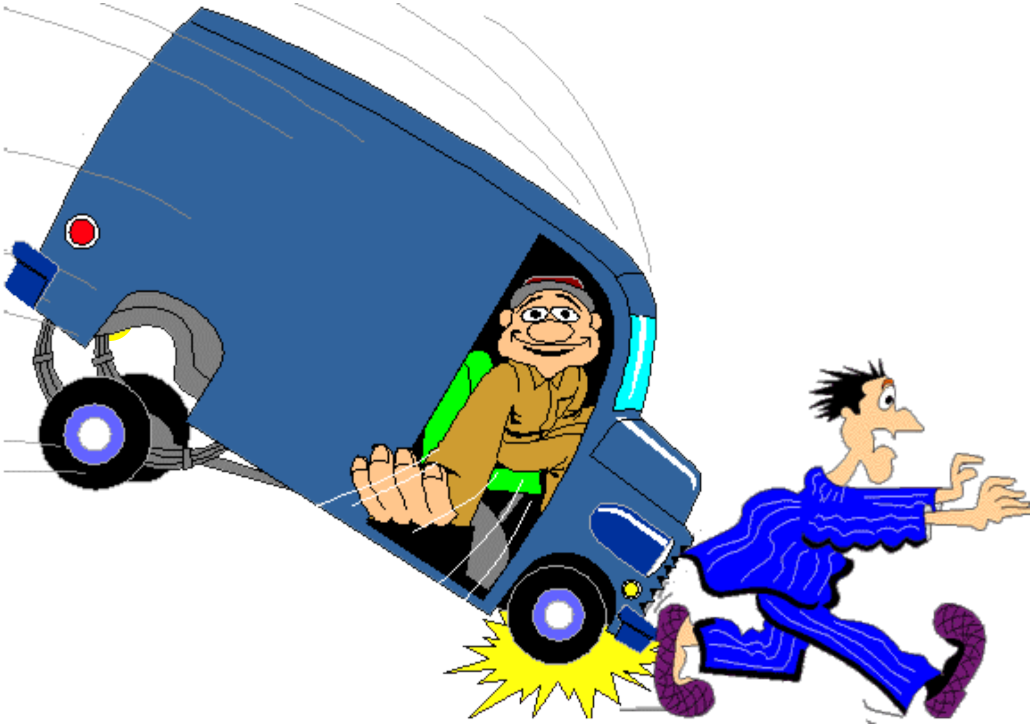
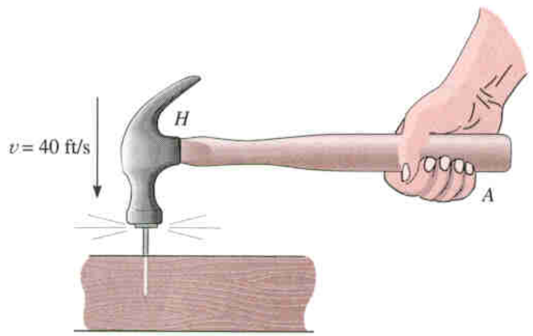
LINEAR MOMENTUM and IMPULSE

Collisions



APPLICATIONS



When a nail is struck by a hammer on a block of wood, a large impulsive force is delivered to the nail which drives it into the wood.

If we know the initial speed of the hammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the nail?

The answer is obtained by determining the impulse of the force from the variation of linear momentum of the hammer.

Linear Momentum of a particle definition

- Momentum can be defined as "*mass in motion*".
- The ***linear momentum*** of a particle of mass $\boldsymbol{m}$ moving with a velocity $\mathbf{v}$ is defined to be the *vector* $\mathbf{p}$

$$\mathrm{p} = m\mathrm{v}$$

or, in component form**:**

$$p_x = m\, v_x$$
$$p_y = m\, v_y$$
$$p_z = m\, v_z$$

For a system of particles

$$\mathrm{P} = \sum m_i \mathrm{v}_{\mathrm{i}}$$

Linear Momentum and Force

$$\mathrm{p} = m\mathrm{v}$$

$$\frac{d\mathrm{p}}{dt} = m\frac{d\mathrm{v}}{dt} = ma = \mathrm{F}$$

$$\sum \mathrm{F} = \frac{d\mathrm{p}}{dt} = \frac{d(m\mathrm{v})}{dt}$$

The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.
<u>This is an equivalent way of writing Newton's second law</u>

We can also write three scalar components of this equation as

$$\sum F_x = \dot{p}_x \qquad \sum F_y = \dot{p}_y \qquad \sum F_z = \dot{p}_z$$

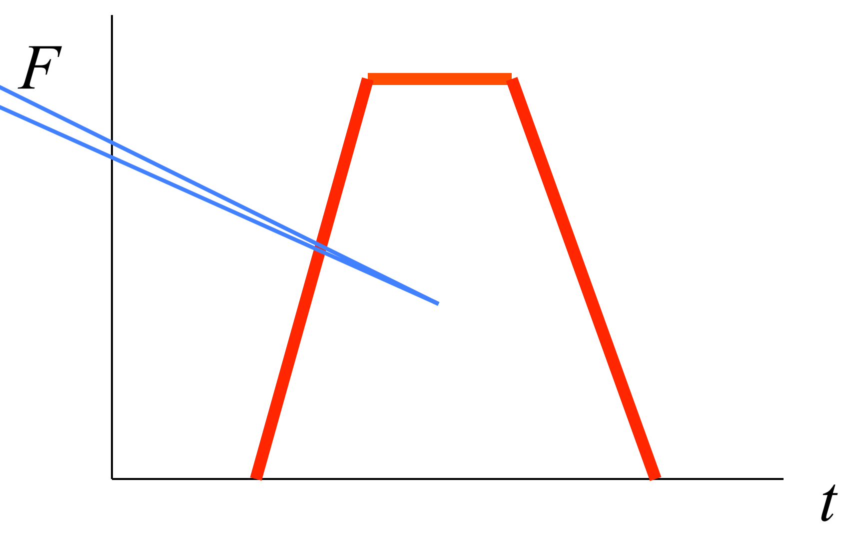
Impulse - definition

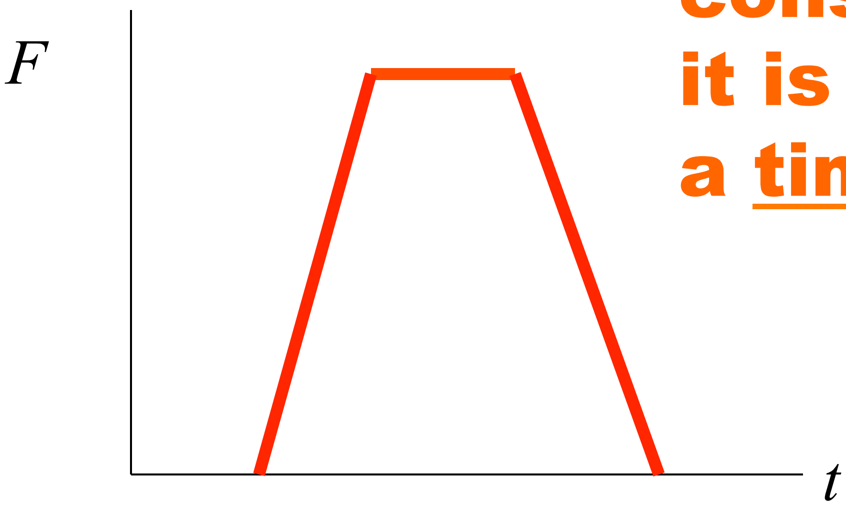
$$I = \int_{t_1}^{t_2} F\,dt = \int_{p_1}^{p_2} dp = p_2 - p_1 = \Delta p$$

Where in the above formula we substituted F by

$$F = \frac{dp}{dt}$$

Impulse is a vector **whose magnitude equals the area under the force-time curve** and **whose direction is the direction of the force**.





