

COMMERCE 290
HOMEWORK #2

Complete by: September 27, 2012

This homework is for **practice only** and is **NOT to be handed in**. However, students should have completed this by the date indicated above

Note: As you know from the course outline, some homework assignments are completed individually and others are completed in groups. **This homework** is to be completed by **individual students only**. Students are reminded not to work in groups, not to discuss your homework with others and this includes using the Course Discussion Forum as a means of communication for this homework. This is NOT a group work homework.

- DIRECTIONS:
1. For Problem #1, write all your answers in the indicated spaces and **show all work on this paper**. No additional pages will be accepted. For Problem #2 and 3, please staple the pages to the end of Problem #1.
 2. Show all work on this homework paper. No credit for answers only!
 3. Be sure to include your units in each answer.

Last Name: _____

First Name: **Solution** _____

Student Number: _____

Signature: _____

Excel Lab Number: _____

Problem	Maximum Possible	Marks Awarded
1	37	
2	43	
3	10	
Total	90	

Problem 1 (37 marks)

Save-On Taxes is a Vancouver firm specializing in preparing Canadian Income Tax forms for their individual and corporate clients. Save-On processes only two types of income tax returns; the T-1 Return for individual clients and the Corporate Return for corporate clients. Each return, whether it is a T-1 or a Corporate Return, is checked by three separate departments within Save-On Tax. These departments are called the Deductions, Income and Arithmetic departments and have respectively 1,168 hours, 952 hours and 362 hours of labour hours available each processing cycle.

Returns not completed during one processing cycle are completed during the next processing cycle. Management requires that the company process at least 25, T-1 Returns each processing cycle. The profit contributions for each T-1 and Corporate Return processed by Save-On Tax are \$10 and \$20 respectively.

A complete and correct formulation and an *incomplete* graph are shown below. Using the following decision variables:

- T = number of T-1 Returns to process each processing cycle
- C = number of Corporate Returns to process each processing cycle

answer the remaining questions below.

max Profit) $10T + 20C$
st
Deductions) $T + 9C \leq 1168$
Income) $4T + 5C \leq 952$
Arithmetic) $2T + C \leq 362$
MinDem) $T \geq 25$
Non-Negativity $T, C \geq 0$

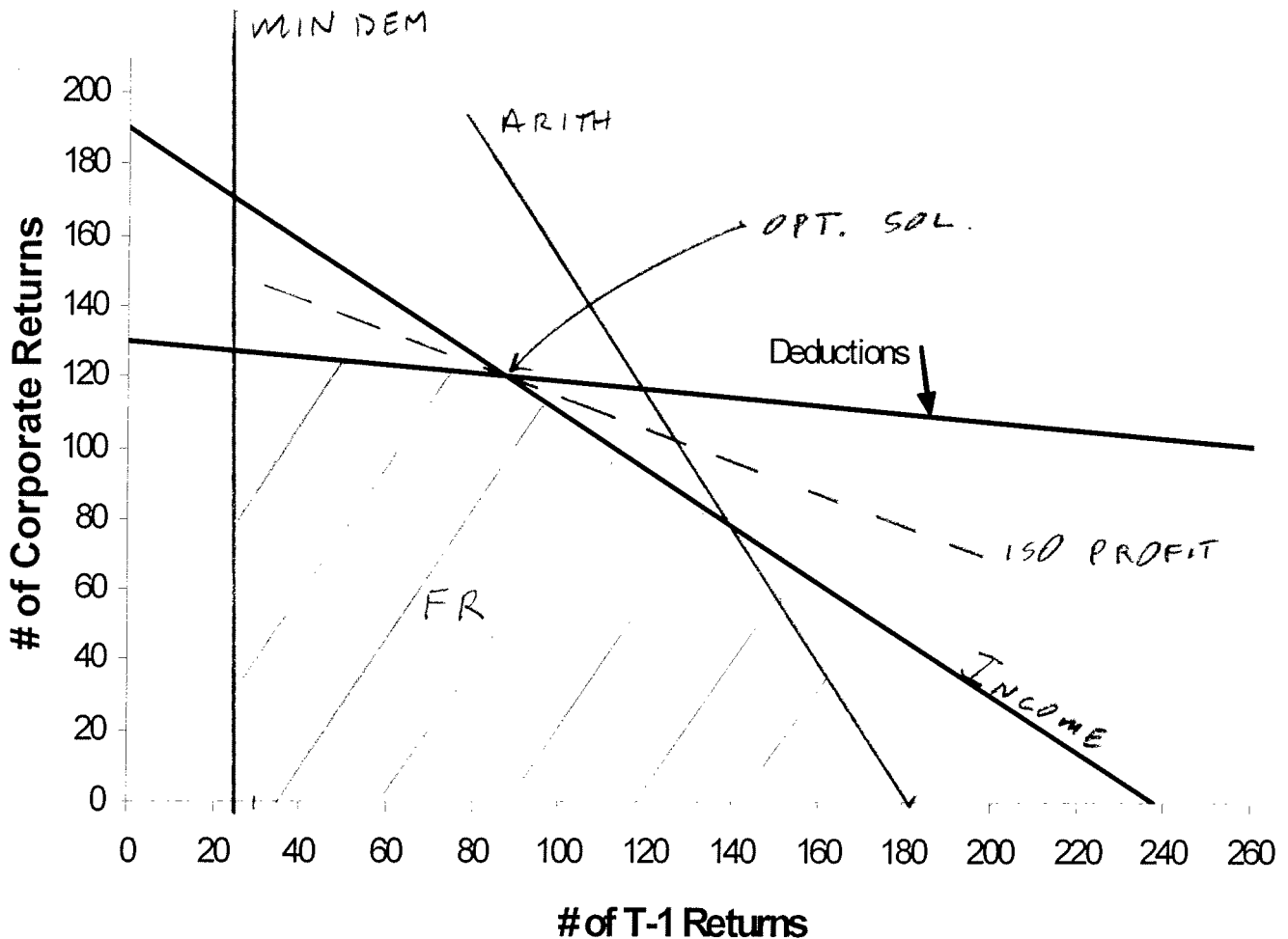
- (a) (1 mark) In the Deduction constraint, describe what the "9" measures.

