

COMMERCE 290  
**Homework # 3B**  
Complete by: October 11, 2012

This homework is for **practice only** and is **NOT to be handed in**. However, students should have completed this by the date indicated above

**This is a Group Homework.**

This homework is to be completed in groups of either 2 people or 3 people. It is expected that each member of the group will contribute in equal parts to all problems. This includes participating in the discussion and debate about the problem, developing appropriate models and writing the homework. Refer to "Rules for Groups" attached to this homework.

- DIRECTIONS:
1. For question #1, write all your answers in the indicated spaces and **show all work on this paper**. No additional pages will be accepted. For question #2, add the pages to the end of this homework.
  2. Show all work for all non multiple choice questions on this homework paper. No credit for answers only!
  3. Be sure to include your units in each answer.

**Group Member Information:**

Lab #	LASTNAME, Firstname	Student #
_____	<b>Solution</b>	_____
_____	_____	_____
_____	_____	_____

Problem	Maximum Possible	Marks Awarded
1	30	
2	30	
Total	60	

**Problem 1 (10 marks)**

**Note: The story below is a slightly modified version of the Lucky Strike problem done in class this term. The only difference is that Lucky now has 3 products, Regular, Mid-Grade and Ultra gasoline compared to the 2 product problem that was discussed in class.**

This week, the Lucky Strike Oil Company has the opportunity to purchase up to 500,000 litres of OilA at a cost of \$0.32 per litre 275,000 litres of OilB at a cost of \$0.38 per litre and 425,000 litres of OilC at a cost of \$0.34 per litre. With this oil, Lucky can produce 3 grades of gasoline - Regular, Mid-Grade and Ultra.

Regular gas must be at least 45% OilA, not over 25% OilB with the remainder being OilC. Mid-Grade gas must be at least 40% OilA, at most 20% OilC with the remainder being OilB. Ultra Gas must be at least 35% OilB, not more than 35% OilC and the remainder being OilA. The selling price of Regular gas is \$0.42 per litre; Mid-Grade sells for \$0.44 per litre and Ultra sells for \$0.51 per litre.

The company would like to know how to blend the ingredients in order to maximize profits.

This problem was correctly formulated as a linear programming problem on Excel and solved using Solver. The solved model, with an optimal solution and Sensitivity Report, are attached. Use only these printouts to answer the following questions.

To receive full marks, be sure to include your units when answering the following questions.  
**QUESTIONS ARE INDEPENDENT.**

(a) (1 mark) How many litres of Regular Gas should be produced?

Your Answer: 191,837

(b) (1 mark) Determine the total number of litres to produce.

191,837		500,000
+ 222,449	OR	275,000
+ 785,714	<u>    </u>	425,000
1,200,000		1,200,000

Your Answer: 1,200,000

- (c) (2 marks) Suppose management had an opportunity to purchase exactly 1,000 additional litres of either OilA or OilB or OilC at the regular cost. Which of these 3 (if any) should they purchase? Explain.

• OIL B

• it has the highest shadow price of the 3 oils

(note: all 3 shadow prices are in effect for the proposed 1000 litre ↑)

- (d) (2 marks) Which of the 3 products, REG, MID or ULTRA contributes more to profit on a per litre basis? Provide convincing evidence.

$$\begin{array}{r} 17073 \\ \hline 191837 \end{array} \quad ) \quad \begin{array}{r} 25804 \\ \hline 222,449 \end{array} \quad ) \quad \begin{array}{r} 127,285 \\ \hline 785,714 \end{array}$$

$$= \quad .088 \quad , \quad .116 \quad , \quad .162$$

Your Answer: ULTRA

- (e) (1 mark) One requirement in this model is that REG Grade gasoline must not have more than 25% OilB in its composition. Is this constraint binding or non-binding?

Your Answer: non-binding

- (f) (1 mark) Show how the value of 0.10 found in cell F9 on the Sensitivity Report was determined.

$$.42 - .32$$

- (g) (4 marks). Suppose management could purchase an additional 100,000 litres of OilB at the regular cost. What can you say about the effect this change will have on:  
 (i) the corresponding total profit?

• Profit will ↑ by \$29,590  
 (100,000 × .2959)

OR

$$\left\{ \begin{array}{l} \uparrow \text{Cost} = 38000 [100,000 \times .38] \\ \uparrow \text{Rev} = 67,590 [100,000 \times (.2959 + .38)] \end{array} \right.$$


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↑ Profit = \$29,590

- (ii) the optimal solution? Explain.