When the force imparting an impulse is not constant (i.e. varies in time**), it is convenient to define a time-average force**

$$F_{av} = \frac{1}{\Delta t}\int_{t_1}^{t_2} F\,dt$$

Then the impulse is defined as

$$I = F_{av}\Delta t$$

Of course if F is constant

$$F_{av} = F$$

$$I = F\Delta t$$

$$I = \int_{t_1}^{t_2} F\,dt = \int_{p_1}^{p_2} dp = p_2 - p_1 = \Delta p$$

This is the mathematical form of the Impulse - Momentum Theorem
(another equivalent way of expressing Newton's Second Law)

The Impulse-Momentum theorem states that

The impulse of the force F acting on a particle equals the change in momentum of the particle

Impulse (*Summary*):

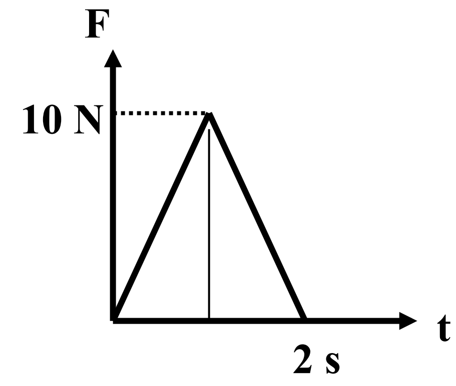
$$I = \int_{t_1}^{t_2} F\,dt = \int_{p_1}^{p_2} dp = p_2 - p_1 = \Delta p$$

- is a vector quantity;
- its magnitude can be determine by calculating the area under the force - time curve for a given interval of time;
- has dimensions and units of momentum (kg m/s);
- is **not** a property of the particle itself;
- is a measure of the degree to which an external force changes the momentum of a particle;

When we say that an impulse is given to a particle, it is implied that momentum is transferred from an external agent to that particle.

Observe the graph and determine the impulse due to the force.

A. 20 kg.m/s
B. 10 kg.m/s
C. 5 kg.m/s
D. 15 kg.m/s



A constant force is applied for 2 seconds to change the particle's velocity from v_1 to v_2 as given in the diagram. Determine the force applied if the mass of the particle is 2 kg.

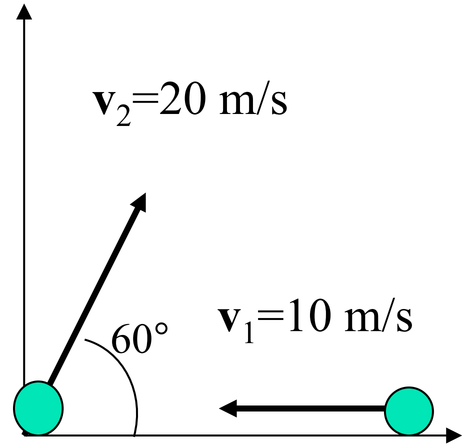
A. (7.3 **i** +17.3 **j**) N

B. (–10 **i**) N

C. (20 **i** +17.3 **j**) N

D. (–20 **i** +17.3 **j**) N

E. (17.3 **j**) N



The internal impulses acting on a system of particles always

A. Equal the external impulses
B. add to zero
C. Equal the impulse of the weight
D. none of the above

Weight is

A. An impulsive force
B. Internal force
C. Explosive force
D. Non-impulsive force

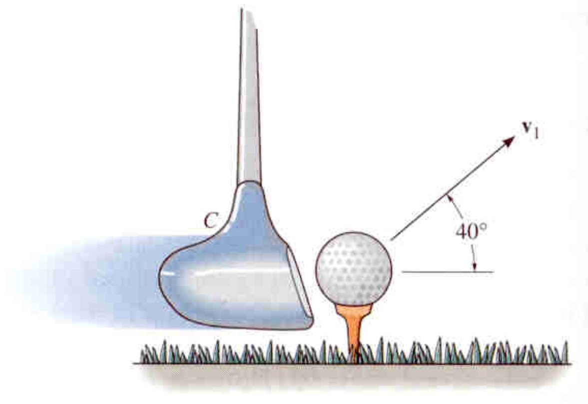
Impulse Approximation:

In many physical situations it happens that one of the forces acting on a particle acts for a short time but is much greater than any of the other forces. ***Thus, as an approximation, one can neglect the action of the other less intense forces.***

When we make this approximation we refer to the force as an impulsive force. ***This approximation is especially useful in treating collisions in which the duration of the collision is very short.***

However, when we use this approximation it is important to remember that $\mathbf{p}_i$ and $\mathbf{p}_f$ represent the momenta immediately before and after the collision, respectively.

EXAMPLE



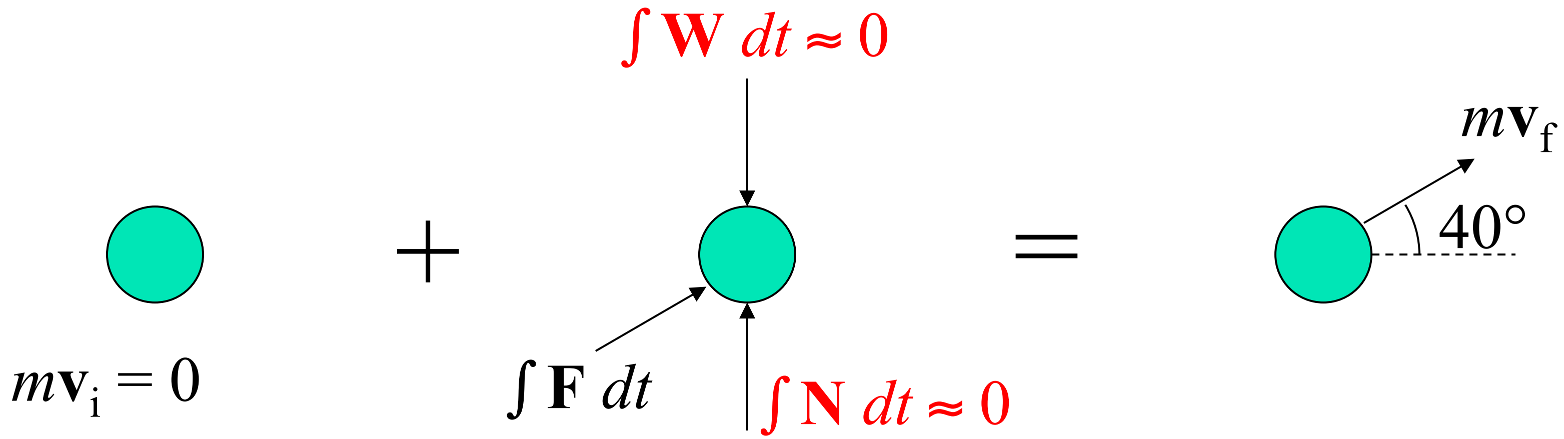
Given: A 40 g golf ball is hit over a time interval of 3 ms by a driver. The ball leaves with a velocity of 35 m/s, at an angle of 40°. Neglect the ball's weight while it is struck.

