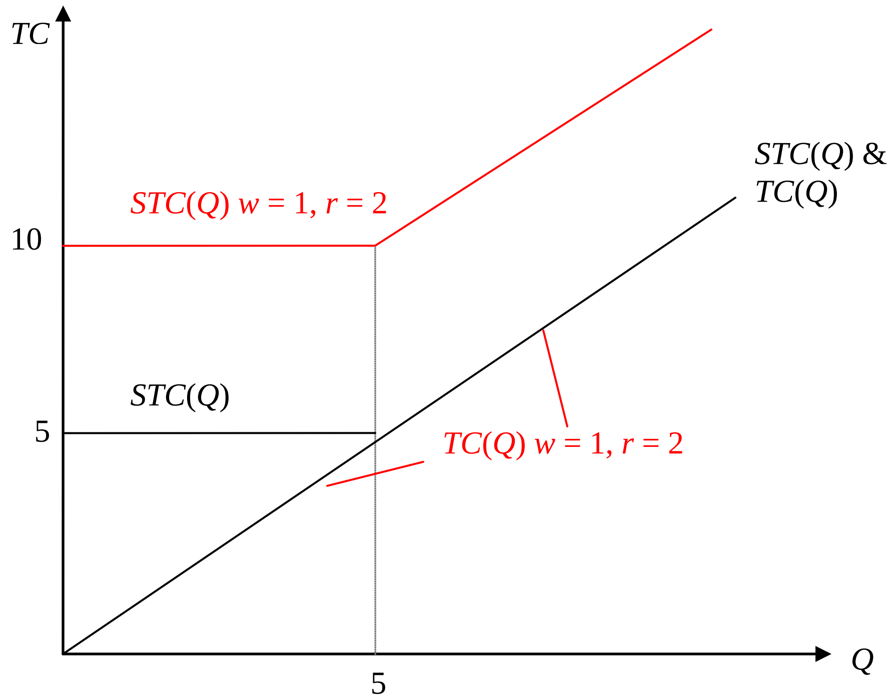
b) When capital is fixed at 5 units, the firm's output would be given by $Q = 5 + L$. If the firm wants to produce $Q < 5$ units of output, it must produce 5 units and throw away $5 – Q$ of them. The total cost of producing fewer than 5 units is constant and equal to $5, the cost of the fixed capital. For $Q > 5$ units, the firm increases its output by increasing its use of labor. In particular, to produce Q units of output, the firm uses $Q – 5$ units of labor, for a cost of $Q – 5$, and 5 units of capital, for a cost of 5. Thus, $STC(Q) = Q – 5 + 5 = Q$



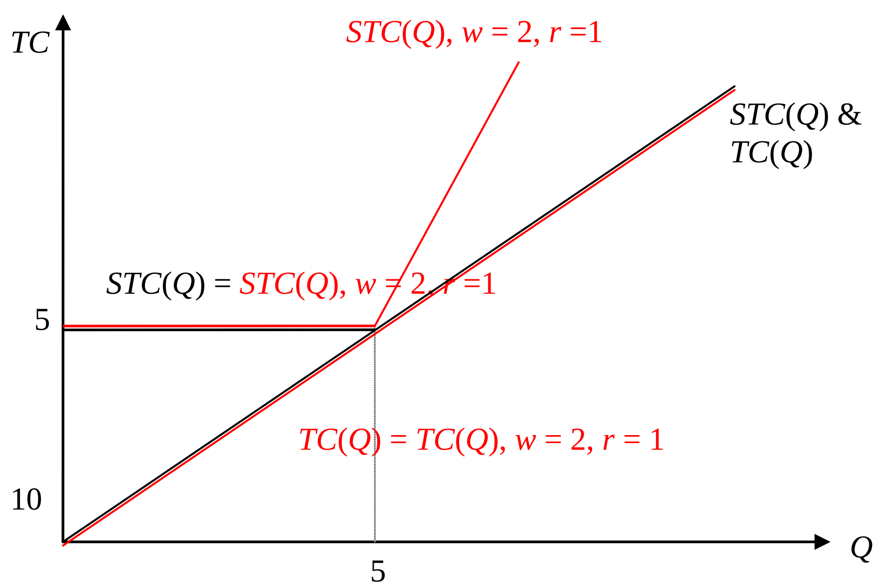
c) In the long run, since $w < r$, the firm produces its output entirely with labor. Thus, $TC(Q) = Q$, just as in part (b). In the short-run, with capital fixed at 5 units, the firm's output would be given by $Q = 5 + L$. If the firm wants to produce $Q < 5$ units of output, it must produce 5 units of output and throw away $5 – Q$ of them. It can produce this output using its fixed stock of 5 units of capital and no labor. The total cost of producing $Q < 5$ units of output when the price of capital is \$2 per unit is \$10.

For $Q > 5$ units, the firm increases its output by increasing its use of labor. In particular, to produce Q units of output, the firm uses $Q – 5$ units of labor, for a cost of $Q – 5$, and 5 units of capital, for a cost of 10. Thus, $STC(Q) = (Q – 5) + 10 = Q + 5$.

Notice that when $\overline{K} = 5$, $w = 1$, and $r = 2$, the *STC* curve strictly lies above the *TC* curve. This is because $K = 5$ is never an optimal capital choice for the firm when $w = 1$ and $r = 2$. As a result the firm's total costs are always higher in the short run than they are in the long run.



d) The total cost curve is the same as in part (b), i.e. $TC(Q) = Q$. This is because the cheaper input (in this case capital) continues to have a price of \$1 per unit. In the short run, with capital being fixed at 5 units, the cost of producing $Q < 5$ is \$5. To produce more than Q units, the firm uses $Q - 5$ units of labor at a total cost of $2(Q - 5) = 2Q - 10$. It also uses 5 units of capital at a total cost of 5. Thus, for $Q > 5$, $STC(Q) = 2Q - 10 + 5 = 2Q - 5$.



8.12 a) Starting with the tangency condition we have

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$
$$\frac{\left[L^{1/2} + K^{1/2}\right]L^{-1/2}}{\left[L^{1/2} + K^{1/2}\right]K^{-1/2}} = \frac{2}{1}$$
$$\frac{K}{L} = 4$$
$$K = 4L$$

Plugging this into the total cost function yields

$$Q = \left[L^{1/2} + (4L)^{1/2}\right]^2$$
$$Q = \left[3L^{1/2}\right]^2$$
$$Q = 9L$$
$$L = \frac{Q}{9}$$

Inserting this back into the solution for K above gives

$$K = \frac{4Q}{9}$$

b)

$$TC = 2\left(\frac{Q}{9}\right) + \frac{4Q}{9}$$
$$TC = \frac{2Q}{3}$$

c)

$$AC = \frac{TC}{Q} = \left(\frac{2Q}{3}\right)\Big/Q$$
$$AC = \frac{2}{3}$$

d) When $Q \le 9$ the firm needs no labor. If $Q > 9$ the firm must hire labor. Setting $\bar{K} = 9$ and plugging in for capital in the production function yields

$$Q = \left[L^{1/2} + 9^{1/2}\right]^2$$
$$Q^{1/2} = L^{1/2} + 3$$
$$L^{1/2} = Q^{1/2} - 3$$
$$L = \left[Q^{1/2} - 3\right]^2$$

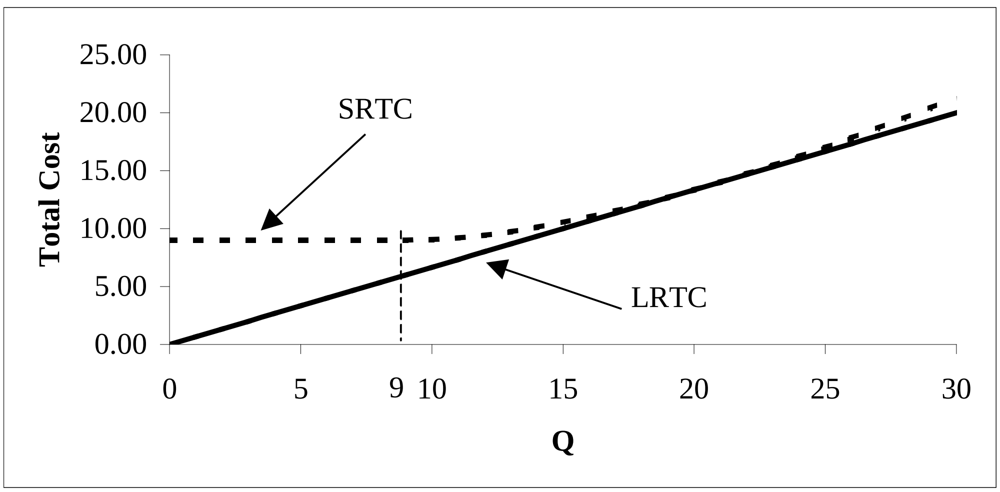
Thus,

$$L = \begin{cases} \left[Q^{1/2} - 3\right]^2 & \text{if } Q > 9 \\ 0 & \text{if } Q \le 9 \end{cases}$$

e)

$$TC = \begin{cases} 2\left(Q^{1/2} - 3\right)^2 + 9 & \text{when } Q > 9 \\ 9 & \text{when } Q \le 9 \end{cases}$$

Graphically, short-run and long-run total cost are shown in the following figure.



f)

$$AC = \frac{TC}{Q} = \begin{cases} \dfrac{2\left(Q^{1/2} - 3\right)^2 + 9}{Q} & \text{if } Q > 9 \\ \dfrac{9}{Q} & \text{if } Q \le 9 \end{cases}$$

8.13 a) Each tricycle requires the purchase of three wheels at price P_W and one frame at price P_F. Thus, $TC(Q, P_W, P_F) = Q(3P_W + P_F)$.

b) Three wheels and one frame are perfect complements in production. Thus the production function is $Q(F, W) = \min\{F, (1/3)W\}$. Notice that $(F, W) = (1, 3)$ yields $Q = 1$, $(F, W) = (2, 6)$ yields $Q = 2$, etc.

8.14 The fixed proportions production function implies that for the firm to be at a cost minimizing optimum, $4L = 7K$ and both of these equal Q. Therefore, $L = Q/4$ and $K = Q/7$. So the firm's total cost is $wL + rK = wQ/4 + rQ/7 = [\frac{w}{4} + \frac{r}{7}]Q$.

The average cost curve is $LRAC = TC/Q = \frac{w}{4} + \frac{r}{7}$. Note that this average cost curve is independent of Q and is simply a straight line.

8.15 Since we can assume an interior solution, the tangency condition must hold. Therefore the optimal bundle must be such that $\frac{K}{L+1} = \frac{w}{r}$. This means $L + 1 = \frac{rK}{w}$. Substituting this back into the production function, we see that $Q = \frac{rK^2}{w}$, so $K = \sqrt{\frac{Qw}{r}}$.

This implies that $L = \sqrt{\frac{Qr}{w}} - 1$. The total cost curve is then $TC = wL + rK = 2\sqrt{wrQ} - w$. If we substitute $2w$ and $2r$ in the place of w and r respectively, we get $TC_2 =$ $2\sqrt{(2w)(2r)Q} - (2w) = 4\sqrt{wrQ} - 2w = 2*TC$, so total cost does indeed double when input prices double.

8.16 As we saw in Chapter 7, linear production functions usually have corner solutions. In this case, the firm will use only labor if

$$MRTS_{L,K} > \frac{w}{r}, \text{ or } \frac{w}{3} < \frac{r}{5}$$

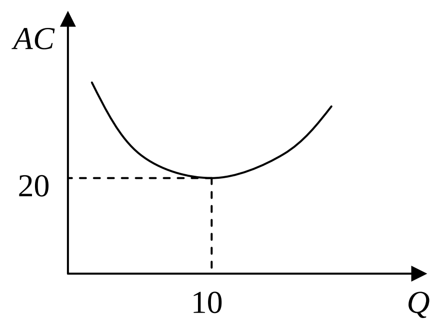
Similarly, it will use only capital if $\frac{w}{3} > \frac{r}{5}$.

If the firm does use labor, then it will use $L = \frac{Q}{3}$ with a total cost of $wQ/3$. Similarly if it uses capital it will use $K = \frac{Q}{5}$ with a total cost of $rQ/5$.

Therefore, the firm's total cost curve can be expressed as $TC = \min\{\frac{w}{3}, \frac{r}{5}\}Q$.

8.17 a) From the production function we see that $Q = 5\sqrt{L}$, so the amount of labor required to produce Q is given by $L = \frac{Q^2}{25}$. The short run total cost function is $C = 25L + 20K = 25\left[\frac{Q^2}{25}\right] + 20(5) = 100 + Q^2$.

b)



8.18 a) Even if the firm hires zero units of labor, with K fixed at 16 it can still produce up to $Q = \sqrt{0} + \sqrt{16} = 4$ units of output. So for $Q \leq 4$, $L = 0$ is the cost-minimizing choice of labor and the short-run total cost function is just the cost of capital: $C = rK + wL = 2(16) + 1(0) = 32$.

b) For $Q > 4$, the firm needs to hire positive amounts of labor, according to $Q = \sqrt{L} + \sqrt{16}$ or $L = (Q - 4)^2$. So for $Q > 4$, the short-run total cost function is $C(Q) = rK + wL = 2(16) + 1(Q - 4)^2 = 32 + (Q - 4)^2$.