- will Δ
- binding constraint

- (h) (4 marks) Suppose management could obtain 100,000 additional litres of OilA at \$0.35 per litre.

- (i) What would be the value of the Target Cell?

$$\begin{aligned} &= 170,163.27 + 100,000 (.1314 - .03) \\ &= 170,163.27 + 10,140 \\ &= \$180,303.27 \end{aligned}$$

OR

Target cell ↑ by \$10,140

Alternate answer:

$$\left\{ \begin{array}{l} \uparrow \text{Cost} = 35,000 [100,000 \times .35] \\ \uparrow \text{Rev} = 45,140 [100,000 \times (.1314 + .32)] \end{array} \right.$$


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↑ Profit = \$10,140

- (ii) What effect would this have on the optimal solution? Explain

- will Δ
- binding constraint

- (i) (3 marks) Suppose the engineers at Lucky suggest that the current MID Grade requirement that OilA be at least 40% of the composition of MID Grade gas be increased from 40% to 50%. What can you say about the effect this change will have on:

(i) the corresponding total profit ?

• no change

(ii) the optimal solution ? Explain

• no  $\Delta$

• non-binding constraint now (at 40%)  
and remains non binding (at 50%) (within allow  $\uparrow$ )  
(or current amount of mid-grade is  $> 50\%$  now)

- (j) (2 marks) In the Sensitivity Report, the row identified as OilB Reg shows an Allowable Decrease of  $1E+30$ . Provide a meaningful and correct interpretation to management.

• profit contribution of .04 can go  $\downarrow$  an infinite amount and the optimal solution will not  $\Delta$ .

OR  
• cost of OilB can  $\uparrow$  by an infinite amount and the optimal solution will not  $\Delta$

- (k) (5 marks) Suppose Lucky could purchase an additional 200,000 litres (or nothing) of OilC at the regular cost. What can you say about the effect this change will have on:

(i) the corresponding profit?

• For the first 173,015 litres, profit  $\uparrow 9394.71$  ( $173015 \times 0.0543$ )  
• for the next 26,985 litres,  $\Delta$  in profit is unknown  
but the cost of these 26,985 litres is  $\$9174.90$  and  
the extra revenue generated by these litres  $\geq 0$ .

So profit  $\uparrow$  by at least  $\$219.81$  ( $9394.71 - 9174.90$ )

(ii) the optimal solution ? Explain.

• will  $\Delta$

• binding constraint.

- (1) (4 marks) Suppose the cost of OilB increased to \$0.40 per litre from its current value of \$0.38 per litre. What effect would this have on:  
(i) the corresponding total profit?

• Profit will ↓ by \$5500 [  $-.02 \times 275,000$  ]

(ii) the optimal solution? {Circle the best response:}

Will Change

Will not Change

Can't tell with the information provided

because: {Choose the best answer:}

- (A) the change is within the allowable increase and this change is binding
- (B) the change is within the allowable decrease and this change is binding
- (C) the change is within the allowable decrease and this change is non-binding
- (D) the change is within the allowable increase and this change is non-binding
- (E) the change is within the allowable increase
- (F) the change is within the allowable decrease

Your Answer:         F

	A	B	C	D	E	F	G	H	I	J	K	L
1	<b>Lucky Strike</b>											
2												
3												
4	<b>Input Data</b>											
5		Cost per	Amount									
6	OilA	litre (\$)	Available									
7	OilB	0.32	500000									
8	OilC	0.38	275000									
9		0.34	425000									
10												
11	<b>Action Plan</b>											
12				Reg	Mid	Ultra	Amount					
13				Used			Available					
14	OilA	86,327	177,959	235,714	500,000		500,000					
15	OilB	0	0	275,000	275,000		275,000					
16	OilC	105,510	44,490	275,000	425,000		425,000					
17	Total	191,837	222,449	785,714								
18												
19	<b>Additional Constraints</b>											
20												
21	Regular gas must be at least			0.45								
22	Regular Gas not more than			0.25								
23	Mid Grade must be at least			0.40								
24	Mid- Grade not more than			0.20								
25	Ultra Gas must be at least			0.35								
26	Ultra Gas not more than			0.35								
27												
28												
29	<b>Revenue/Cost Data</b>											
30												
31	Selling Price Regular			\$	0.42	litre						
32	Selling Price Mid Grade			\$	0.44	litre						
33	Selling Price Ultra			\$	0.51	litre						
34												
35		Cost per										
36	OilA	litre (\$)										
37	OilB	0.32										
38	OilC	0.38										
39		0.34										
40												