Find: The average impulsive force exerted on the ball and the momentum of the ball 1 s after it leaves the club face.

Plan:
1) Draw the momentum and impulsive diagrams of the ball as it is struck.
2) Apply the principle of impulse and momentum to determine the average impulsive force.
3) Use kinematic relations to determine the velocity of the ball after 1 s. Then calculate the linear momentum.

Solution:

1) The impulse and momentum diagrams can be drawn for the ball:



EXAMPLE (continued)

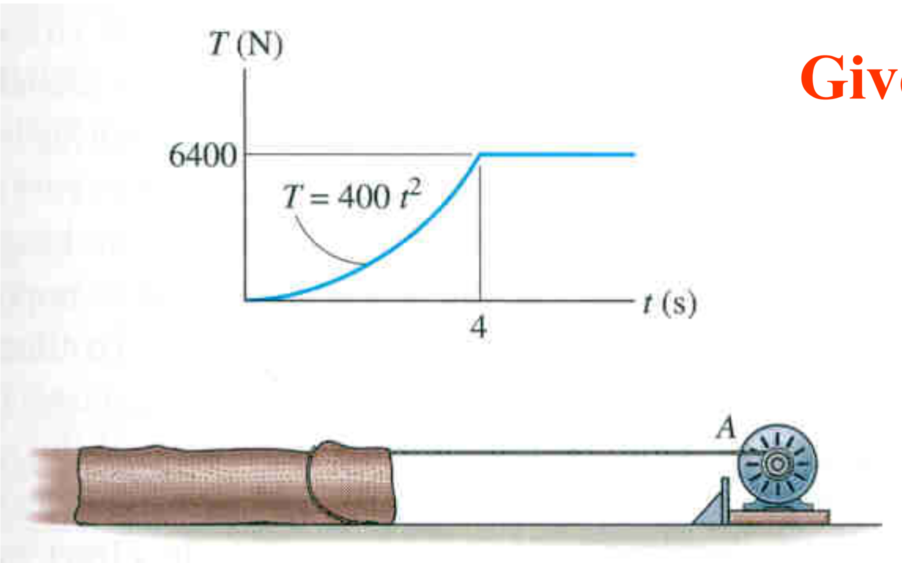
2) The principle of impulse and momentum can be applied along the direction of motion:

EXAMPLE (continued)

3) After impact, the ball acts as a projectile undergoing free-flight motion. Using the constant acceleration equations for projectile motion:

The linear momentum is calculated as $\mathbf{p} = m\,\mathbf{v}$.

GROUP PROBLEM SOLVING



Given: The 500 kg log rests on the ground (coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$). The motor delivers a towing force **T** to its cable at A as shown. Note from the graph that after 4 seconds the force T becomes constant

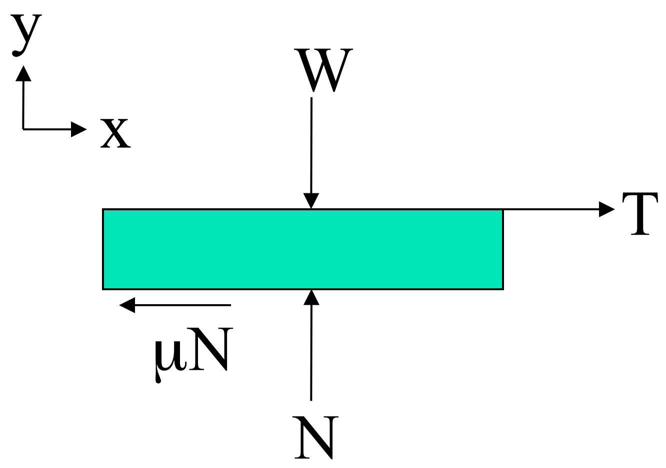
Find: The speed of the log when $t = 5$ s.

Plan:
1) Draw the FBD of the log.
2) Determine the force needed to begin moving the log, and the time to generate this force.
3) After the log starts moving, apply the principle of impulse and momentum to determine the speed of the log at $t = 5$ s.

GROUP PROBLEM SOLVING (continued)

Solution:

1) Draw the FBD of the log:

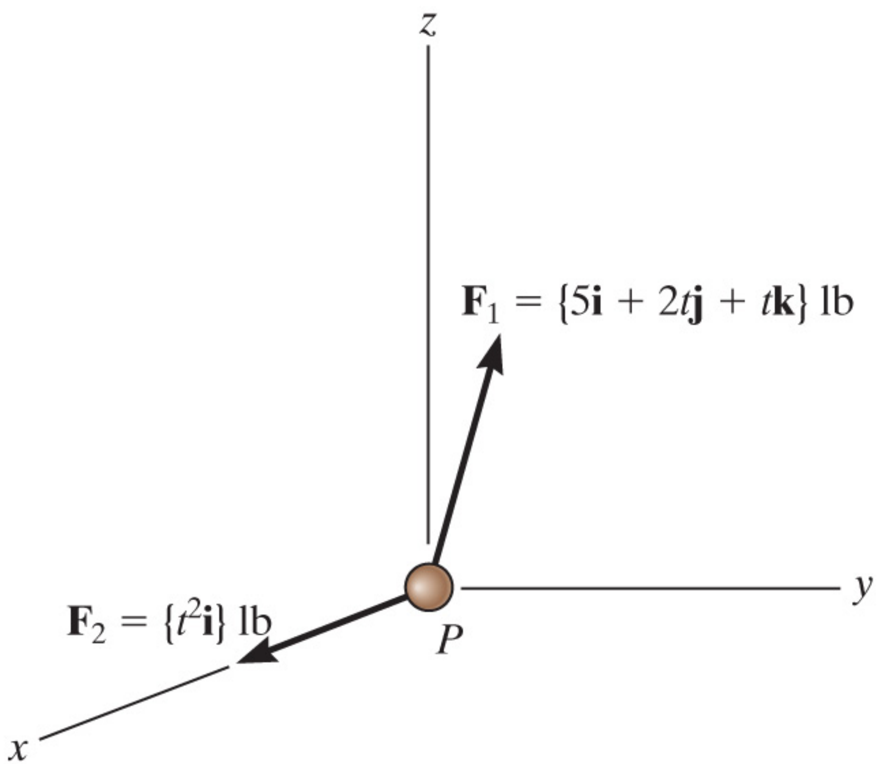


2) The log begins moving when the towing force T exceeds the friction force $\mu_s N$. Solve for the force, then the time.