8.19 a) Equating the bang for the buck between labor and capital implies

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\frac{KM}{LM} = \frac{5}{1}$$

$$K = 5L$$

Equating the bang for the buck between labor and materials implies

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$
$$\frac{KM}{KL} = \frac{5}{2}$$
$$M = \frac{5L}{2}$$

Plugging these into the production function yields

$$Q = L(5L)\left(\frac{5L}{2}\right)$$
$$Q = \frac{25L^3}{2}$$
$$L^3 = \frac{2Q}{25}$$
$$L = \left(\frac{2Q}{25}\right)^{1/3}$$

Substituting into the tangency condition results above implies

$$K = 5\left(\frac{2Q}{25}\right)^{1/3}$$

and

$$M = \frac{5}{2}\left(\frac{2Q}{25}\right)^{1/3}$$

b)

$$TC = 5\left(\frac{2Q}{25}\right)^{1/3} + 5\left(\frac{2Q}{25}\right)^{1/3} + 2\left(\frac{5}{2}\right)\left(\frac{2Q}{25}\right)^{1/3}$$
$$TC = 15\left(\frac{2Q}{25}\right)^{1/3}$$

c)

$$AC = \frac{TC}{Q} = \frac{15}{Q}\left(\frac{2Q}{25}\right)^{1/3}$$

d) Beginning with the tangency condition

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$
$$\frac{KM}{KL} = \frac{5}{2}$$
$$M = \frac{5L}{2}$$

Setting $\bar{K} = 50$ and substituting into the production function yields

$$Q = L(50)\left(\frac{5L}{2}\right)$$
$$Q = 125L^2$$
$$L = \sqrt{\frac{Q}{125}}$$

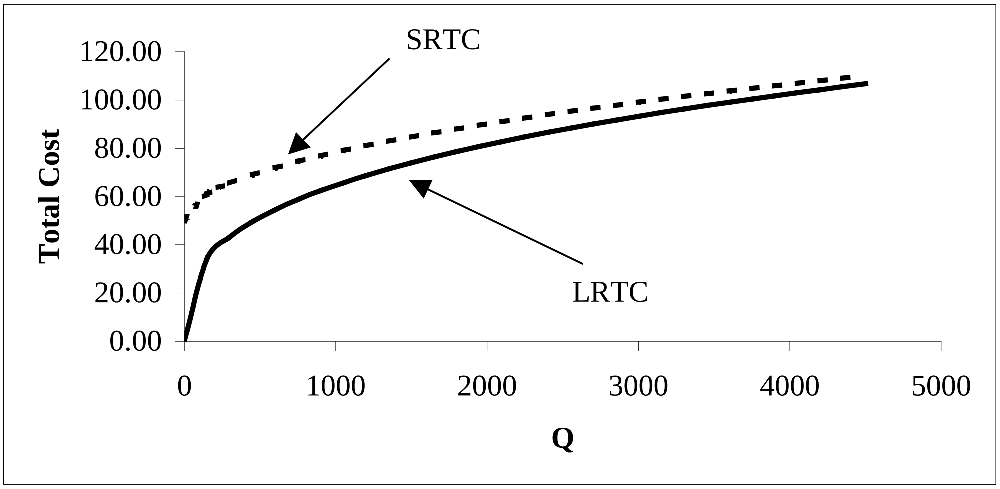
Substituting this result into the tangency condition result above implies

$$M = \frac{5\sqrt{\frac{Q}{125}}}{2}$$
$$M = \sqrt{\frac{Q}{20}}$$

e) In the short run,

$$TC = 5\sqrt{\frac{Q}{125}} + 50 + 2\sqrt{\frac{Q}{20}}$$
$$TC = 2\sqrt{\frac{Q}{5}} + 50$$

Graphically, short-run and long-run total cost curves are shown in the following figure.



f) Short run average cost is given by

$$AC = \frac{TC}{Q} = \frac{2\sqrt{\frac{Q}{5}} + 50}{Q}$$

8.20 First, notice that if the firm uses L it must necessarily use K and vice versa; there is no point using a positive amount of one of these inputs and zero of the other. Thus, there are three possible solutions to the long-run cost minimization problem: (i) interior, using positive amounts of K, L, and M; (ii) a corner with $K = L = 0$ and $M > 0$; or (iii) a corner with $M = 0$ but positive amounts of *both* K and L.

Using approach (i), we equate $MP_K/r = MP_M/s$ and $MP_L/w = MP_M/s$ to get $K = 16$ and $L = 4$. Using the production constraint then yields $M = Q – 64$. Total cost using this approach will be $C_{KLM}(Q) = 16(4) + 4(16) + 1(Q – 64) = Q + 64$.

Using approach (ii), we have $K = L = 0$ and the input demand for M comes from the production constraint: $M = Q$. Total cost will be $C_M(Q) = Q$.

Using approach (iii), $M = 0$ and the tangency condition between K and L yields $MP_L/w = MP_K/r$, or $K = 4L$. Combined with the production function, we get the input demand functions $K = 2\sqrt{Q}$ and $L = \frac{1}{2}\sqrt{Q}$. Total cost will be

$$C_{KL}(Q) = 16\left(\tfrac{1}{2}\sqrt{Q}\right) + 4\left(2\sqrt{Q}\right) + 1(0) = 16\sqrt{Q}\,.$$

Comparing the three approaches, it is easy to see that $C_M(Q) < C_{KLM}(Q)$ for all values of Q. Hence, a cost-minimizing firm will never use K, L, and M simultaneously; it could produce the same output at less cost by just using M. Furthermore, $C_M(Q) < C_{KL}(Q)$ only for $Q < 256$. So for $Q = 400$, the firm should set $M = 0$ and, following approach (ii), set $K = 40$ and $L = 10$. Total cost will be $C_{KL} = 320$.

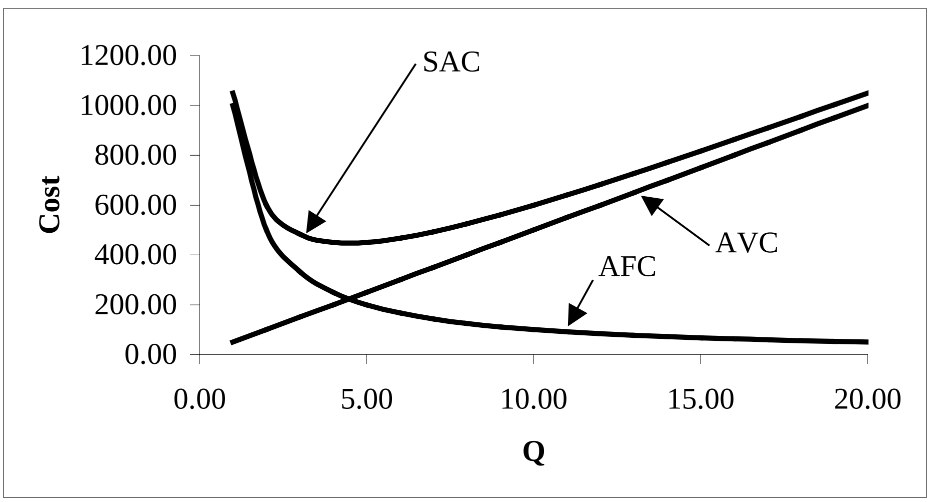
8.21 With K fixed at 20 units, the production function becomes $Q = 20L + M$. Thus, L and M are perfect substitutes. Since $MP_L/w = 1.25 > MP_M/s = 1$, the marginal product per dollar spent on labor is always higher than that on materials. So the cost-minimizing input combination is $M = 0$ with L solving $400 = 20L + 0$, or $L = 20$. The short run total cost is $C = 16(20) + 4(20) + 1(0) = 400$.

8.22 With K fixed at 20 and M fixed at 40, the production function becomes $Q = 20L + 40$. To produce 400 units, the firm needs to hire labor until $400 = 20L + 40$, or $L = 18$. The short-run total cost is $C = 16(18) + 4(20) + 1(40) = 408$.

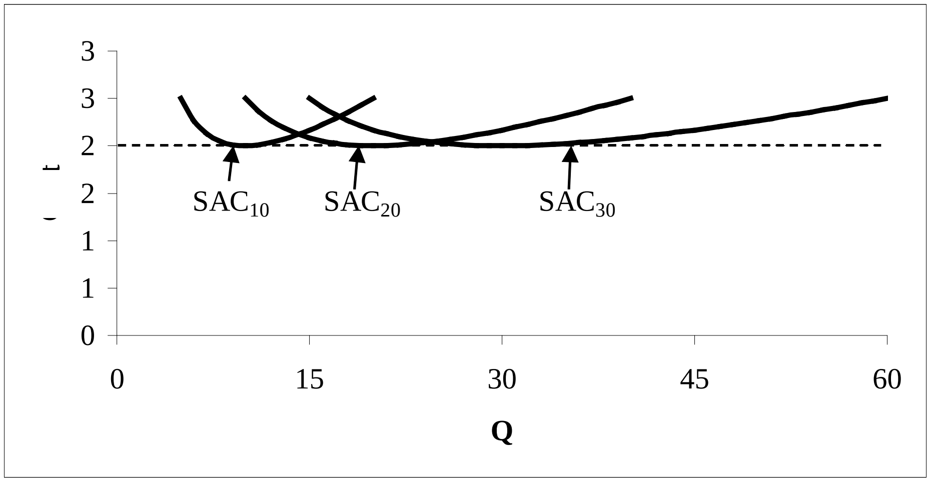
8.23

$$STC(Q) = 1000 + 50Q^2$$
$$SAC(Q) = \frac{STC(Q)}{Q} = \frac{1000}{Q} + 50Q$$
$$AVC(Q) = 50Q$$
$$AFC(Q) = \frac{1000}{Q}$$

Graphing $SAC(Q)$, $AVC(Q)$, and $AFC(Q)$ yields



8.24



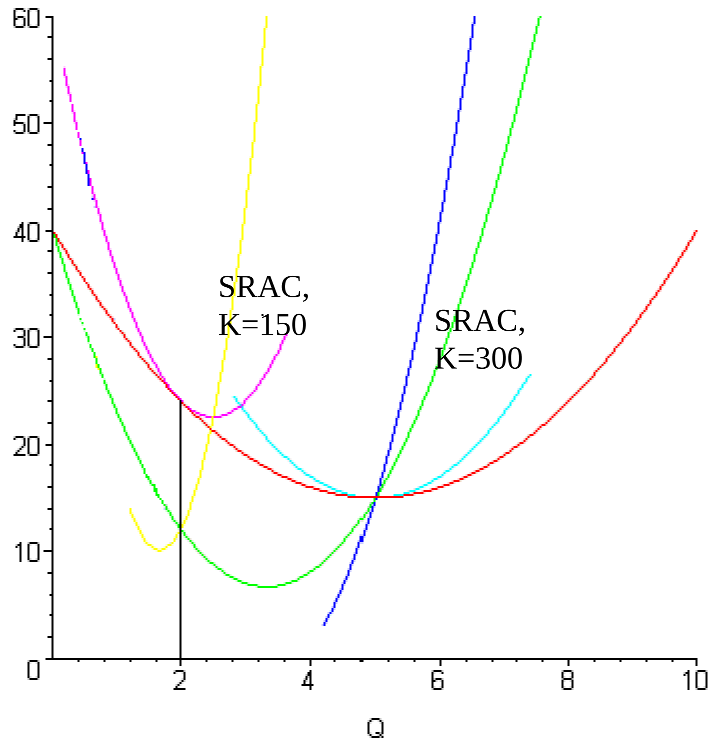
Since each of these short-run average cost curves reaches a minimum at an average cost of 2.0, the long-run average cost curve associated with these short-run curves will be a horizontal line, tangent to the bottom of each of these curves, at a long-run average cost of 2.0.

8.25 With some inputs fixed, it is likely that the fixed level is not optimal given the firm's size. Therefore, it may be more expensive to produce additional units in the short run than in the long run when the firm can employ the optimal, *i.e.*, cost minimizing, quantity of the fixed input.

8.26 The SRAC curves are shown below. Each curve must satisfy two requirements- (i) the SRAC curve must be tangent to the LRAC curve at the output level at which the SRMC curve for that particular plant size intersects the LRMC curve; and (ii) the SRAC curve must reach a minimum at the output level at which it intersects its own SRMC curve.

For the plant size of 300 this is easily achieved by just drawing a curve tangent to the LRAC curve at its minimum point, since this is also the point at which the LRMC and the corresponding SRMC curves intersect (at the output level Q = 5).