Output	Req't	units
86,327	>=	86,327 litres
0	<=	47,959 litres
177,959	>=	88,980 litres
44,490	<=	44,490 litres
275,000	>=	275,000 litres
275,000	<=	275,000 litres

	Reg	Mid Grade	Ultra	Total
Total Revenue	\$ 80,571.43	\$ 97,877.55	\$ 400,714.29	\$ 579,163.27
Total Costs				
OilA	\$ 27,624.49	\$ 56,946.94	\$ 75,428.57	\$ 160,000.00
OilB	\$ -	\$ -	\$ 104,500.00	\$ 104,500.00
OilC	\$ 35,873.47	\$ 15,126.53	\$ 93,500.00	\$ 144,500.00
Total Profit	\$ 17,073.47	\$ 25,804.08	\$ 127,285.71	\$ 170,163.27

	A	B	C	D	E	F	G	H
1	Microsoft Excel 9.0 Sensitivity Report							
2								
3								
4								
5								
6	Adjustable Cells							
7		Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
9		\$C\$14	OilA Reg	86,327	0	0.10	0.0250	0.0528
10		\$D\$14	OilA Mid	177,959	0	0.12	0.0528	0.0250
11		\$E\$14	OilA Ultra	235,714	0	0.19	0.0571	0.1209
12		\$C\$15	OilB Reg	0	-0.282	0.04	0.2816	1E+30
13		\$D\$15	OilB Mid	0	-0.224	0.06	0.2245	1E+30
14		\$E\$15	OilB Ultra	275,000	0	0.13	1E+30	0.2245
15		\$C\$16	OilC Reg	105,510	0	0.08	0.0364	0.0432
16		\$D\$16	OilC Mid	44,490	0	0.10	0.2111	0.0364
17		\$E\$16	OilC Ultra	275,000	0	0.17	1E+30	0.0571
18								
19	Constraints							
20		Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
22		\$F\$14	OilA Used	500,000	0.1314	500,000	335714.29	141558.44
23		\$F\$15	OilB Used	275,000	0.2959	275,000	106818.18	275000
24		\$F\$16	OilC Used	425,000	0.0543	425,000	173015.873	83928.57
25		\$G\$21	OilA Output	86,327	-0.0571	0	77857.14	37767.86
26		\$G\$22	OilB Output	0	0.0000	0	1E+30	47959.18
27		\$G\$25	OilB Output	275,000	-0.2245	0	126923.08	136250
28		\$G\$26	OilC Output	275,000	0.0571	0	67142.86	77857.14
29		\$G\$23	OilA Output	177,959	0.0000	0	88979.59	1E+30
30		\$G\$24	OilC Output	44,490	0.0571	0	67142.86	28311.69

## Problem 2 ( 30 marks)

Do the Textbook Problem 87 (Chapter 4, Springfield City Police Department) with the changes in the box below and *answer only the following questions*:

Changes:

1. **Change:** Saturday, 28 to Saturday, 29
2. **Replace the sentence** "Each officer works 5 days per week " **with:** "Each officer works 5 consecutive days per week ."
3. **Delete the last 2 sentences and replace with:** Assuming that all 5-day shifts cost the same, what is the minimum cost solution that uses all 30 officers?

- (a) Set up the above problem in Excel using the style that we used in class. Make sure that your spreadsheet model is logical, well organized and easy to understand. Name this model, Model 1.
- (b) Solve Model 1 using Solver. What would be your recommendation to management in terms of staffing? Write your *brief answer* to part (b) in a single text box placed on your Model 1 page.
- (c) Create a second model. Call this model, Model 2. In this model, remove the restriction that all 30 officers must be scheduled. This means that the Police department is no longer bound to use all 30 officers; it may choose for example, to use less than 30 or more than 30 officers. Solve Model 2 using Solver. What would be your recommendation to management now in terms of staffing? Write your *brief answer* to part (c) in a single text box placed on your Model 2 page.
- (d) Print out Model 1 on one page in landscape orientation and include it with your answers.
- (e) Print out Model 2 on one page in landscape orientation and include it with your answers.
- (f) Print out the cell formulas for Model 2 on one page in landscape orientation showing Row and Column Headings and include it with your answers.