GROUP PROBLEM SOLVING (continued)

3) Apply the principle of impulse and momentum in the x-direction from the time the log starts moving at $t_1 = 2.476$ s to $t_2 = 5$ s.

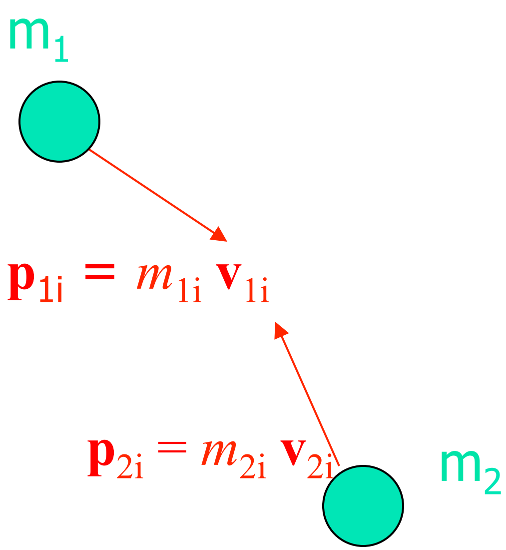
Particle at P is acted upon by its weight of 3 lb and forces $\mathbf{F_1}$ and $\mathbf{F_2}$ where t is in seconds. If the particle originally has a velocity $\mathbf{v_1}$=(3**i**+1**j**+6**k**) ft/s, determine its speed after 2 s.



Linear Momentum of a Two-Particle System

Consider two particles that

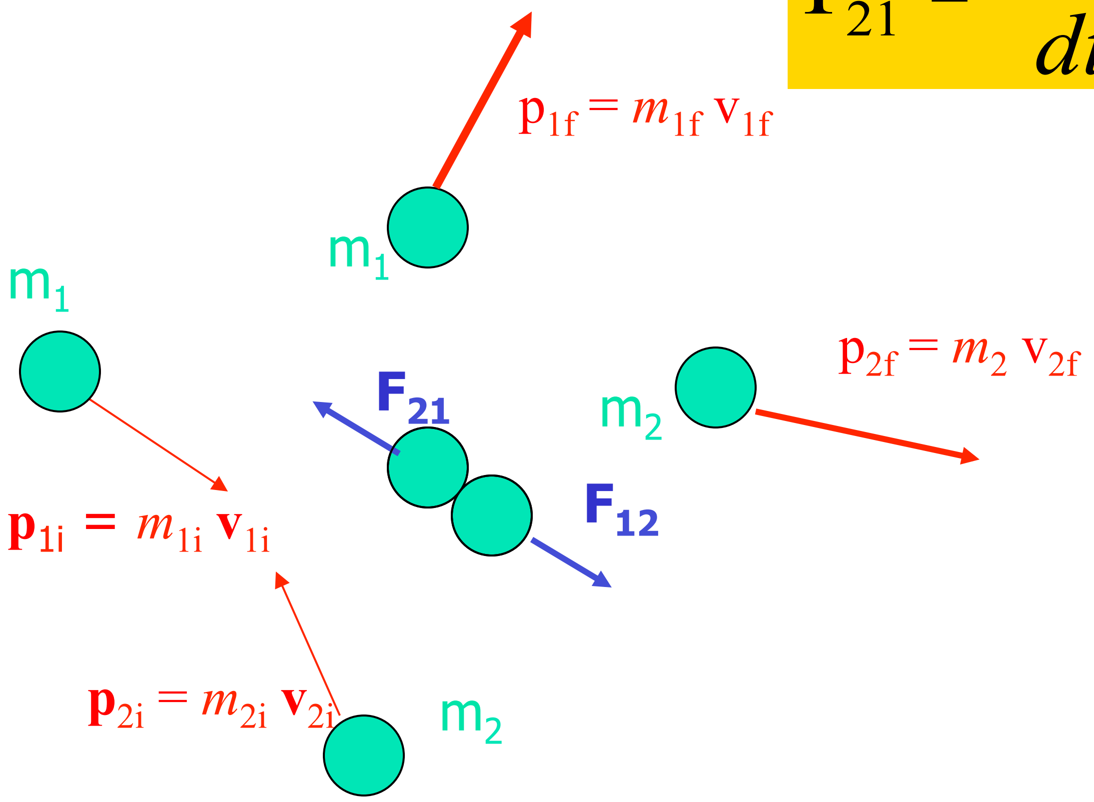
- are moving towards each other
- will interact with each other only
- are isolated from their surroundings i.e. there are no external forces



Linear Momentum of a Two-Particle System

- From *Newton's Second Law* applied to each particle

$$F_{21} = \frac{dp_1}{dt} \quad \text{and} \quad F_{12} = \frac{dp_2}{dt}$$



where $\mathbf{F}_{21}$ is the force exerted by particle 2 on particle 1, and $\mathbf{F}_{12}$ is the force exerted by particle 1 on particle 2.

Newton's Third Law implies:

That the forces on these 2 particles are equal in magnitude and opposite in direction (action-reaction), therefore:

$$F_{12} = -F_{21}$$

$$F_{12} + F_{21} = 0$$

$$\frac{dp_1}{dt} + \frac{dp_2}{dt} = \frac{d(p_1 + p_2)}{dt} = 0$$

- **This result can be extended to any number of particles in an isolated system.**

$$\mathbf{p_{total} = p_1 + p_2 = const}$$

or

$$\mathbf{p_{1i} + p_{2i} = p_{1f} + p_{2f}}$$

where i and j stands for initial and final momentum.

Law of Conservation of Linear Momentum

- Whenever two (or more) isolated, uncharged particles interact with each other, their total momentum remains constant.

Or,

- The total momentum of an isolated system at all times equals its initial momentum.

- **Note:** no assumption on the nature of the **internal** forces acting on the system.

Conservation of Linear Momentum

$$\Delta p_1 + \Delta p_2 = 0$$

$$\Delta(p_1 + p_2) = 0$$

The variation of the total momentum is zero; thus,

$$p_{system} = p_1 + p_2 = \text{constant}$$

The total momentum of an isolated system just before collision equals the total momentum of the system just after the collision.

For two particles:

$$p_1{}^b + p_2{}^b = p_1{}^a + p_2{}^a$$

Collisions

We use the term **collision** to represent the event of two particles coming together for a short time, thereby exerting impulsive forces on each other. These forces are assumed to be much greater than any external forces.