For a plant size of 150, these two points must be kept in mind and the curve must be drawn carefully to comply with both. First, SRAC is tangent to LRAC at Q = 2, where LRMC intersects SRMC for K = 150. Second, SRAC reaches its minimum where it intersects SRMC, near Q = 2.4.



8.27 Economies of scope exist if

$$TC(Q_1, Q_2) - TC(Q_1, 0) < TC(0, Q_2) - TC(0, 0)$$

In this case

$$TC(Q_1, Q_2) = 1000 + 2Q_1 + 3Q_2$$
$$TC(Q_1, 0) = 1000 + 2Q_1$$
$$TC(0, Q_2) = 1000 + 3Q_2$$
$$TC(0, 0) = 0$$

So, economies of scope exist if

$$(1000 + 2Q_1 + 3Q_2) - (1000 + 2Q_1) < 1000 + 3Q_2$$
$$3Q_2 < 1000 + 3Q_2$$
$$0 < 1000$$

which is certainly true. Thus, in this case the cost of adding a movie channel when the firm is already providing a sports channel is less costly (by $1000) than a new firm supplying a movie channel from scratch. Economies of scope exist for this satellite TV company.

8.28 $TC(10, 300) = 1000$ while $TC(10, 0) + TC(0, 300) = 500 + 400 = 900$. Thus $TC(10, 300) > TC(10, 0) + TC(0, 300)$ so economies of scope do not exist at this output level.

8.29

$$TC = \frac{100Q^3\sqrt{r}}{\sqrt{w}}$$

This TC function implies that for a fixed Q and r, increasing w would lower long-run total cost. If the firm were minimizing cost in the long run, by using the optimal combination of K and L, it would not be possible to reduce total cost when w is increased.

As Figures 8.3 and 8.4 in the text illustrate, when one input price increases, the total long-run cost will increase. Therefore, this long-run total cost function is not consistent with long-run cost minimization by the firm.

8.30

a) Essentially, the firm's production function is

$$\begin{aligned} Q &= Q_1 + Q_2 \\ &= \sqrt{25L_1} + \sqrt{100L_2} \\ &= 5\sqrt{L_1} + 10\sqrt{L_2} \end{aligned}$$

That is, the firm has two variable inputs, L_1 and L_2. The marginal products are $MP_{L1} = 2.5(L_1)^{-0.5}$ and $MP_{L2} = 5(L_2)^{-0.5}$. Using the tangency condition, we see that $L_2 = 4L_1$. Using the production functions at each plant, we see

$$Q_2 = 10\sqrt{L_2} = 20\sqrt{L_1} = 4Q_1$$

Since total output $Q = Q_1 + Q_2$, we have $Q_2 = 4(Q - Q_2)$ or $Q_2 = .8Q$. Similarly, $Q_1 = .2Q$. So the firm should produce 80 percent of output at plant 2 and 20 percent at plant 1.

b) Combining the above tangency condition and the production constraint, we find the input demands are $L_1 = Q^2/625$ and $L_2 = 4Q^2/625$. Including the cost of capital, total cost is then $C = 125 + (Q^2/125)$. Average cost is $AC = (125/Q) + (Q/125)$. Marginal cost is $MC = 2Q/125$. $MC(100) = 1.6$, $MC(125) = 2$, and $MC(200) = 3.2$.

c) In the long run, the plants are identical so the entrepreneur should split production equally between the two plants (i.e. $Q_1 = Q_2$). Thus the total production function can be written as

$$\begin{aligned} Q &= Q_1 + Q_2 \\ &= 2Q_1 \\ &= 2\sqrt{K_1 L_1} \end{aligned}$$

Again, we can view total output as depending on the choice of only two inputs. (Since the plants are identical the entrepreneur will hire equal amounts of capital at each plant; and similarly for labor.) Minimizing cost implies $MRTS_{L_1,K_1} = w/r$ or $K_1 = L_1$. Input demands are then $L_1 = K_1 = 0.5Q$ so that total cost is $C = w(L_1 + L_2) + r(K_1 + K_2) = 2Q$. Long-run average cost is $AC = 2$ and long-run marginal cost is $MC = 2$.

Chapter 9

Perfectly Competitive Markets

Solutions to Review Questions

1. The difference between accounting profit and economic profit is in how total cost is measured. With accounting profit, total cost is measured as total accounting cost while with economic profit, total cost is measured as total economic cost. Accounting cost measures the historical expenses the firm incurred to produce and sell its product while economic cost measured the opportunity cost of the resources that the firm uses to produce and sell its product.

   If a firm chose to produce and sell a product it could earn a positive accounting profit but negative economic profit. This would occur if the economic cost of the resources used was greater than the accounting cost of the resources used. For example, the firm might purchase resources for $1 million and use these to produce a product when instead the firm could have resold the resources for $2 million. In this case the economic cost exceeds the accounting cost and economic profit would be less than accounting profit.

2. The law of one price ensures that all transactions will take place at a single market price. A perfectly competitive firm cannot affect the market price by increasing or decreasing production. Therefore, for each unit produced and sold, the firm will receive the market price as revenue. Revenue will increase with each unit sold by the market price, implying the market price is equal to marginal revenue.

3. A perfectly competitive firm would not produce if the market price is below the minimum of its average variable cost. If the firm shuts down, the "bad news" is that it loses revenue. The "good news" is that the firm avoids non-sunk costs (including variable costs). If the market price is below the minimum of the firm's average variable cost, the good news from shutting down outweighs the bad news.

   If the market price is below the minimum of the firm's short-run average cost, the decision as to whether the firm should shut down depends on how much of the fixed costs are non-sunk (avoidable). Suppose first that all of the fixed costs are non-sunk. If the firm shuts down, the "bad news" is that it loses revenue. The "good news" is that the firm avoids variable costs, as well as all of the fixed costs. If the market price is below the minimum of the firm's short-run average cost, the good news from shutting down outweighs the bad news.

   Now suppose that some of the fixed costs are sunk. Then for at least some levels of market price below the minimum of short-run average cost, the revenue lost may be greater than the costs that can be avoided if the firm shuts down. For such a market price, the firm would be better off continuing to operate in the short-run, because its losses from operating would be less than the losses it would sustain if it were to shut down.

4. When all fixed costs are sunk, the shut-down price is the minimum level of average variable cost. When all fixed costs are non-sunk, the shut-down price is the minimum level of short-run average cost.

5. The supply elasticity can be used to determine the extent to which the equilibrium price will change when demand shifts exogenously. If supply is elastic, then a shift in demand will have a smaller impact on the equilibrium price than when supply is inelastic.

6. Because the minimum efficient scale is higher in industry 1 than in industry 2, and since in a perfectly competitive market each firm must produce at minimum efficient scale in the long-run, there will be fewer firms in industry 1 since each firm is producing more units.

7. Economic rent measures the economic surplus that is attributable to an extraordinarily productive input whose supply is limited. It is equal to the difference between the maximum amount a firm is willing to pay for the services of the input and the input's reservation value.

   Economic rent and economic profit are closely related. A firm's economic profit depends on the price it pays for the extraordinarily productive input. If the firm pays the input its reservation value, the firm can earn a positive economic profit, but if the firm pays the input the reservation value plus the economic rent, as defined above, the firm will earn zero profit. Economic rent is thus the potential increase in the firm's economic profit from employing the productive input, and the firm's economic profit will depend on how much of the economic rent gets allocated to the firm and how much gets allocated to the input.

8. The producer surplus for an individual firm is the difference between the total revenue the firm receives and the non-sunk cost. In general, it is the area below price and above the supply curve.

   Producer surplus for a market of firms when the number of firms is fixed is the sum of the producer surplus for each of the individual firms.

   Producer surplus for a firm will equal economic profit if the firm has no sunk fixed costs.

   If producer surplus and economic profit are not equal, producer surplus equals the difference between total revenue and total non-sunk costs while economic profit equals the difference between total revenues and all total costs. Therefore, producer surplus will exceed economic profit if the two are not equal.

9. In a market in which the long-run industry supply curve is upward sloping, the area between the price and the long-run supply curve measures the economic rents of inputs that are in scarce supply and whose price is bid up as more firms enter the industry.

10. Producer surplus and economic profit may be equal in the short run. In particular, in the short run,

Producer Surplus = Economic Profit + Sunk Fixed Costs.

Thus, in the short run producer surplus and economic profits differ by the level of sunk fixed costs. However, in the long run, since no fixed costs are sunk, producer surplus and economic profit will be equal. In general, producer surplus measures the difference between total revenue and total non-sunk costs while economic profit measures the difference between total revenue and all total costs.

These two measures differ from economic rent. Economic rent is the economic surplus that is attributable to an extraordinarily productive input whose supply is limited. Essentially, economic rent measures the potential increase in economic profit attributable to the scarce input above and beyond the economic profit the firm would enjoy if the firm paid suppliers of the input an amount equal to their reservation value. However, it is possible that the input, because of its scarcity, can extract the economic rent from the firm so that the firm still earns zero economic profit.

Solutions to Problems

9.1 The accounting costs are

Supplies	$25,000
Employee Salaries	$170,000
Total Accounting Cost	$195,000

Accounting profit = Revenue – Accounting Cost = $250,000 - $195,000 = $55,000

The economic costs are

Supplies	$25,000
Employee Salaries	$170,000
Opportunity cost of land	$100,000
Total Economic Cost	$295,000

Economic profit = Revenue – Economic Cost = $250,000 - $295,000 = - $45,000

The negative economic profit indicates that the owners would be better off by $45,000 if they shut down the shop and rent out the land.

9.2 Her accounting profit equals revenues less all of the expenses reflected in the ledger: $1,000,000 - $300,000 - $20,000 - $500,000 - $100,000 = $80,000.

All of the accounting costs are also economic costs. The first three expense items (wages paid to hired labor, utilities, and purchases of drugs and supplies) are expenses in competitive markets, so the opportunity cost is reflected in the market prices. Further, the wages she pays herself are the same as the opportunity cost of her time, because the most she could earn if she exits the drug store business is $100,000 working as a lawyer.

The economic costs of the business include all of the accounting costs, plus the $200,000 opportunity cost of the building because she could earn that if she exits the drug store business. Her economic profit is her accounting profit ($80,000) less the additional opportunity cost ($200,000) not included in the accounting cost. So her economic profit is actually -$120,000.

We can look at this another way. If she continues to work at the grocery store, she earns an accounting profit of $80,000, plus the salary she pays herself ($100,000). But if she exits the business, her salary as a lawyer would be $100,000, and she would receive $200,000 rent for the building. She would therefore be better off by $120,000 if she worked as a lawyer.

9.3 The table is as follows:

Output (Units)	Total Revenue (($/unit)	Total Cost ($/unit)	Profit ($)	Marginal Revenue ($/unit)	Marginal Cost ($/unit)
0	0	30	-30	50	
1	50	80	-30	50	50
2	100	100	0	50	20
3	150	130	20	50	30
4	200	172	28	50	42
5	250	226	24	50	54
6	300	296	4	50	70

When the firm is producing a positive amount of output, profit is maximized when Q = 4, regardless of the fixed cost. The firm will produce another unit when MR > MC, and cut back production when MR < MC. The relationship between MR and MC is unaffected by fixed cost.

9.4 The table is as follows:

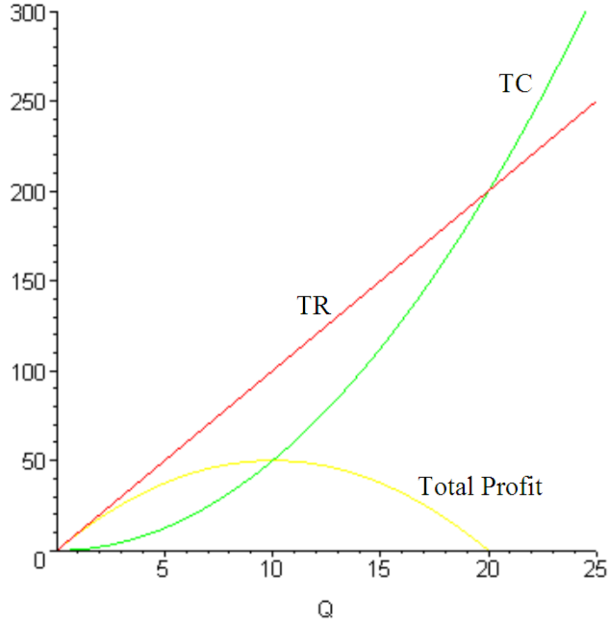
Q	TC	TVC	AFC	AC	MC	AVC
1	**200**	80	120	200	80	80
2	220	**100**	60	110	20	50
3	240	120	40	80	**20**	40
4	360	**240**	30	90	120	60
5	500	380	**24**	100	140	76
6	**660**	540	20	110	**160**	90

The firm should produce 5 units. (Up to that level of output $P > MC$, but $P < MC$ for the sixth unit.) Profit = $PQ - C$ = 150(5) – 550 = 250.

9.5 If the firm operates at a point where its SRAC curve is rising, it must mean that the SRMC curve lies above the SRAC curve. And since the firm will choose an output such that price=SRMC, it means that price is greater than SRAC. Therefore the firm is earning positive economic profit.