## Group Homework

Over the years, Commerce 290 students have requested that some of the homework in the course be of a group variety. In addition, teaching and learning models have moved in the direction of using peer to peer discussion and debate as a viable and important part of a university students overall education.

With this in mind, some homework in Commerce 290 this year will be in group form. Naturally the purpose in doing homework (group or individual) is to develop and hone your skills. This in turn will prepare students for the more major parts of the course evaluation procedure; midterms and final exams.

It is also important to remember that while we may work on our own occasionally, most work performed after university is done in groups. Groups are very powerful tools in the solving of problems. It is expected that these group homework assignments in Commerce 290 will help contribute to the development of students in this course.

### Rules for Groups:

- (a) Students must form groups of either 2 people or 3 people.
- (b) **Groups may be formed by any people registered in the course.** This means that you can choose people from your own section or any of the other sections.
- (c) Students who are having difficulty forming groups should use the discussion forum to find suitable group members.
- (d) Each member of the group will receive the same grade for this homework.
- (e) Each group member must contribute equal amounts to each question in the homework.
- (f) Each group member must participate in the discussion and debate about the problem, develop appropriate models and share in the writing of the homework.
- (g) Each group must submit 1 homework, with a cover page that closely resembles the suggested format for a group homework.
- (h) Group members cannot discuss the homework with other groups. The only discussion that is allowed is for members of one group to talk with others in their group.
- (i) In the extreme case where groups cannot be formed in teams of either 2 people or 3 people, the following rule will be applied to homework submitted :

If your group is smaller than 2 or larger than 3 people, please provide a satisfactory explanation for why it was absolutely necessary to violate the group size constraint. Otherwise you will lose 10% for each additional or missing member.

**Problem 2 (Model 1)**

		Days of Week						
		sat	sun	mon	tue	wed	thur	fri
Starting Shift on:	Sat	9	9	9	9	9		
	Sun		0	0	0	0	0	
	Mon			4	4	4	4	4
	Tue	6			6	6	6	6
	Wed	6	6			6	6	6
	Thur	0	0	0			0	0
Fri	5	5	5	5			5	
Supply		26	20	18	24	25	16	21
		>=	>=	>=	>=	>=	>=	>=
Minimum Demand		29	18	18	24	25	16	21

# Employees
9
0
4
6
6
0
5
30
=
30

Answers will vary

5 marks

**Note:**  
All 30 officers must be used

1 mark for the constraint and 2 marks for the changing cell

**Note:** Since all the 5 day shifts cost the same, minimizing cost is exactly the same as minimizing the number of officers

Part (b)  
Staffing:  
**There is no solution to this problem.** It is **impossible** (infeasible) to assign all 30 officers and meet the minimum daily demand.  
The answer shown above in the changing cells is infeasible because at least one of the constraints is violated.

2 marks

3 marks

Marking Key: part (a)  
This model page is worth 10 marks as indicated (plus 3 marks for the text box(Part (b)))

Marking Key: Part (d)  
2 marks for printing this page in landscape

**Problem 2 (Model 2)**

		Days of Week						
		sat	sun	mon	tue	wed	thur	fri
Starting Shift on:	Sat	10	10	10	10	10		
	Sun		0	0	0	0	0	
	Mon			2	2	2	2	2
	Tue	11			11	11	11	11
	Wed	2	2			2	2	2
	Thur	1	1	1			1	1
Fri	5	5	5	5			5	
Supply		29	18	18	28	25	16	21
Minimum Demand		>= 29	>= 18	>= 18	>= 24	>= 25	>= 16	>= 21

# Employees
10
0
2
11
2
1
5
31

Answers will vary. This problem has multiple optima

3 marks

2 marks

**Note:** Since all the 5 day shifts cost the same, minimizing cost is exactly the same as minimizing the number of officers

**Marking Key: Part (e)**  
2 marks for printing this page in landscape

**Part (c) Recommendation**

**You should schedule:**  
 10 officers to start on Saturday  
 0 officers to start on Sundays  
 2 officers to start on Mondays  
 11 officers to start on Tuesdays  
 2 officers to start on Wednesdays  
 1 officer to start on Thursdays  
 5 officers to start on Fridays

This schedule will minimize the number of officers needed per week at 31.

2 marks

3 marks

**Note:** There are multiple optima in this problem so student answers may have different numbers but they should all have the same value (31) in the Target Cell.

**Marking Key:part (c)**

This model page is worth 7 marks for the model as indicated and 3 marks for the recommendation in the text box