If we assume that no external forces act on the particles, then the change in linear momentum of particle 1 is only due to the impulse imparted by particle 2

$$\Delta p_1 = \int_{t_1}^{t_2} F_{21} dt$$

Likewise for partice 2

$$\Delta p_2 = \int_{t_1}^{t_2} F_{12} dt$$

Since $F_{12} = -F_{21}$ Action-reaction

$$\Delta p_1 = -\Delta p_2$$

$$\Delta p_1 + \Delta p_2 = 0$$

Types of Collisions

Elastic

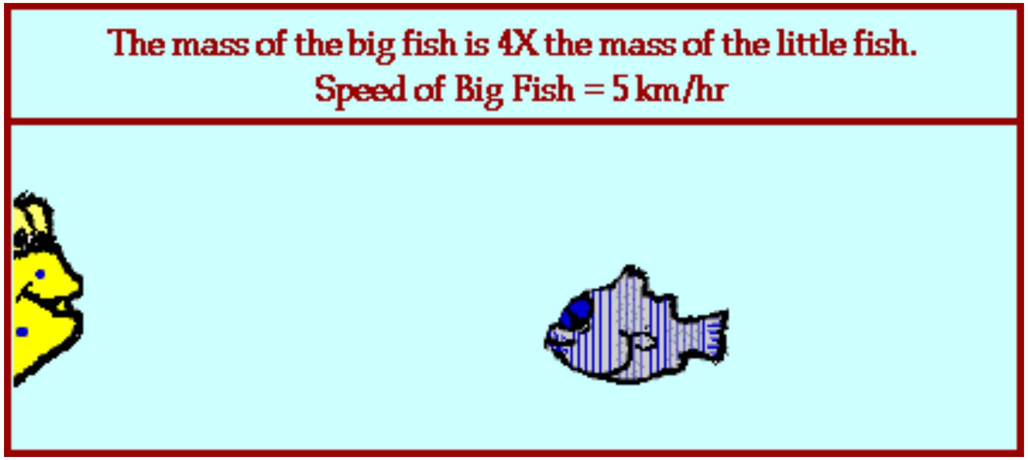
Inelastic

An **elastic collision** is one in which total kinetic energy as well as linear momentum is conserved.

An **inelastic collision** is one in which total kinetic energy is not constant but linear momentum is.

A **perfectly inelastic collision** is a collision in which the two bodies after the collision have the same velocity (i.e. move together).

Big Fish eats Little Fish



Little fish is at rest

$$m_b v_b + m_l v_l = (m_b + m_l)v \quad (m_l v_l = 0)$$

$$m_b = 4m_l \qquad v_b = 5\ \text{km/h}$$

$$4m_l(5\ \text{km/h}) = 5m_l v$$

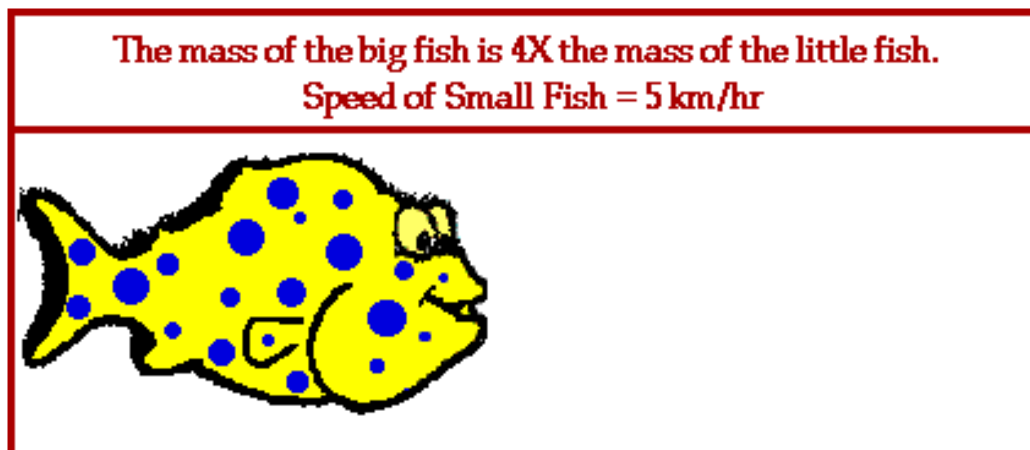
$$v = \frac{20}{5} = 4\ \text{km/h}$$

Big fish is at rest

$$m_b v_b + m_l v_l = (m_b + m_l)v \quad (m_b v_b = 0)$$

$$m_l(5\ \text{km/h}) = 5m_l v$$

$$v = \frac{5}{5} = 1\ \text{km/h}$$



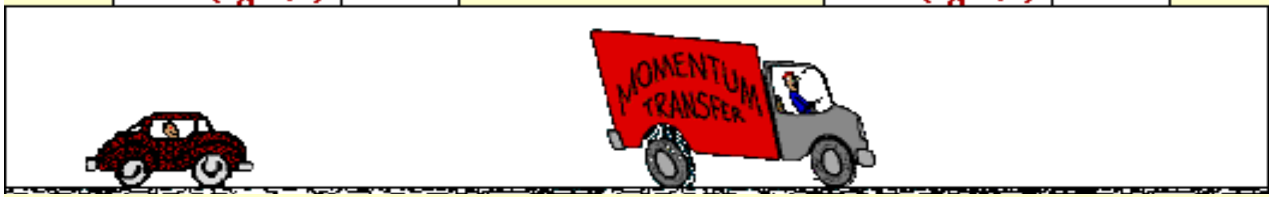
Determine the type of collision from the tabulated values

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



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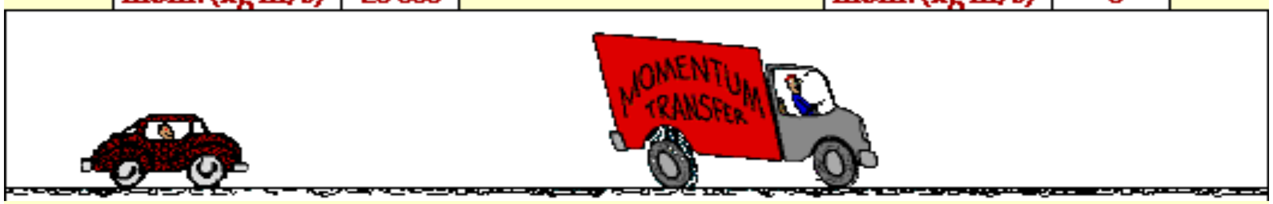
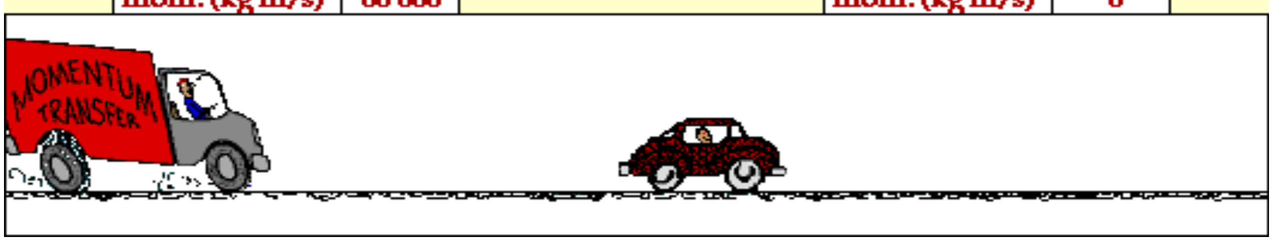