9.6 a) Since the firm is producing in a perfectly competitive market, the firm views the output price as exogenous. It should produce up to the point at which $P = SMC(Q)$, that is, so that 10 = Q. So it should produce 10 units of output.

b) The graph is shown below.



The total cost function increases in Q, and at an increasing rate. Total Profit at first increases in Q and then decreases. From the graph, it appears that Profit is maximized when Q is about 10, which we found in (a).

9.7 Producer surplus equals revenue less all non-sunk costs. Thus:

Producer surplus = 200 – 160 = 40

The non-sunk costs of 160 include the variable cost of 120 and the non-sunk fixed cost of 40.

Profit = Revenue minus all costs = 200 – 120 – 60 – 40 = –20.

To decide whether to operate or shut down, the firm should look at producer surplus (rather than profit). Producer surplus (40) shows how much better off he would be operating (with a profit = –20) than shutting down (with a profit = –60). So he should stay in business in the short run; he will lose money, but not as much as if her were to shut down.

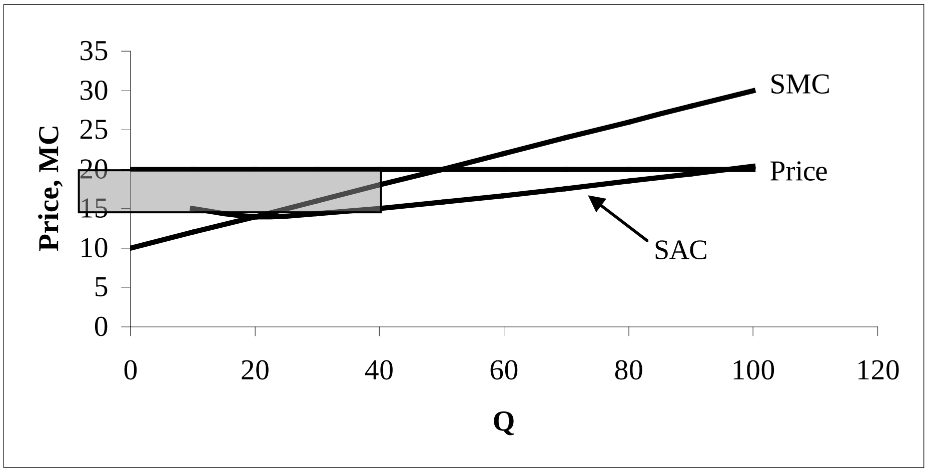
9.8 a) In order to maximize profit Ron should operate at the point where $P = MC$.

$$20 = 10 + 0.20Q$$
$$Q = 50$$

b) Ron's profit is given by $\pi = TR - TC$.

$$\pi = 20(50) - (40 + 10(50) + 0.10(50)^2)$$
$$\pi = 210$$

c) The firm's profit is equal to the shaded area in the graph below. It is a rectangle whose height is the market price and the average cost of the 50th unit, and whose width is the 50 units being produced.



d) If all fixed costs are sunk, then *ANSC* = *AVC* = $(10Q + 0.1Q^2)/Q = 10 + 0.1Q$. So the first step is to find the minimum of *ANSC* by setting *ANSC* = *SMC*, or $10 + 0.1Q = 10 + 0.2Q$ which occurs when $Q = 0$. The minimum level of *ANSC* is thus 10. For prices below 10 the firm will not produce and for prices above 10, its supply curve is found by setting $P = SMC$:

$$P = 10 + .2Q$$
$$Q = 5P - 50$$

The firm's short-run supply curve is thus

$$s(P) = \begin{cases} 0 & \text{if } P < 10 \\ 5P - 50 & \text{if } P \geq 10 \end{cases}$$

e) If all fixed costs are non-sunk, as in this case, then *ANSC* = *ATC* = $(40/Q) + 10 + 0.1Q$. The minimum point of *ANSC* occurs where *ANSC* = *SMC*:

$$\frac{40}{Q} + 10 + .1Q = 10 + .2Q$$
$$Q = 20$$

The minimum level of *ANSC* is thus 14. For prices below 14 the firm will not produce and for prices above 14, its supply curve is found by setting $P = SMC$ as before.

$$s(P) = \begin{cases} 0 & \text{if } P < 14 \\ 5P - 50 & \text{if } P \geq 14 \end{cases}$$

9.9 a) First, find the minimum of AVC by setting $AVC = SMC$.

$$AVC = \frac{TVC}{Q} = \frac{Q^2}{Q}$$
$$AVC = Q$$

$$Q = 2Q$$
$$Q = 0$$

The minimum level of AVC is thus 0. When the price is 0 the firm will produce 0, and for prices above 0 find supply by setting $P = SMC$.

$$P = 2Q$$
$$Q = \tfrac{1}{2}P$$

Thus,

$$s(P) = \tfrac{1}{2}P$$

b) Market supply is found by horizontally summing the supply curves of the individual firms. Since there are 20 identical producers in this market, market supply is given by

$$S(P) = 20s(P)$$
$$S(P) = 10P$$

c) Equilibrium price and quantity occur at the point where $S(P) = D(P)$.

$$10P = 110 - P$$
$$P = 10$$

Substituting $P = 10$ back into $D(P)$ implies equilibrium quantity is $Q = 100$. So at the equilibrium, $P = 10$ and $Q = 100$.

9.10 a) The firm will not produce any output when the price falls below the point where *SMC* = *ANSC*, i.e. the minimum of the *ANSC* curve. Therefore
$50/Q + 40 + 0.5Q = 40 + Q$

This implies $Q = 10$. The corresponding price, below which the firms will not produce, is equal to $MC(10) = ANSC(10) = 50$.

b) Each firm will produce according to the relation, $P = MC$, or $P = 40 + Q$. This means that each firm's supply curve is $Q = P - 40$ if $P \geq 50$ and zero if $P < 50$. Therefore market supply equals $12(P - 40)$ and in equilibrium this must equal market demand, $360 - 2P$. Therefore the equilibrium price is $P = 60$. At this price, each firm produces 20 units of output. The firm's profit is $PQ - V(Q) - F$ and this equals 30. Substituting $Q = 20$ and $P = 60$, we get total fixed costs, $F = 170$. Since non-sunk fixed costs are 50, sunk fixed costs must total up to 120.

9.11 The firm's *ANSC* curve is given by $32/Q + 2Q$. To find the shut-down price, we find the minimum level of *ANSC*. This occurs at the quantity at which *ANSC* equals *MC*, or $32/Q + 2Q = 4Q$. Solving for Q yields $Q = 4$, and substituting this into the expression for *ANSC* tells us that the minimum level of *ANSC* is equal to $32/4 + 2(4) = \$8$. At prices below \$8, a firm's supply is 0. At prices above \$8, a firm produces a quantity at which $P = SMC$: $P = 4Q$, or $Q = P/4$. Thus, the short-run supply curve for a firm is:

$$s(P) = \begin{cases} 0 & \text{if } P < 8. \\ \dfrac{P}{4} & \text{if } P \geq 8 \end{cases}$$

Since there are 60 identical producers, each with this supply curve, the short-run market supply curve $S(P)$ is 60 times $s(P)$, or:

$$S(P) = \begin{cases} 0 & \text{if } P < 8. \\ 15P & \text{if } P \geq 8 \end{cases}$$

To find the equilibrium price, we equate market supply to market demand and solve for P: $15P = 400 - 5P$, or $P = 20$.

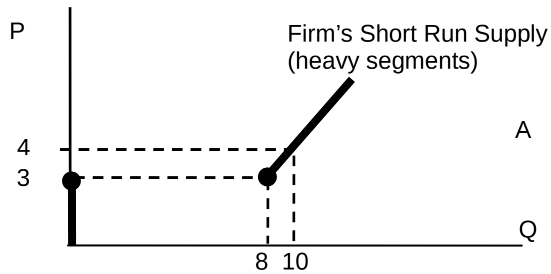
9.12 a) $C = F + 2Q^2$. $MC = 4Q$.
Breakeven price = 40. When P = 40, the firm would produce Q so that MC = P; 40 = 4Q; Q = 10.
Profit = $PQ - F - 2Q^2 = 40(10) - F - 2(10)^2 = 200 - F = 0$. So F = 200.

b) The total nonsunk fixed cost is $NSC = 128 + 2Q^2$.
The firm will shut down if the market price is less than the minimum of ANSC.
$ANSC = (128 + 2Q)^2/Q$.

At minimum of ANSC, we know that ANSC = MC, or that $[128 + 2Q^2]/Q = 4Q$, so the quantity at the shutdown price is Q = 8.
The shutdown price will be where Q = 8; MC = 4Q = 4(8) = 32. So the shutdown price is P = 32.
(Alternatively, you can verify that when Q = 8, then ANSC = 32.)

c) The firm's supply schedule will be Q = 0 when P < 32.
When P > 32, the firm will supply according to the optimal quantity choice rule P = MC; thus P = 4Q, so that Q = 4/P.
When P = 32, the firm will be indifferent between shutting down (Q = 0) or operating with Q = 8.

To summarize, $$Q = \begin{cases} P/4 & \text{when } P > 32 \\ 0 & \text{when } P < 32 \\ 0 \text{ or } 8 & \text{when } P = 32 \end{cases}$$



d) With 10 firms in the market, total market supply will be 10(P/4) = 2.5P. Market demand is 180 – 2.5P.
In equilibrium 2.5P = 180 – 2.5P, so P = 36 (note: P > 32, so the firms do produce).
e) For profits to be zero, the price would be P = 40, and each firm would produce 40/4 = 10 units.
The quantity demanded in the market would be 180 – 2.5(40) = 80 units. Thus, there is room for only 80/10 = 8 firms.

9.13 Total industry supply is the sum of the supply curves of the individual firms. Since we have 100 type *A* firms, total supply from type *A* firms is $100s_A(P) = 200P$, and since we have 30 type *B* firms, total supply from type B firms is $30s_B(P) = 300P$. The short-run industry supply curve is thus $S(P) = 200P + 300P = 500P$. The short-run market equilibrium occurs at the price at which quantity supplied equals quantity demanded, or $5000 – 500P = 500P$, or $P = 5$. At this price, a type *A* firm supplies 10 units, while a type *B* firm supplies 50 units.

9.14 To determine the quantity supplied for a given price, set $P = SMC$.

$$P = 2Q$$

$$Q = \tfrac{1}{2}P$$

Thus the supply curve for each firm is $s(P) = \frac{1}{2}P$. If each firm is producing 20 units, then

$$20 = \tfrac{1}{2}P$$
$$P = 40$$

So the market price is 40. Substituting into demand reveals

$$D(P) = 240 - \tfrac{1}{2}(40)$$
$$D(P) = 220$$

If each firm is producing 20 units, the market will have $^{220}\!/_{20} = 11$ firms.

9.15 Since each firm is earning zero economic profit, we know $P = SAC$. Since each firm supplies where $P = SMC$, set $SAC = SMC$.

$$\frac{400}{Q} + 5 + Q = 5 + 2Q$$
$$Q = 20$$

Since $P = 5 + 2Q$, $P = 45$. If market price is $P = 45$, $D(P) = 240$. Finally, if total market demand is $Q = 240$ and each firm is producing 20 units, there will be $^{240}\!/_{20} = 12$ firms in the market.

9.16 If the firm operates at a point where its *SAC* curve is rising, it must mean that the marginal cost curve is above the *SAC* curve. And since the firm must set price=MC, it means that price is greater than average cost. Therefore the firm earns positive economic profit.

If it operates at a point where the *SAC* curve is falling, it means S*MC* < *SAC* and therefore price is less than average cost. Therefore the firm is making negative economic profit in the short run. However, the fact that the firm is still operating means that marginal cost must be above the average non-sunk cost curve, so that it is better for the firm to continue operating, albeit at a loss, than to shut down.

9.17 Given a market price P, the firm will produce from each plant so that MC = P.
The profit maximizing quantity supplied at plant 1 will be $2Q_1 = P$, or $Q_1 = P/2$.
The profit maximizing quantity supplied at plant 2 will be $4Q_2 = P$, or $Q_2 = P/4$.
The quantity supplied by the whole firm will $Q_{Firm} = Q_1 + Q_2$. Thus $Q_{Firm} = 3P/4$.
So 1/3 of the firm's total production will come from plant 2.

9.18 a) When P <10, only Type B firms will operate, and the market supply will be 4(2P) = 8P.
When P >10, both types of firms will operate, and the market supply will be 4(2P) + 6(-10 + P) = -60 + 14P.
To summarize, the market supply will be $Q^{Supply}_{Market} = \begin{cases} -60 + 14P, & \text{when } P > 10 \\ 8P, & \text{when } P \le 10 \end{cases}$

Let's first assume the equilibrium price exceeds 10, so that all firms are producing. If this is true, setting market supply equal to market demand: -60 + 14P = 108 – 10P, so that P = 7; however, the market supply we have used is valid for P>10, but not valid for P = 7.
So the equilibrium price must be less than 10, with only Type B firms producing (and Type A firms not producing).
Setting market supply equal to market demand: 8P = 108 – 10P, so that P = 6.
We have found that in equilibrium, only Type B firms produce, and the equilibrium price is 6.

b) Let's first assume the equilibrium price exceeds 10, so that all firms are producing. If this is true, setting market supply equal to market demand: -60 + 14P = 228 – 10P, so that P = 12; the market supply we have used is valid for P=12. At this equilibrium both types of firms will be producing.

