UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/1331 FALL 2012
Dr. A. Czajkowski

CLASS: PHY_____

STUDENT #: ________________

NAME: ______________________

Assignment 2:
KINEMATICS 2-D Motion

Assigned: Sept 20 14:30 Due: September 28 10:00

1 An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. If *x* is horizontal and *y* vertical, find the (*x*, *y*) coordinates where the shell explodes.

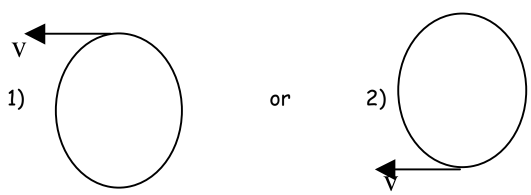
$$x_f = x_i + v_x t = x_i + v_0(\cos\theta)t = 0 + 300(\cos 55)42 = 7227m$$

$$y_f = y_i + v_y t + \tfrac{1}{2} a_y t^2 = 0 + v_0(\sin\theta)t - \tfrac{1}{2} g t^2 = 0 + 300(\sin 55)42 - 4.9(42)^2 = 1677m$$

ANS: 7.23 km, 1.68 km

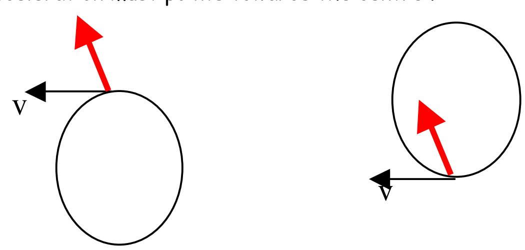
2. A car travels on a flat circular road of radius R. At a certain instant the velocity of the car is 24 m/s west, and the total acceleration of the car is 2.5 m/s² 53° north of west. Find the radial and tangential components of the acceleration of the car. How long will it take for the car to make a one full circle from the point at which its velocity is 24m/s west?

It seems that the information about the position is missing. The only possible realizations of the described situation from the point of view of the velocity direction are these two:



since the velocity is always tangent to the trajectory (circle)!!!

If the direction of the acceleration is considered it becomes clear that only the second position is physically possible. (radial component of the acceleration must points towards the centre !



Impossible due to the direction of acceleration vector!

this is our case!

The rest of the solution follows the same lines as the problem solved in class.

$$a_r = 2.5\frac{m}{s^2}\sin 53 = 2.00 \qquad a_t = 2.5\frac{m}{s^2}\cos 53 = 1.50$$

There is also no information about radius given directly, but we can obtain it from v and radial component of acceleration:

$$a_r = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_r} = 288(m)$$

Using a_t, v and r in kinematic equation

$$2\pi r = vt + \tfrac{1}{2} a_t t^2 \Rightarrow 1810 = 24t + 0.75t^2$$

$$t = 35.7s$$

The projectile motion is fired with velocity of magnitude v_0 at the angle θ. Find for which the maximum elevation of the projectile is

3. **Find the angle of the projection for which the maximum height is equal half of the range.**

We will use the equation for projectile motion trajectory y(x) for x=R/2and R/2=y(R/2)

$$y = (\tan\theta)x - \frac{g}{2(v_0\cos\theta)^2}x^2 \Rightarrow \frac{R}{2} = (\tan\theta)(\frac{R}{2}) - \frac{g}{2(v_0\cos\theta)^2}(\frac{R}{2})^2 \Rightarrow \frac{1}{2} = (\tan\theta)(\frac{1}{2}) - \frac{g}{2(v_0\cos\theta)^2}\frac{R}{4}$$

$$\frac{1}{2} = (\tan\theta)(\frac{1}{2}) - \frac{g}{2(v_0\cos\theta)^2}\frac{v_0^2\sin 2\theta}{4g} \Rightarrow \frac{1}{2} = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{8}\frac{\sin 2\theta}{\cos^2\theta} \Rightarrow \frac{1}{2} = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{8}\frac{2\sin\theta\cos\theta}{\cos^2\theta}$$

$$\frac{1}{2} = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{4}\frac{\sin\theta}{\cos\theta} \Rightarrow 2 = \tan\theta \Rightarrow \theta = 63.43^o$$

$$\theta = 64.4^o$$

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Assignment 2: KINEMATICS 3-D Motion CONT

4 A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? Compare this with the time the trip would take if the water were still.

Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}$.

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1\,000}{(1.20 - 0.500)} = 1.43 \times 10^3 \text{ s}$$

$$t_{\text{down}} = \frac{1\,000}{1.20 + 0.500} = 588 \text{ s}.$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3 \text{ s}}$.

5 A plane has a velocity of 280 m/s at an angle θ below the horizontal. When the altitude of the aircraft is 2.15km, it releases a water bomb, which subsequently hits a target on the ground. The magnitude of the displacement from the point of release of the bomb to the target is 3.25 km. Find the angle θ .

When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25 \text{ km})^2 - (2.15 \text{ km})^2} = 2.437 \text{ km}$$

$$y_f = x_f \tan\theta - \frac{g x_f^2}{2 v_i^2 \cos^2\theta_i}$$

$$-2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - \frac{(9.8 \text{ m/s}^2)(2\,437 \text{ m})^2}{2(280 \text{ m/s})^2 \cos^2\theta_i}$$

$$\therefore -2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - (371.19 \text{ m})(1 + \tan^2\theta_i)$$

$$\therefore \tan^2\theta - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945.$$

Select the negative solution, since θ_i is below the horizontal. $\therefore \tan\theta_i = -0.662$, $\boxed{\theta_i = -33.5°}$

6 In the experimental setup similar to the one shown during the class demonstration, the small ball is launched from the corner of inclined plane. When the inclination of the plane □ is 70deg, the maximum range of the projectile is 50cm. Later the inclination of the plane changes to new unknown angle and the same projectile launcher is oriented at 30 deg to the horizontal line. The ball lands 1m away from the launcher. Find the new angle of inclination.

SOLUTION:

Let R be the range and g' and g'' the effective g value acting along the plane of motion of the projectile when the plane inclination with respect to horizontal is 70 deg and □ respectively.

$$R = \frac{v_0^{\,2} \sin 2\theta}{g'}$$

We may use this to find the initial speed of the projectile given by the launcher (same in both cases).

$$R_{\max} = \frac{v_0^{\,2} \sin 90}{g'} = \frac{v_0^{\,2}}{g'} = \frac{v_0^{\,2}}{g\cos 20} \Rightarrow v_0^{\,2} = gR_{\max}\cos 20 = (9.8)(0.5)\cos 20(\frac{m^2}{s^2}) = 4.60\frac{m^2}{s^2}$$

Having v_0 we may use the equation for range in the second case (inclination angle □ and projectile orientation 30 deg)

$$90 - \alpha = 66.0$$

$$\alpha = 24.0$$