Truck

mass (kg)	3000
vel. (m/s)	20.0
mom. (kg m/s)	60 000

Car

mass (kg)	1000
vel. (m/s)	0.0
mom. (kg m/s)	0



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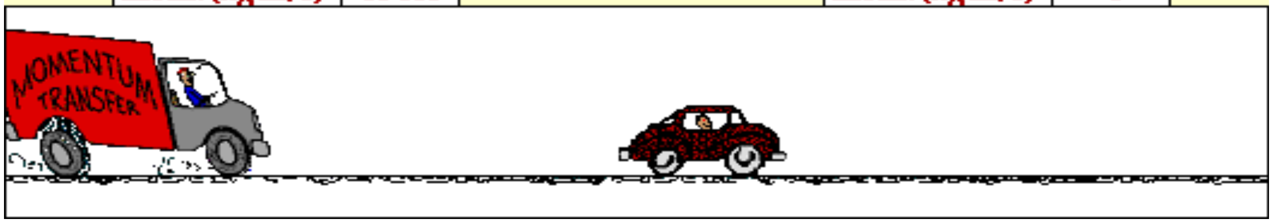
vel. (m/s)	0.0
mom. (kg m/s)	0



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mass (kg)	1000
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Truck

mass (kg)	3000
vel. (m/s)	-20.0
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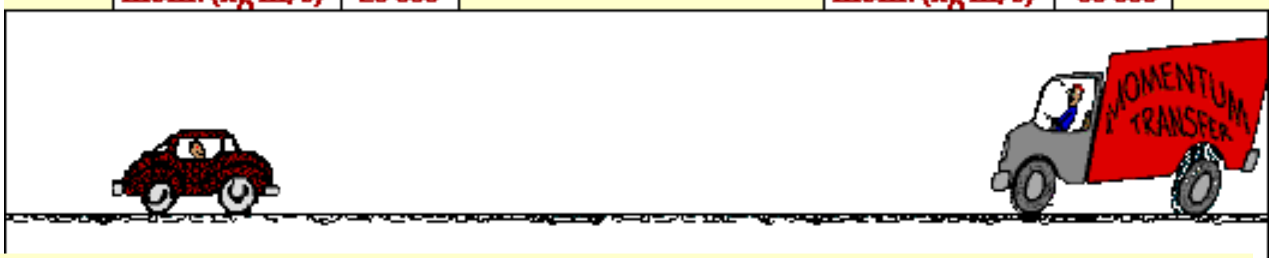

Car

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mass (kg)	3000
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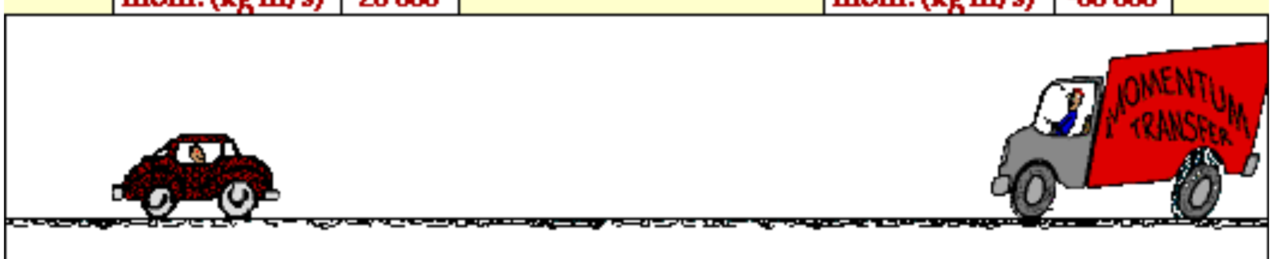
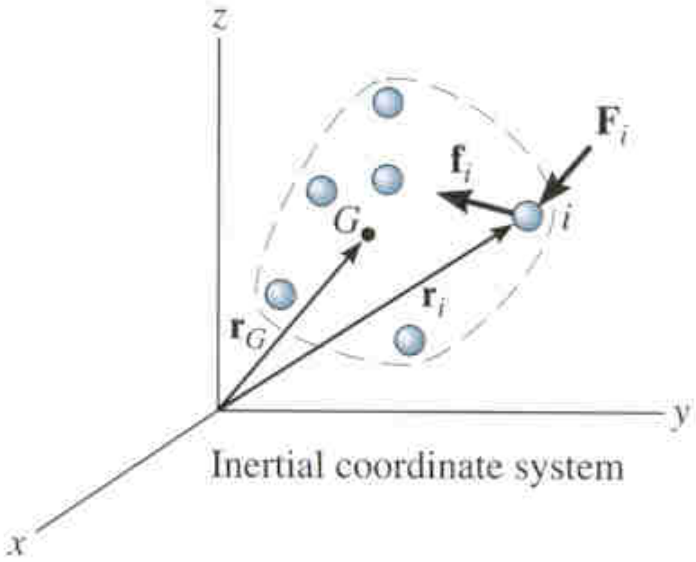


PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES



For the system of particles shown, the internal forces f_i between particles always occur in pairs with equal magnitude and opposite directions. Thus the internal impulses sum to zero.

The linear impulse and momentum equation for this system only includes the impulse of external forces.

$$\sum m_i(v_i)_1 + \sum \int_{t_1}^{t_2} F_i \, dt = \sum m_i(v_i)_2$$

CONSERVATION OF LINEAR MOMENTUM FOR A SYSTEM OF PARTICLES

When the sum of external impulses acting on a system of objects is zero, the linear impulse-momentum equation simplifies to

$$\sum m_i(\mathbf{v}_i)_1 = \sum m_i(\mathbf{v}_i)_2$$

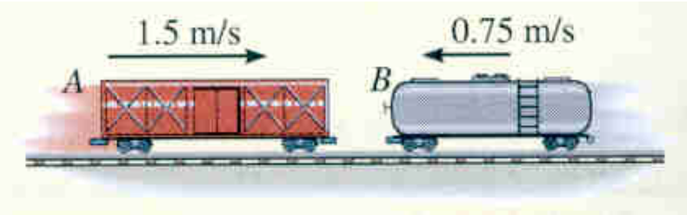
This important equation is referred to as the conservation of linear momentum. Conservation of linear momentum is often applied when particles collide or interact. When particles impact, only impulsive forces cause a change of linear momentum.

The hammer applies an impulsive force to the stake. The weight of the stake can be considered negligible, or non-impulsive, as compared to the force of the hammer. Also, provided the stake is driven into soft ground with little resistance, the impulse of the ground's reaction on the stake can also be considered negligible or non-impulsive.