9.19 We know that if a firm produces positive output, it produces where P=SMC. In this case, when the firm produces positive output, $Q = 3P - 30$, or $P = \frac{Q}{3} + 10$. This means that the equation of the firm's short run marginal cost is $SMC(Q) = \frac{Q}{3} + 10$.

9.20 a) When the supply curve is vertical at a positive quantity, the quantity supplied is unresponsive to price, and the price elasticity of supply is equal to 0 (supply is perfectly inelastic).

b) When the supply curve is horizontal at a positive quantity, price elasticity of supply is infinite (supply is perfectly elastic).

c) When the supply curve is a straight line going through the origin, the price elasticity of supply must equal 1. We can this as follows. The equation of a straight line supply curve through the origin takes the form $Q = aP$, where a is the slope of the supply curve. Thus $\Delta Q/\Delta P = a$. The price elasticity of supply is equal to $(\Delta Q/\Delta P)(P/Q) = a(P/Q) = a(P/aP) = 1$.

9.21 As with example 9.4 in the text, we can estimate the slope of the supply curve as

$$\frac{\Delta Q}{\Delta P} = \frac{4,000,000 - 3,800,000}{1.00 - 0.20}$$
$$\text{Slope} = 250,000$$

Elasticity can then be estimated as

$$\varepsilon_{Q,P} = \frac{\Delta Q}{\Delta P}\left(\frac{P}{Q}\right)$$
$$\varepsilon_{Q,P} = 250,000\left(\frac{1.00}{4,000,000}\right)$$
$$\varepsilon_{Q,P} = \frac{1}{16}$$

This implies the market supply of roses is quite inelastic.

9.22 The long-run equilibrium price in a perfectly competitive equilibrium equals the minimum level of long-run average cost. This is given as $5 per ton. Each producer supplies a quantity of output equal to the point at which long-run average is minimized. This is given as 2 million tons per year. Market demand at the long-run equilibrium price of $5 per ton is equal to 205 – 5 = 200 million tons per year. This implies that there must be 100 active firms in the long-run equilibrium because (200 million tons per year)/(2 million tons per year per firm) = 100.

9.23 In a long-run equilibrium all firms earn zero economic profit implying $P = AC$ and each firm produces where $P = MC$. Thus,

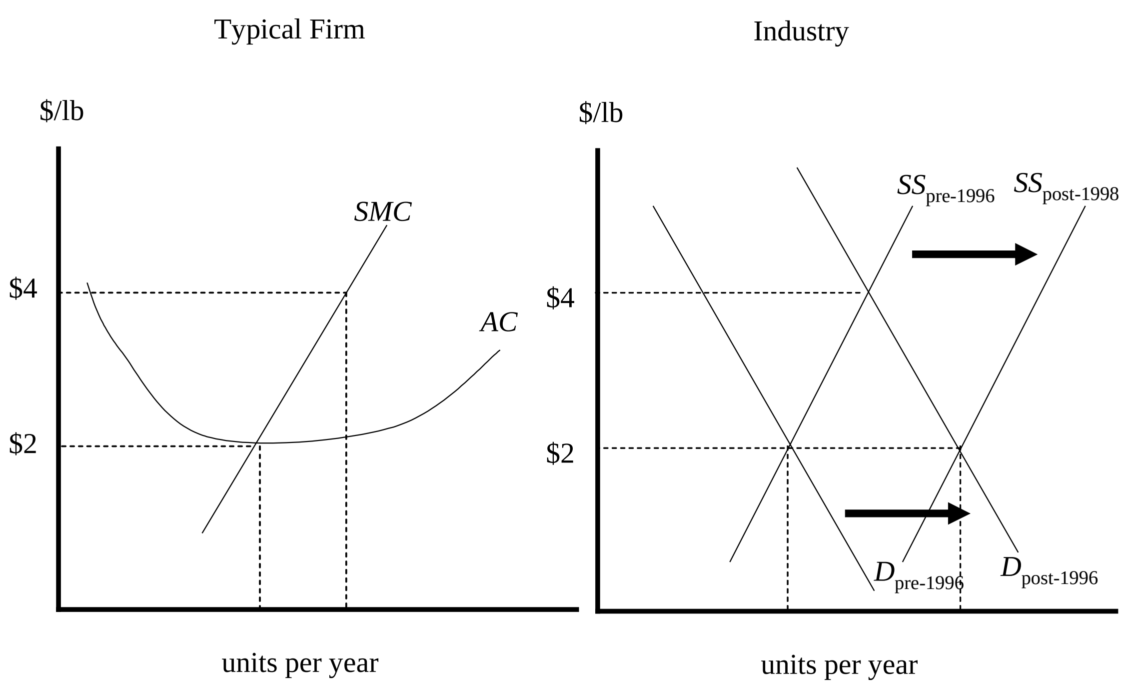
$$40 - 12Q + Q^2 = 40 - 6Q + \tfrac{1}{3}Q^2$$
$$Q = 9$$

So each individual firm produces $Q = 9$, and the long-run equilibrium price must be $P = 40 - 12(9) + 9^2 = 13$. Since $D(P) = 2200 - 100P$,

$$D(P) = 2200 - 100(13)$$
$$D(P) = 900$$

If each firm produces 9 units, the market will have 100 firms in equilibrium.

9.24 The scenario described in the problem can be explained as a constant-cost perfectly competitive industry that experienced an increase in demand (i.e., rightward shift in the demand curve) in early 1996 as shown in the figure below. The price between 1990-1995 reflects a market that is in long-run equilibrium. The increase in price in early 1996 reflects the movement to a short-run equilibrium following the increase in demand. Once price stabilizes at the new short-run equilibrium, firms earn positive economic profits, which attracts new entry. As new entry occurs during 1997 and 1998, the short-run supply curve shifts rightward, causing price to fall. Entry is no longer profitable once price is reestablished at the minimum level of long-run average cost for a typical firm. As a result of the increase in demand, the market now contains more active producers in 2002 than it did in 1990.



9.25 For this total cost function, $MC = c$. Since each firm will supply where $P = MC$, in equilibrium $P = c$. If in equilibrium $P = c$,

$$D(P) = a - bc$$

Equilibrium market quantity is $a - bc$.

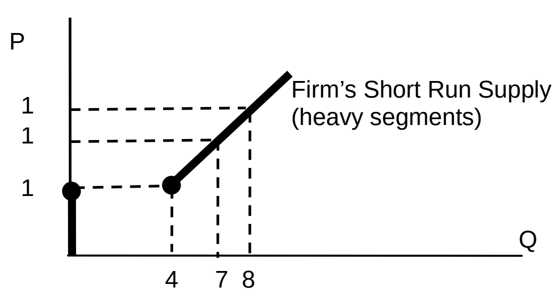
In order to determine the number of firms we need to know the quantity that each individual firm will produce. In this case marginal cost is constant implying perfectly elastic supply. Thus, at $P = c$ a firm may produce any quantity. Therefore, the number of firms cannot be determined.

9.26 This statement is true.
Profit = Revenue – Total Cost = Revenue – Nonsunk Cost – Sunk Cost
Producer Surplus = Revenue – Nonsunk Cost
In the long run, there are no sunk costs, so profit is the same as producer surplus.

9.27 a)

Q	1	2	3	4	5	6	7	8
MC	4	6	8	10	12	14	16	18
V	3	8	15	24	35	48	63	80
Prod surplus =PQ– V - 16	4(1)-3 -16 =-15	6(2) – 8 - 16 = -12	8(3) – 15- 16 = -7	10(4) – 24- 16 = 0			16(7) – 63-16 =33	
Profit = PQ – V – 64	4(1) -3 -64 =-63	6(2) – 8 - 64 = -60	8(3) – 15 - 64 = -55	10(4) – 24 - 64 = -48	12(5) – 35 - 64 = -39	14(6) – 48 - 64 = -28	16(7) – 63- 64 = -15	18(8) – 80- 64 = 0

We know that the supply curve for the firm is just the marginal cost curve for all prices greater than the shut down price. At the shut down price: Producer surplus = Revenue – V – $F_{Nonsunk}$ = 0.
$F_{Nonsunk} = F_{Total} – F_{Sunk}$ = 64 – 48 = 16.
Simple calculations from the table show that **the shut down price is P = 10**. If the firm elects to produce when P = 10, it chooses Q so that MC = P, that is, Q = 4.
Producer surplus = Revenue – V – $F_{Nonsunk}$ = 10(4) – 24 – 16 = 0.
The graph of the firm's supply function is as follows. Note that it is the same as the firm's supply function when P > 10.



b) When P = 16, the firm chooses Q so that MC = P, that is, Q = 7.
Producer surplus = Revenue – V – $F_{Nonsunk}$.
Producer surplus = Revenue – V – $F_{Nonsunk}$ = 16(7) – 63 – 16 = 33.

c) Again, calculations from the table show that **the breakeven price is P = 18**.
When P = 18, the firm chooses Q so that MC = P, that is, Q = 8.
Profit = Revenue – V – F_{Total} = 18(8) – 80 – 64 = 0.

9.28 In the short run, as demand increases, price is driven up and firms can earn positive economic profits. In the short run, however, the number of firms is fixed, so total market supply is simply the sum of the supply of each individual firm. In the long run, though, the firms cannot continue to earn positive economic profit. New firms will enter, driving the price back down until economic profit is zero. In a constant cost industry this occurs at the same equilibrium price as prior to the increase in market demand. Thus, in the long run, any quantity will be supplied and the number of firms will adjust so that each firm earns zero economic profit. The primary difference in the derivation then is that in the short run the number of firms is fixed, but in the long run the number of firms will adjust to maintain zero economic profit.

9.29 a) In a long-run competitive equilibrium $P = MC$ and $P = AC$, implying $MC = AC$.

$$\sqrt{wr}(120 - 40Q + 3Q^2) = \sqrt{wr}(120 - 20Q + Q^2)$$
$$Q = 10$$

b) In a long-run competitive equilibrium $P = MC$ so that (with $r = 1$ and $Q = 10$)

$$P = \sqrt{w(1)}(120 - 40(10) + 3(10)^2)$$
$$P = 20\sqrt{w}$$

c) Given demand for labor and setting $r = 1$ and $Q = 10$

$$L(Q, w, r) = \frac{\sqrt{r}\left(120Q - 20Q^2 + Q^3\right)}{2\sqrt{w}}$$
$$L(Q, w) = \frac{100}{\sqrt{w}}$$

d) Given market demand and setting $r = 1$

$$D(P) = \frac{10000}{P}$$
$$Q = \frac{10000}{20\sqrt{w}}$$
$$Q = \frac{500}{\sqrt{w}}$$

e) Since each firm will produce 10 units,

$$N = \frac{500/\sqrt{w}}{10}$$
$$N = \frac{50}{\sqrt{w}}$$

f) From part c), the labor demand for an individual firm is $L(Q,w) = 100/\sqrt{w}$. Overall demand for labor is then

$$\text{Demand for Labor} = \frac{50}{\sqrt{w}}\left(\frac{100}{\sqrt{w}}\right)$$
$$\text{Demand for Labor} = \frac{5000}{w}$$

g) Setting the supply of skilled labor equal to the demand for skilled labor,

$$50w = \frac{5000}{w}$$
$$w = 10$$

h) Plugging $w = 10$ into the solution for price implies $P = 63.25$; plugging $w = 10$ in market demand implies $Q = 158.10$; and plugging $w = 10$ into the solution for the number of firms and rounding down to the nearest integer implies $N = 15$.

i) If

$$D(P) = \frac{20000}{P}$$
$$Q = \frac{20000}{20\sqrt{w}}$$
$$Q = \frac{1000}{\sqrt{w}}$$

The number of firms will be

$$N = \frac{1000/\sqrt{w}}{10}$$
$$N = \frac{100}{\sqrt{w}}$$

Overall labor demand will be

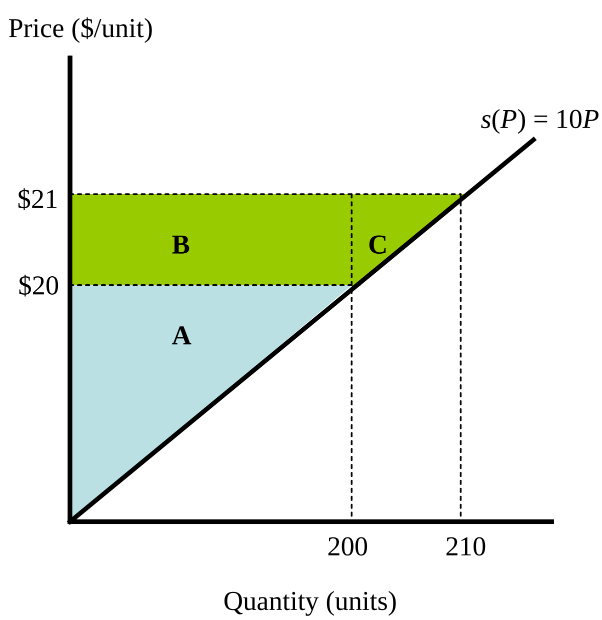
$$\text{Labor} = \frac{100}{\sqrt{w}}\left(\frac{100}{\sqrt{w}}\right)$$
$$\text{Labor} = \frac{10000}{w}$$

Setting the supply of labor equal to the demand for labor implies

$$50w = \frac{10000}{w}$$
$$w^2 = 200$$
$$w = 14.14$$

Plugging $w = 14.14$ into the solution for price implies $P = 75.21$; plugging $w = 14.14$ into market demand implies $Q = 265.92$; and plugging $w = 14.14$ into the solution for the number of firms and rounding down to the nearest integer implies $N = 26$.

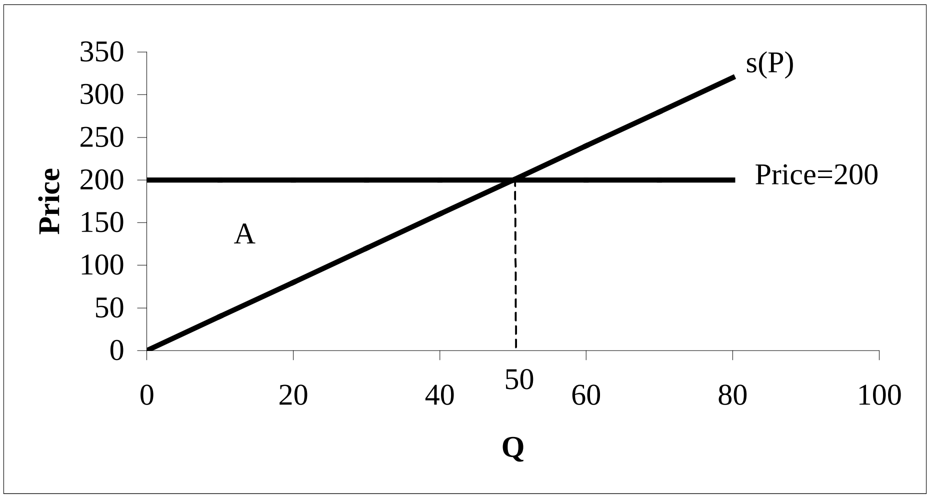
9.30 The solution is shown in the figure below. The producer surplus at a price of \$20 is equal to the area of triangle *A*, or (1/2)(20)(200) = \$2,000. When the price increases to \$21, producer surplus increases by area *B* (\$200) plus area *C* (\$5), or \$205.



9.31 Since an individual firm will supply where $P = SMC$,

$$P = 4Q$$
$$Q = \tfrac{1}{4}P$$

Assuming a firm will supply for any positive price this implies $s(P) = \tfrac{1}{4}P$. Graphically we have



Producer surplus for an individual firm is given by area A in the figure above which is $\tfrac{1}{2}(200)50 = 5000$. Since all firms are identical, overall producer surplus will be $100(5000) = 500,000$.

9.32 a) Minimum efficient scale occurs at the point where average cost reaches a minimum. This point occurs where $MC = AC$.

$$2Q = \frac{144}{Q} + Q$$
$$Q = 12$$

At $Q = 12$,

$$AC = \frac{144}{Q} + Q$$
$$AC = 24$$

b) In the long-run, the equilibrium price will be determined by the minimum level of average cost for firms with average CEOs. Thus, $P = 24$. At this price, firms having average CEOs will earn zero economic profit and firms with exceptional CEOs will earn positive economic profit.

c) At the price, the firms with an average CEO will produce where $P = MC$

$$24 = 2Q$$
$$Q = 12$$

The firms with an exceptional CEO will also produce where $P = MC$

$$Q = 24$$

d) At this price

$$D(P) = 7200 - 100P$$
$$D(P) = 4800$$

e) Since there are 100 exceptional CEOs and assuming they are all employed, the total supply from exceptional CEO firms will be

$$S_E = 100(24)$$
$$S_E = 2400$$

This leaves $Q = 4800 - 2400 = 2400$ units to be supplied by firms with average CEOs. Thus,

$$N_A = \frac{2400}{12}$$
$$N_A = 200$$

f) To calculate the exceptional CEO's economic rent we must compute the highest salary the firm would pay this CEO. This salary is the amount that would drive economic profit to zero. Call this amount S^*. Since the exceptional CEO firm is producing $Q = 24$, the firm's average cost is

$$AC = \frac{144}{24} + \frac{1}{2}(24)$$
$$AC = 18$$

Since $P = 24$, the exceptional CEO has produced a $6 per unit cost advantage. This implies

$$\frac{S^*}{24} - \frac{144}{24} = 6$$
$$S^* = 288$$

Economic rent is the difference between this salary, $288,000, and the reservation wage of $144,000. Thus, the exceptional CEO's economic rent is $144,000.

g) Firms that hire exceptional CEOs for $144,000 will gain all of the CEO's economic rent and will therefore earn economic profit of $144,000.

h) In a long-run competitive equilibrium, exceptional CEO salaries should be bid up as other firms compete for the exceptional CEOs. This should bid up the salary of the CEOs until economic profits for firms with exceptional CEOs are driven to zero. Thus, exceptional CEO salaries should approach $288,000 in a long-run equilibrium.

Chapter 11

Monopoly and Monopsony

Solutions to Review Questions

1. A monopoly market consists of a single seller facing many buyers. Because the firm is by definition supplying the entire market, it faces the entire set of buyers making up the market demand curve.

2. Marginal revenue is less than price for a monopolist. This is because as it lowers its price two things happen. First, the firm's revenue increases from the additional units it sells (these are the marginal units). Second, the firm's revenue decreases because it loses revenue from selling units at a lower price than it could have had it chosen a lower quantity of output (these are the inframarginal units). The change in revenue is the sum of the increase from the marginal units and the decrease from the inframarginal units. This change can be summarized as

$$MR = \frac{\Delta TR}{\Delta Q} = P + Q\frac{\Delta P}{\Delta Q}$$

   Since demand is downward sloping, the second term will be negative implying marginal revenue will be less than price.

3. The firm's marginal revenue could be negative if the increase in revenue the firm gets from selling additional (marginal) units at a lower price is more than offset by the decrease in revenue from selling (inframarginal) units at a lower price than if it had chosen a lower quantity of output.

If demand is price inelastic then

$$\frac{\%\Delta Q}{\%\Delta P} > -1$$
$$\frac{\Delta Q / Q}{\Delta P / P} > -1$$
$$\frac{\Delta Q}{\Delta P}\left(\frac{P}{Q}\right) > -1$$
$$P < -Q\left(\frac{\Delta P}{\Delta Q}\right)$$
$$P + Q\left(\frac{\Delta P}{\Delta Q}\right) < 0$$

But,

$$MR = P + Q\frac{\Delta P}{\Delta Q}$$

Thus, when demand is price inelastic, marginal revenue is negative.

4. a) True. Because the firm is operating on the inelastic region of the demand curve marginal revenue is negative. Thus, decreasing output will increase total revenue. And, since output is lower, total cost will be lower. Thus, by decreasing output and increasing price the firm can increase profits.

   b) False. When the firm operates on the elastic portion of the market demand curve, increasing output will increase total revenue. In addition, increasing output will increase total costs. Thus, the effect on profit will depend on how costs increase in relation to revenue.

5. The firm will not be maximizing total revenue at the point where the firm maximizes total profit. The firm maximizes revenue at the point where $MR = 0$ and the firm maximizes profit at the point where $MR = MC$. Thus, unless $MC = 0$, the firm will not maximize both revenue and profit at the same point.

6. IEPR is the Inverse Elasticity Pricing Rule. This rule states that a profit- maximizing firm that sets $MR = MC$ will satisfy the condition that

$$\frac{P^* - MC^*}{P^*} = -\frac{1}{\varepsilon_{Q,P}}$$

where the asterisks indicate the price and marginal cost at the profit-maximizing level of output.

7. While perfectly competitive firms do not have market power, it is not true that any firm that faces competition does not have market power. In this particular example, while Toyota clearly has competition, Toyota also sells a differentiated product from the other automobile manufacturers. This will allow Toyota to control its price since no other manufacturer is producing the identical product. Thus, Toyota will have some market power. It is true, however, that this market power may be limited by the prices other manufacturers set for their automobiles. If Toyota's product is not seen as being much different from other autos, it will not be able to set a price far out of line with the rest of the auto market.

8. A multi-plant monopolist will choose a level of output and then allocate output between plants so that marginal costs are equalized across plants. If a perfectly competitive firm had multiple plants it would follow the same rule. To see why, imagine it did not and allowed marginal costs to be different across plants. If marginal costs were different then it reallocate one unit of output from the high marginal cost plant to the low marginal cost plant. This would reduce total cost without changing revenue. Thus, profit would increase. Therefore, to maximize profit the firm should allocate output between plants to equalize marginal cost.

9. A monopolist creates a deadweight loss because it produces a lower level of output and charges a higher price than would occur in perfect competition. This choice allows the monopolist to generate economic profits and increase producer surplus, essentially extracting surplus away from consumers. The firm, however, will not be able to gain as much surplus as consumers lose, lowering total net benefits, and creating a deadweight loss.

10. A monopsonist is a firm that is a single buyer that can purchase from many sellers, whereas a monopolist is a firm that is a single seller that can sell to many buyers. It is possible for a firm to be both a monopolist and a monopsonist. Using the text example, suppose some local area had only one hospital. It would be a monopolist in the provision of some hospital services, e.g. emergency room services, and it would be a monopsonist in the purchase of some hospital inputs, e.g. nurses.

11. The monopsonist's marginal expenditure function is the rate at which the monopsonist's total cost goes up, per unit of input, as it hires more units of the input. Marginal expenditure will exceed unit cost because as the monopsonist increases the price it pays for units of input it must pay this higher price for the units it could have purchased at lower prices. This marginal expenditure can be summarized as

$$ME_L = w + L\frac{\Delta w}{\Delta L}$$

Since the second term is positive (the monopsonist must pay a higher wage to increase the supply of labor) the marginal expenditure will exceed the wage.

12. The monopsonist creates a deadweight loss. This occurs because the monopsonist hires a lower quantity and pays a lower price for its input than would occur in perfect competition. This allows the monopsonist to extract surplus away from suppliers, but the monopsonist is unable to earn as much additional surplus as suppliers lose, lowering net total benefits, and creating a deadweight loss.

Solutions to Problems

11.1 a) If demand is given by $Q = 100 - 5P$, inverse demand is found by solving for P. This implies inverse demand is $P = 20 - \frac{1}{5}Q$.

b) Average revenue is given by

$$AR = \frac{TR}{Q} = \frac{PQ}{Q} = P$$

Therefore, average revenue will be $P = 20 - \frac{1}{5}Q$.

c) For a linear demand curve $P = a - bQ$, marginal revenue is given by $MR = a - 2bQ$. In this instance demand is $P = 20 - \frac{1}{5}Q$ implying marginal revenue is $MR = 20 - \frac{2}{5}Q$.

11.2 a) Since the demand curve is written in inverse form and is linear, the *MR* curve has the same vertical intercept and twice the slop as the demand curve. Thus, $MR = 40 - 4Q$.

b) Total revenue will be maximized when $MR = 0$, or when $Q = 10$. At that quantity, the price will be $P = 40 - 2Q = 20$. Total revenue is $PQ = 20(10) = 200$.

11.3 $MR = P + Q\frac{\Delta P}{\Delta Q} = P\left[1 + \frac{Q}{P}\frac{\Delta P}{\Delta Q}\right] = P\left[1 + \frac{1}{\varepsilon_{Q,P}}\right]$. Since $P > 0$, $MR = 0$ if and only if $1 + (1/\varepsilon_{Q,P}) = 0$, which is equivalent to $1/\varepsilon_{Q,P} = -1$ or $\varepsilon_{Q,P} = -1$.

11.4 If demand is $P = 9 - Q$, then $MR = 9 - 2Q$. If the firm sets $Q = 7$, then $MR = -5$. At this point, if the firm lowered its output it would increase total revenue, and with the lower level of output total cost would fall. Thus, decreasing output would increase profit. Therefore, a profit-maximizing monopolist facing this demand curve would never choose $Q = 7$.

11.5 Recall that the *MR* curve can easily be derived from the demand curve when the latter is written in the *inverse* form. The inverse demand curve is $P = 50 - (Q/20)$ so the marginal revenue curve is $P = 50 - (Q/10)$ (using the fact that the slope of the *MR* curve is twice that of the inverse demand curve, with the same intercept). Using the rule *MR*=*MC*, we get 50 – (Q/10) = 8, so Q = 420. Plugging this back into the demand curve (or the inverse demand curve) we can calculate the profit maximizing price, P = 29.