EXAMPLE

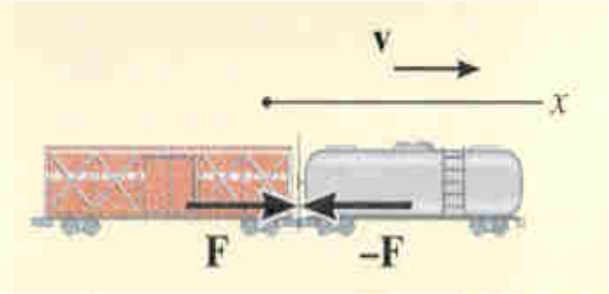
Given: Two rail cars with masses of m_A = 15 Mg and m_B = 12 Mg and velocities as shown.

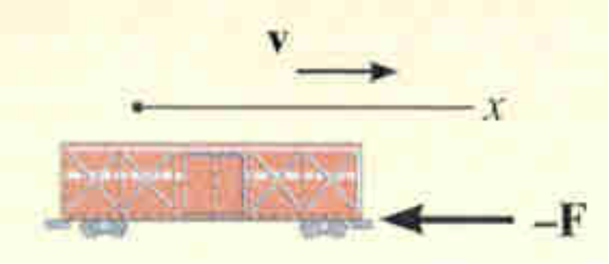
Find: The speed of the cars after they meet and connect. Also find the average impulsive force between the cars if the coupling takes place in 0.8 s.



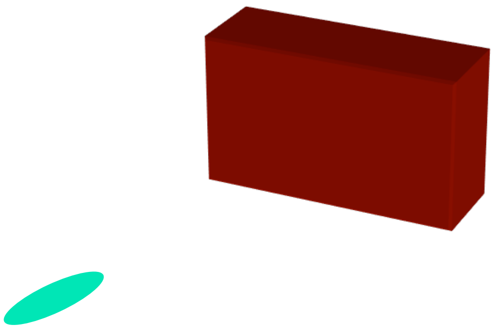
EXAMPLE (continued)

Solution:

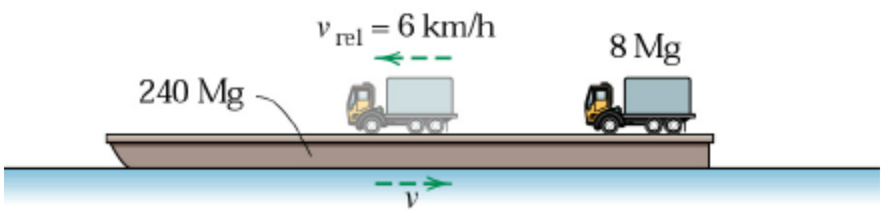




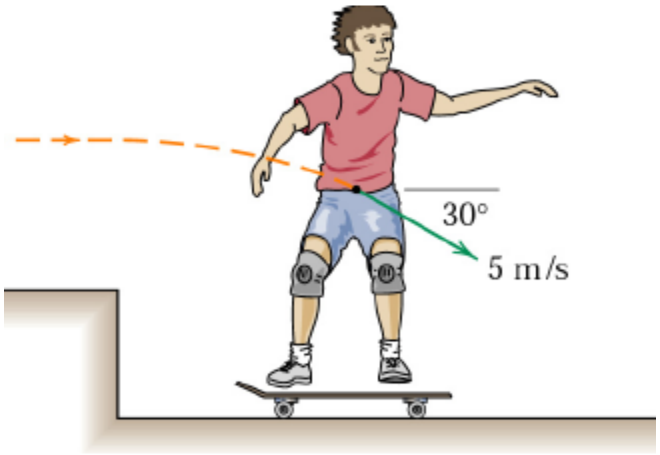
<u>Example :</u> **A 10.0 g bullet is fired into a stationary block of wood (*m*=5.00 kg). The relative motion of the bullet stops inside the block. The speed of bullet-plus-wooden block immediately after the collision is 0.600 m/s. What was the original speed of the bullet?**



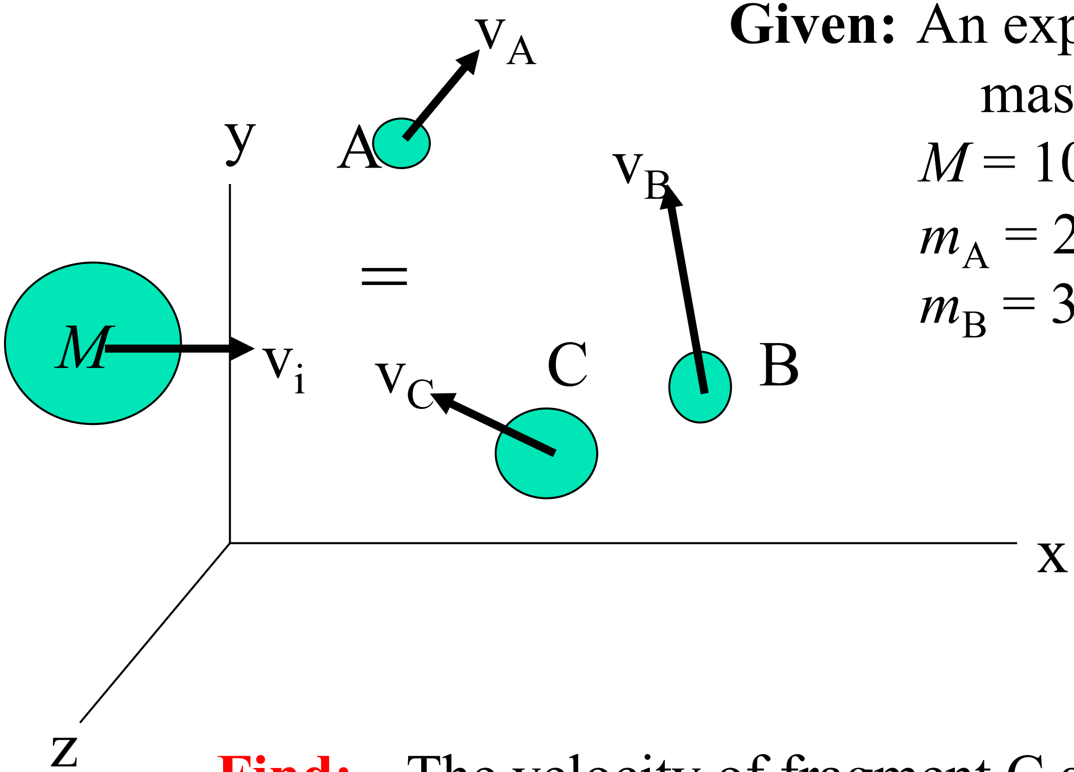
Problem: An 8-Mg truck is resting on the deck of a barge which displaces 240 Mg and is at rest in still water. If the truck starts and drives toward the bow at a speed relative to the barge v_{rel} = 6 km/h, calculate the speed v of the barge. The resistance to the motion of the barge through the water is negligible at low speeds.



Problem: The 40-kg boy has taken a running jump from the upper surface and lands on his 5-kg skateboard with a velocity of 5 m/s in the plane of the figure as shown. If his impact with the skateboard has a time duration of 0.05 *s*, determine the final speed *v* along the horizontal surface and the total normal force ***N*** exerted by the surface on the skateboard wheels during the impact.