11.6 If marginal cost is independent of Q, then marginal cost is constant. Assume $MC = c$. Then in the winter the firm will produce where $MR = MC$.

$$a_1 - 2bQ = c$$
$$Q = \frac{a_1 - c}{2b}$$

At this quantity the price charged will be

$$P = a_1 - b\left(\frac{a_1 - c}{2b}\right)$$
$$P = \frac{a_1 + c}{2}$$

In the summer the firm will also produce where $MR = MC$.

$$a_2 - 2bQ = c$$
$$Q = \frac{a_2 - c}{2b}$$

At this quantity the price charged will be

$$P = a_2 - b\left(\frac{a_2 - c}{2b}\right)$$
$$P = \frac{a_2 + c}{2}$$

Since we are told that $a_2 > a_1$, the price charged during the summer months will be greater than the price charged during the winter months.

11.7 The monopolist chooses Q so that MR = MC: 120 – 4Q = 2Q => Q = 20.
P = 120 – 2(20) = 80.
Profit = PQ – V – F = 80(20) – 20^2 – 1400 = - 200.
The firm has nonsunk fixed costs: $F_{Nonsunk}$ = F - F_{Sunk} = 1400 – 600 = 800.
Producer surplus = PQ – V – $F_{Nonsunk}$ = 80(20) – 20^2 – 800 = 400. So the firm should continue to operate in the short run. If it operates, its profit is -200. But if it shuts down, its profit = -F_{Sunk} = -600. So it can lessen its losses by 400 if it continues to operate (and this is why producer surplus is +400 annually.)

11.8 A profit-maximizing monopolist would choose the output at which MR = MC. A revenue-maximizing monopolist would choose the output at which MR = 0. The two would therefore choose the same output (and set the same price) when MC = 0.

11.9 When the P = 30, the demand function shows that Q = 30.
At that price, profit = 0 = PQ – C = (30)(30) – F – 20(30); therefore F = 300.
So total cost is C = 300 – 20Q.
Now find the quantity that maximizes profit. Set MR = MC. MR = 60 – 2Q and MC = 20.
60 – 2Q = 20 implies that Q = 20 and P = 40.
So, the profit-maximizing profit will be PQ – C = (40)(20) – 300 – (20)(20) = 100.

11.10 a) If demand is given by $P = 300 - Q$ then $MR = 300 - 2Q$. To find the optimum set $MR = MC$.

$$300 - 2Q = Q$$
$$Q = 100$$

At $Q = 100$ price will be $P = 300 - 100 = 200$. At this price and quantity total revenue will be $TR = 200(100) = 20,000$ and total cost will be $TC = 1200 + .5(100)^2 = 6,200$. Therefore, the firm will earn a profit of $\pi = TR - TC = 13,800$.

b) The price elasticity at the profit-maximizing price is

$$\varepsilon_{Q,P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$$

With the demand curve $Q = 300 - P$, $\Delta Q / \Delta P = -1$. Therefore, at the profit-maximizing price

$$\varepsilon_{Q,P} = -1\left(\frac{200}{100}\right)$$
$$\varepsilon_{Q,P} = -2$$

The marginal cost at the profit-maximizing output is $MC = Q = 100$. The inverse elasticity pricing rule states that at the profit-maximizing price

$$\frac{P - MC}{P} = -\frac{1}{\varepsilon_{Q,P}}$$

In this case we have

$$\frac{200-100}{200} = -\frac{1}{-2}$$
$$\frac{1}{2} = \frac{1}{2}$$

Thus, the IEPR holds for this monopolist.

11.11 a) With demand $P = 210 - 4Q$, $MR = 210 - 8Q$. Setting $MR = MC$ implies

$$210 - 8Q = 10$$
$$Q = 25$$

With $Q = 25$, price will be $P = 210 - 4Q = 110$. At this price and quantity total revenue will be $TR = 110(25) = 2,750$.

b) If $MC = 20$, then setting $MR = MC$ implies

$$210 - 8Q = 20$$
$$Q = 23.75$$

At $Q = 23.75$, price will be $P = 115$. At this price and quantity total revenue will be $TR = 115(23.75) = 2,731.25$. Therefore, the increase in marginal cost will result in lower total revenue for the firm.

c) Competitive firms produce until $P = MC$, so in this case we know the market price would be $P = 10$ and the market quantity would be:

$$210 - 4Q = 10$$
$$Q = 50$$

d) In this case, the market price will be $P = MC = 20$, implying that the industry quantity is given by

$$210 - 4Q = 20$$
$$Q = 47.50$$

At this quantity, price will be $P = 20$. When $MC = 10$, total industry revenue is $10(50) = 500$. With $MC = 20$, total industry revenue is $20(47.50) = 950$. Thus, total industry revenue increases in the perfectly competitive market after the increase in marginal cost.

11.12 The marginal revenue curve is $MR = 120 - 4Q$. Initially we are not sure whether the optimal quantity will be less than 15 units (in which case $MC = 10$), or more than 20 units (where $MC = 20$).

There are two regions of output:

Region I: where $MC = 10$ and $0 \leq Q \leq 15$

Region II: where $MC = 20$ and $15 < Q$

Let's assume that the $MC = 10$ and optimal quantity is less than or equal to 15 units. In that case, setting $MR = MC$, we find that $120 - 4Q = 10$, or that $Q = 27.5$. But when $Q = 27.5$, MC is not 10, so the assumption that the optimal quantity is in Region I is not correct.

Now let's assume that the $MC = 20$ and optimal quantity is greater than 15 units. In that case, setting $MR = MC$, we find that $120 - 4Q = 20$, or that $Q = 25$. When $Q = 25$, MC is 20, so that marginal cost we have assumed is correct at the optimal output level we have calculated. The market price is $P = 120 - 2(25) = 70$.

Revenue = $PQ = 70(25) = 1750$

Variable cost = 10(15) + 20 (25 – 15) = 350

Fixed Cost = 300

Profit = 1750 – 350 – 300 = 1100.

11.13 If demand is initially $P = 100 - Q + I$, then initially $MR = 100 + I - 2Q$. Setting $MR = MC$ implies

$$100 + I - 2Q_1 = MC_1$$
$$Q_1 = \frac{100 + I - MC_1}{2}$$

where Q_1 is the profit-maximizing quantity when income equals I and MC_1 is the corresponding level of marginal cost.

With this quantity, price will be

$$P_1 = 100 - \left(\frac{100 + I - MC_1}{2}\right) + I$$
$$P_1 = \frac{100 + I + MC_1}{2}$$

Now suppose income increases by a factor K where $K > 1$. Then setting $MR = MC$ implies

$$100 + KI - 2Q_2 = MC_2$$
$$Q_2 = \frac{100 + KI - MC_2}{2}$$

where Q_2 is the profit-maximizing quantity when income equals KI and MC_2 is the corresponding level of marginal cost. This quantity must be greater than the quantity when income equals I, *i.e.*, $Q_2 > Q_1$. If it were not, *i.e.*, if $Q_2 \le Q_1$, then the marginal cost MC_2 would be less than or equal to MC_1 (since we know marginal cost is not decreasing). But that would mean that

$$Q_2 = \frac{100 + KI - MC_2}{2} > \frac{100 + I - MC_2}{2} = Q_1$$

contradicting the assumption that $Q_2 \le Q_1$

At quantity Q_2, price will be

$$P_2 = 100 + KI - \left(\frac{100 + KI - MC_2}{2}\right)$$
$$P_2 = \frac{100 + KI + MC_2}{2}$$

In this case the new price charged by the monopolist will be greater than the initial price. Clearly $KI > I$ since $K > 1$, and because the marginal cost function is assumed to not be downward sloping, the increase in Q at the higher income level will result in a marginal cost at least as high as the initial marginal cost, *i.e.*, $MC_2 \ge MC_1$. Therefore, the price will increase when consumer income increases.

11.14 a) If the two demand curves are linear and parallel they differ only by a constant; call this constant c. Then

$$P_1 = a - bQ_1$$
$$P_2 = a + c - bQ_2$$

In this instance demand for the second firm will be further from the origin assuming $c > 0$. Now assume that both firms have identical constant marginal cost e. Then the first firm will maximize profit where $MR = MC$.

$$a - 2bQ_1 = e$$
$$Q_1 = \frac{a - e}{2b}$$

At this quantity price will be

$$P_1 = a - b\left(\frac{a-e}{2b}\right)$$
$$P_1 = \frac{a+e}{2}$$

The second firm will also maximize profit where $MR = MC$.

$$a + c - 2bQ_2 = e$$
$$Q_2 = \frac{a+c-e}{2b}$$

At this quantity price will be

$$P_2 = a + c - b\left(\frac{a+c-e}{2b}\right)$$
$$P_2 = \frac{a+c+e}{2}$$

For the first monopolist

$$\frac{P_1}{MC} = \frac{(a+e)/2}{e}$$

and for the second monopolist

$$\frac{P_2}{MC} = \frac{(a+c+e)/2}{e}$$

Here $P_2/MC > P_1/MC$ implying the firm with the demand curve further from the P axis will have the higher mark-up ratio.

b) Suppose the first monopolist faces demand $P_1 = a - bQ_1$ and the second monopolist faces demand $P_2 = a - kbQ_2$ where $k > 1$. In this case the demand curve for the second monopolist is steeper. As in part a), the first monopolist will maximize profit at

$$Q_1 = \frac{a-e}{2b}$$
$$P_1 = \frac{a+e}{2}$$

For the second monopolist profit will be maximized where $MR = MC$.

$$a - 2kbQ_2 = e$$
$$Q_2 = \frac{a-e}{2kb}$$

At this quantity price will be

$$P_2 = a - kb\left(\frac{a-e}{2kb}\right)$$
$$P_2 = \frac{a+e}{2}$$

Since both monopolists will charge the same price and since marginal cost is constant, both monopolists will have the same mark-up ratio.

c) Suppose the first monopolist faces demand $P_1 = a - bQ_1$ and the second monopolist faces demand $P_2 = k(a - bQ_2)$ where $k > 1$. In this case both firms face linear demand curves with the same horizontal intercept (at $Q = a/b$) but the demand for monopolist 2 is steeper.

The first firm maximizes profit as in parts A and B at

$$Q_1 = \frac{a-e}{2b}$$
$$P_1 = \frac{a+e}{2}$$

For the second monopolist, profit will be maximized where $MR = MC$.

$$ak - 2kbQ_2 = e$$
$$Q_2 = \frac{ak-e}{2b}$$

At this quantity price will be

$$P_2 = ak - kb\left(\frac{ak-e}{2kb}\right)$$
$$P_2 = \frac{ak+e}{2}$$

Since $k > 1$, $P_2 > P_1$. Since marginal cost is constant, the monopolist with the steeper demand function will have the higher mark-up ratio.

11.15 a) The monopolist will operate where $MR = MC$. With demand $P = a - bQ$, marginal revenue is given by $MR = a - 2bQ$. Setting this equal to marginal cost implies

$$a - 2bQ = c + eQ$$

$$Q = \frac{a-c}{2b+e}$$

At this quantity price is

$$P = a - b\left(\frac{a-c}{2b+e}\right)$$

$$P = \frac{ab + ae + bc}{2b+e}$$

b) Since

$$Q = \frac{a-c}{2b+e}$$

increasing c or decreasing a will reduce the numerator of the expression, reducing Q.

c) Since $e \geq 0$ and

$$P = \frac{ab + ae + bc}{2b+e}$$

increasing a will increase the numerator for this expression. This will therefore increase the equilibrium price.

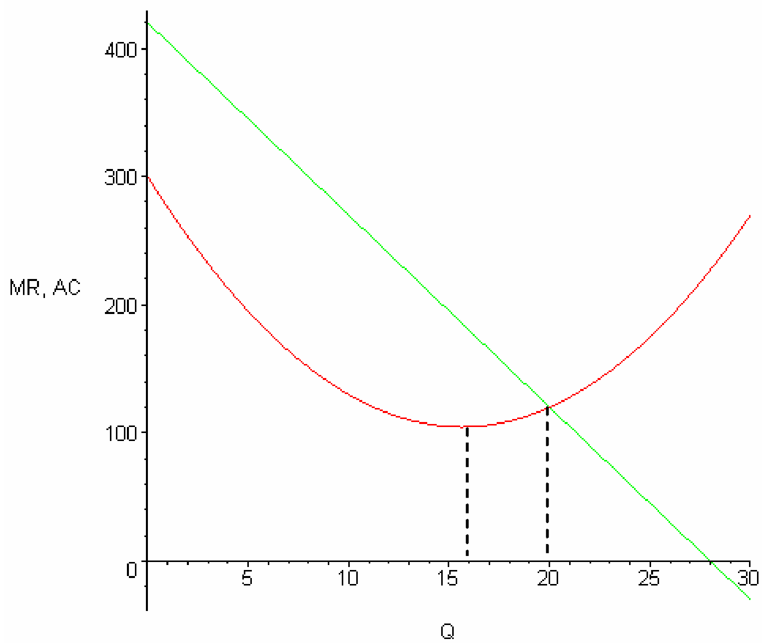
11.16 With demand $Q = 1000P^{-3}$, elasticity along the demand curve is constant at -3. Employing the inverse elasticity pricing rule implies

$$\frac{P - MC}{P} = -\frac{1}{-3} = \frac{1}{3}$$

Therefore, the optimal percentage mark-up of price over marginal cost is $1/3$, or 33 percent.

11.17 Remember that the demand elasticity in a constant elasticity demand function is the exponent on P when the demand function is written in the regular form, i.e. $Q = f(P)$. We can manipulate the inverse demand function to get the regular demand function, $Q = 10{,}000P^{-2}$. This implies that the demand elasticity is –2. Therefore, using the IEPR, $\frac{P - MC}{P} = \frac{1}{2}$. So the optimal percentage mark-up of price over marginal cost is ½, or 50 percent.

11.18 The graph is reproduced below. The *MES* appears to be at about 16 units of output, and the point where the *MR* curve intersects the *AC* curve is at about 20 units.



The monopolist's profit maximizing output must fall between 16 and 20 units. To see this, remember that the firm will produce where $MR = MC$. This cannot happen at any point less than 16 units because the *AC* curve is decreasing for $Q < 16$. Therefore the *MC* curve lies below the *AC* curve and clearly $MR > MC$ for $Q > 16$. Similarly, since the *MC* curve must lie above the *AC* curve to the right of 16 units, it must intersect the *MR* curve before the *MR* curve intersects the *AC* curve. That is, the profit maximizing quantity must be less than 20 units.

11.19 a) Profit-maximizing firms generally allocate output among plants so as to keep marginal costs equal. But notice that $MC_2 < MC_1$ whenever $1 + 0.5Q_2 < 8$, or $Q_2 < 14$. So for small levels of output, specifically $Q < 14$, Gillette will only use the first plant. For $Q > 14$, the cost-minimizing approach will set $Q_2 = 14$ and $Q_1 = Q - 14$.

Suppose the monopolist's profit-maximizing quantity is $Q > 14$. Then the relevant $MC = 8$, and with $MR = 968 - 40Q$ we have

$$968 - 40Q = 8$$
$$Q = 24$$

Since we have found that $Q > 14$, we know this approach is valid. (You should verify that had we supposed the optimal output was $Q < 14$ and set $MR = MC_2 = 1 + 0.5Q$, we would have found $Q > 14$. So this approach would be invalid.) The allocation between plants will be $Q_2 = 14$ and $Q_1 = 10$. With a total quantity $Q = 24$, the firm will charge a price of $P = 968 - 20(24) = 488$. Therefore the price will be $4.88 per blade.

b) If $MC = 10$ at plant 1, by the logic in part (a) Gillette will only use plant 2 if $Q < 18$. It will produce all output above $Q = 18$ in plant 1 at $MC = 10$. Assuming $Q > 18$, setting $MR = MC$ implies

$$968 - 40Q = 10$$
$$Q = 23.95$$

(So again, this approach is valid. You can verify that setting $MR = MC_2$ would again lead to $Q > 18$.) The firm will allocate production so that $Q_2 = 18$ and $Q_1 = 5.95$. At $Q = 23.95$, price will be $4.89.

11.20 a) Equating the marginal costs at MC_T, we have $Q = Q_1 + Q_2 + Q_3 = 0.25MC_T + 0.5MC_T - 1 + MC_T - 6$, which can be rearranged as $MC_T = (4/7)Q + 4$. Setting $MR = MC$ yields

$$64 - (2/7)*Q = (4/7)*Q + 4$$

or $Q = 70$ and $P = 54$. At this output level, $MC_T = 44$, implying that $Q_1 = 11$, $Q_2 = 21$, and $Q_3 = 38$.

b) In this case, using plant 3 is inefficient because its marginal cost is *always* higher than that of plant 2. Hence, the firm will use only plants 1 and 2. Moreover, the firm will not use plant 1 once its marginal cost rises to $MC_2 = 4$, so we can immediately see that it will only produce $4Q_1 = 4$ or $Q_1 = 1$ unit at plant 1. Its total production can be found by setting $MR = MC_2$, yielding

$$64 - (2/7)*Q = 4$$

or $Q = 210$ and $P = 34$. So it produces $Q_1 = 1$ unit in plant 1 and $Q_2 = 209$ units in plant 2, while producing no units in plant 3 (i.e. $Q_3 = 0$).

11.21 The firm will be maximizing its profit when the marginal costs are equal for the two plants. (Otherwise, the firm could take the last unit produced at the high-cost plant and instead produce that same unit at the low cost plant, not changing revenues and reducing costs). When Q_2 =4, MC_2 = 30. So plant 1 must be operating with MC_1 = 30. This means that Q_1 = 5.

11.22 Because the firm needs to charge the same price in both markets, it needs to set its marginal cost equal to the marginal revenue associated with the aggregate demand curve.

To get the aggregated demand curve, it must sum the demands "horizontally," i.e., add the quantities when $P_1 = P_2$ (= P).

$Q_1 = 100 – 0.5P$ and $Q_2 = 140 – P$. The aggregate quantity demanded is $Q = Q_1 + Q_2$.
Then the aggregate demand is Q = 240 – 1.5P.
Now find the inverse aggregate demand curve: P = 160 - (2/3)Q.
The marginal revenue associated with the aggregate demand curve has the same vertical intercept and twice the slope as the demand curve: MR = 160 - (4/3)Q.
The marginal cost is MC = 20 + Q.
Set MR = MC. 160 - (4/3)Q = 20 + Q. Thus the profit-maximizing total quantity to produce is Q = 60.
The optimal price is P = 160 - (2/3)(60) = 120.

11.23 a) Set MR = MC in Europe. The inverse demand is P = 120 – Q, so MR = 120 – 2Q. MR = MC implies that 120 – 2Q = 20, or Q = 50. P = 120 – 50 = 70.

b) Because the firm needs to charge the same price in both markets, it needs to set its marginal cost equal to the marginal revenue associated with the aggregate demand curve. To get the aggregated demand curve, it must sum the demands "horizontally," i.e., add the quantities.
$Q_1 = 120 – P$ and $Q_2 = 240 – 2P$. The aggregate quantity demanded is $Q = Q_1 + Q_2$.
Then the aggregate demand is Q = 360 – 3P.
Now find the inverse aggregate demand curve: P = 120 - (1/3)Q.
The marginal revenue associated with the aggregate demand curve has the same vertical intercept and twice the slope as the demand curve: MR = 120 - (2/3)Q.
The marginal cost is MC = 20. Set MR = MC. 120 - (2/3)Q = 20. Thus the profit-maximizing total quantity to produce is Q = 150.
The optimal price is P = 120 - (1/3)(150) = 70.

c) The demand in Europe is linear and has a choke price of 120. The aggregate demand in part (b) is also linear, with a choke price of 120. The marginal cost is constant at 20. The Monopoly Midpoint Rule states that with a linear demand and a constant marginal cost, the profit maximizing price will be (choke price + marginal cost)/2, or (120 + 20)/2 = 70. This is the same in parts (a) and (b).

11.24 a) With demand $P=100-2Q$, $MR=100-4Q$. Setting $MR=MC$ implies

$$100-4Q=.5Q$$
$$Q=22.2$$

(All figures are rounded.) At this quantity, price will be $P=55.6$.

b) A perfectly competitive market produces until $P = MC$, or

$$100-2Q=.5Q$$
$$Q=40$$

At this quantity, price will be $P = 20$.

c) Under monopoly, consumer surplus is 0.5(100 – 55.6)(22.2) = 493. Since *MC*(22.2) = 11.1, producer surplus is 0.5(11.1)(22.2) + (55.6 – 11.1)(22.2) = 1111. Net benefits are 1604. (All figures are rounded.)

Under perfect competition, consumer surplus is 0.5(100 – 20)(40) = 1600, and producer surplus is 0.5(20)(40) = 400. Net benefits are 2000. Therefore, the deadweight loss due to monopoly is 396.

d) Now setting $MR = MC$ gives

$$180-8Q=0.5Q$$
$$Q=21.2$$

At this quantity, price is 95.2. Consumer surplus is 0.5(100 – 95.2)(21.1) = 51 and producer surplus is 0.5(10.6)(21.2) + (95.2 – 10.6)(21.2) = 1906. Net benefits are 1957.

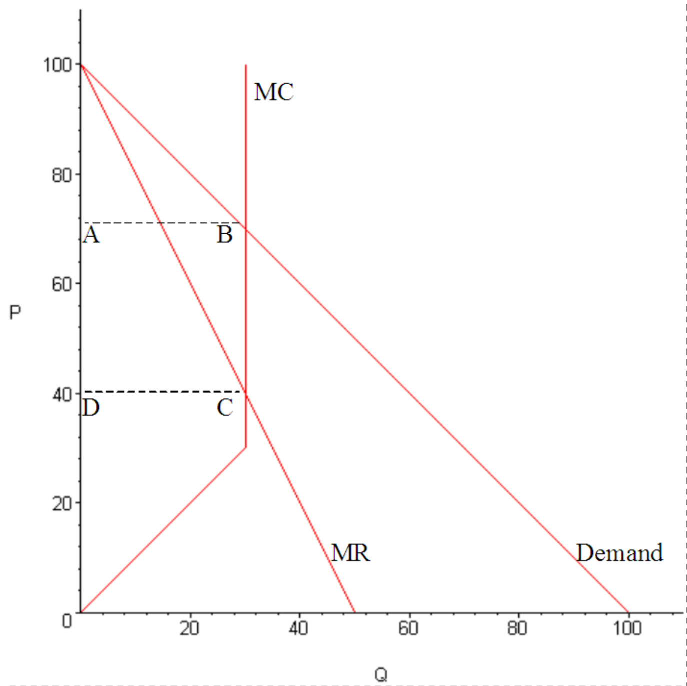
Setting $P = MC$ as in perfect competition yields

$$180-4Q=.5Q$$
$$Q=40$$

At this quantity, price is 20. Consumer surplus is 0.5(180 – 20)(40) = 3200 and producer surplus is 0.5(20)(40) = 400. Net benefits with perfect competition are 3600. Therefore, the deadweight loss in this case is 1643.

While the competitive solution is identical with both demand curves, the deadweight loss in the first case is far greater. This difference occurs because with the second demand curve demand is less elastic at the perfectly competitive price. If consumers are less willing to change quantity as price increases toward the monopoly level, the firm will be able to extract more surplus from the market.

11.25 a) See the figure below. The monopolist will produce the quantity that corresponds to $MR = MC$. However, because the MC curve is vertical at $Q = 30$, this is also the quantity corresponding to the point where the MC curve intersects the demand curve. The monopolist produces 30 units and sells at a price of 70.



b) The deadweight loss is zero. To see this, notice that the price and quantity are the same in the case of monopoly and the case of a competitive market, if $P = MC$. Therefore, there is no deadweight loss from monopoly.

11.26 a) For this monopsonist

$$ME_L = w + L\frac{\Delta w}{\Delta L}$$
$$ME_L = 4L + L(4)$$
$$ME_L = 8L$$

b) The monopsonist will maximize profit at the point where $MRP_L = ME_L$, where

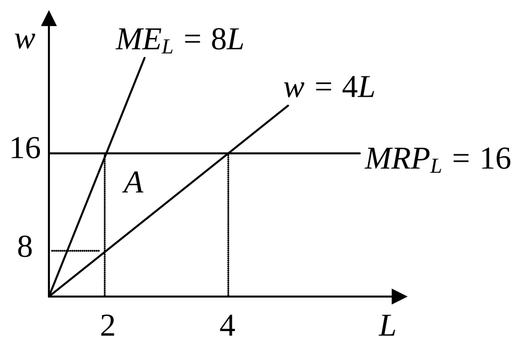
$$MRP_L = P\frac{\Delta Q}{\Delta L}$$

In this example, $\Delta Q / \Delta L = 0.5$, so $MRP_L = 0.5P$. Since $P = 32$, $MRP_L = 16$. Now setting $MRP_L = ME_L$ implies

$$16 = 8L$$
$$L = 2$$

At this quantity of labor, $w = 4L = 8$.

c) In a competitive labor market, $w = MRP_L$. So the competitive supply of labor satisfies $4L = 16$ or $L = 4$, with $w = 4L = 16$. The deadweight loss due to monopsony is equal to area A in the graph below, or 0.5(16 – 8)(4 – 2) = 8.



11.27 We can use the IEPR condition for monopsony: $\frac{MRP_L - w}{w} = \frac{1}{\in_{L,w}}$. Since labor supply is unit elastic, it means that $MRP_L - w = w$ or that $MRP_L = 2w$. So the marginal revenue product of labor is twice as much as the wage rate.