EXAMPLE 1



Given: An explosion has broken the mass M into 3 smaller particles.

$M = 100$ kg, $\mathbf{v}_i = 20\mathbf{j}$ (m/s)

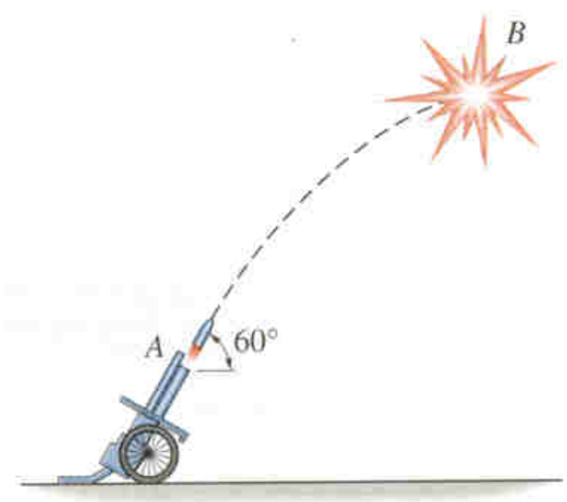
$m_A = 20$ kg, $\mathbf{v}_A = 50\mathbf{i} + 50\mathbf{j}$ (m/s)

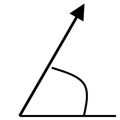
$m_B = 30$ kg, $\mathbf{v}_B = -30\mathbf{i} - 50\mathbf{k}$ (m/s)

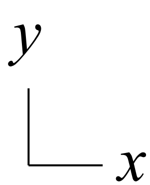
Find: The velocity of fragment C after the explosion.

Plan: Since the internal forces of the explosion cancel out, we can apply the conservation of linear momentum to the SYSTEM.

GROUP PROBLEM SOLVING



Given: A 10-lb projectile is fired from point A. It's velocity is 80 ft/s @  60°
The projectile explodes at its highest point, B, into two 5-lb fragments. One fragment moves vertically upward at $v_y = 12$ ft/s.

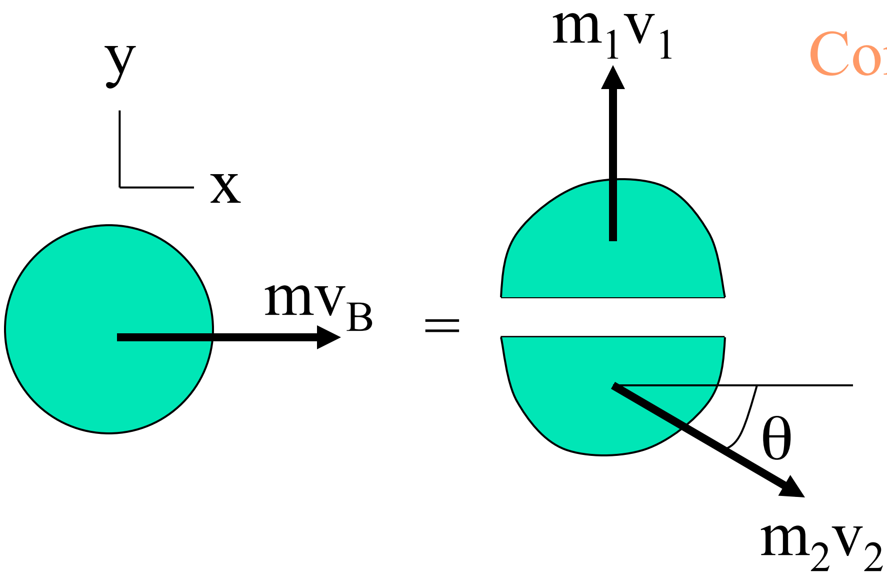


Find: Determine the velocity of the other fragment immediately after the explosion.

Plan: Since we know $(v_B)_y = 0$ just before the explosion, we can determine the velocity of the projectile fragments immediately after the explosion.

GROUP PROBLEM SOLVING (continued)

Solution:



Conservation of linear momentum:

Since the impulse of the explosion is an internal impulse, the system's linear momentum is conserved. So

$$m\mathbf{v}_B = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

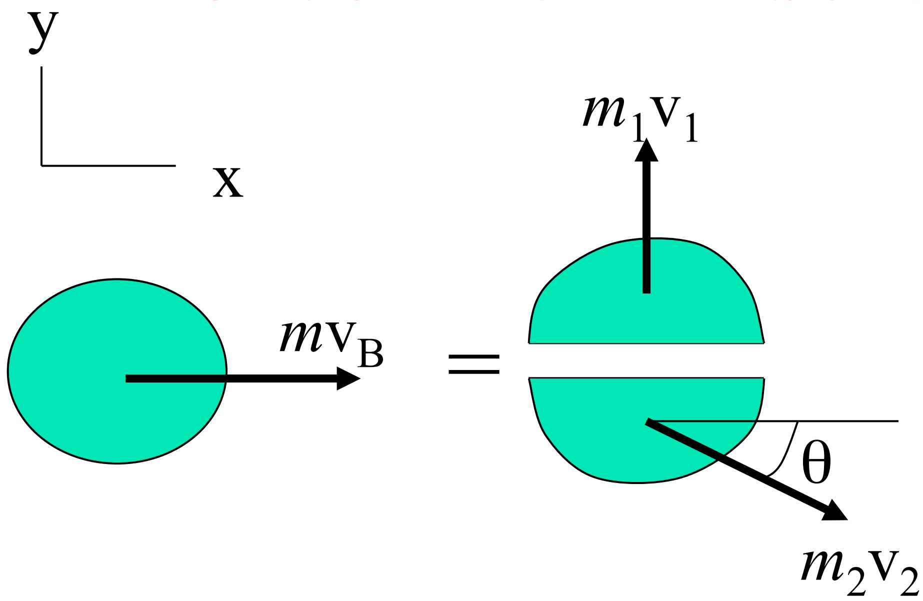
We know $(v_B)_y = 0$. Use projectile motion equations to calculate $(v_B)_x$:

$$(v_B)_x = (v_A)_x = v_A \cos 60° = 80 \cos 60° = 40 \text{ ft/s}$$

Therefore, substituting into the linear momentum equation

$$(10/g)(40)\,\mathbf{i} = (5/g)(12)\,\mathbf{j} + (5/g)(v_2)(\cos\theta\,\mathbf{i} - \sin\theta\,\mathbf{j})$$

GROUP PROBLEM SOLVING (continued)



$$(10/g)(40)\,\mathbf{i} = (5/g)(12)\,\mathbf{j} + (5/g)(v_2)(\cos\theta\,\mathbf{i} - \sin\theta\,\mathbf{j})$$

Eliminating g, dividing by 5 and creating **i** & **j** component equations yields: $80 = v_2 \cos\theta$

$$0 = 12 - v_2 \sin\theta$$

Solving for v_2 and θ yields $\theta = 8.53°$ and $v_2 = 80.9$ ft/s