• each corporate return takes 9 hours

- (b) (2 marks) The Boss at Save-On Taxes has said that the company should only prepare Corporate returns since each Corporate return translates into \$20 profit, whereas each T-1 Return only provides \$10 per return. Without referring to the, yet unknown, optimal solution, how would you respond to this argument?

• Boss is ignoring resources needed
in Ded, Inc, Arith

- (c) (5 marks) The partially completed graph below correctly identifies the Deductions constraint and **either** the Income or Arithmetic constraint.
- Add any missing constraints.
 - Label all the constraints by name.
 - Shade and identify the feasible region.
 - Draw and identify the iso-profit line on this graph showing where the optimal solution is located.
 - Identify at which intersection the optimal solution exists.



- (d) (2 marks) Save-On management is hoping to produce 120 T-1 Returns and 110 Corporate Returns the next processing cycle. Comment on their plans.

not possible; outside FR

(e) (2 marks) What is the optimal solution?

$$\begin{aligned}
 \text{D) } T + 9C &= 1168 \Rightarrow 4T + 36C = 4672 \\
 \text{I) } 4T + 5C &= 952 \Rightarrow \frac{4T + 5C = 952}{31C = 3720}
 \end{aligned}$$

$$31C = 3720$$

$$C = 120$$

$$\Rightarrow T = 88$$

Your Answer: _____

(f) (1 mark) If this problem was formulated in Excel, what would be the value of the target cell?

$$\begin{aligned}
 \text{MAX } 10T + 20C \\
 10(88) + 20(120)
 \end{aligned}$$

Your Answer: \$ 3,280

(g) (1 mark) How many hours will Save-On use in the Arithmetic Department?

$$\begin{aligned}
 \text{A) } 2T + C &= ? \\
 2(88) + 120 &= 296
 \end{aligned}$$

Your Answer: 296 hours

(h) (2 marks) What is the allowable increase and allowable decrease for the Arithmetic constraint?

Allow ↓

$$\begin{aligned}
 \text{A) } 2T + C &= 362 \\
 2(88) + 120 &= 296 \\
 \hline
 &66
 \end{aligned}$$

Allow ↑ = ∞
Allow ↓ = 66 hours

Your Answer: _____

(i) (3 marks) **FOR THIS QUESTION ONLY**, suppose the **MinDem** constraint was changed so that the minimum was now 40 units. **{Circle the correct responses.}**

- (I) Would the feasible region change? Yes No
- (ii) Would the optimal solution change? Yes No
- (iii) Would the set of binding constraints change? Yes No

(j) (4 marks) Determine the **allowable increase** and **allowable decrease** on the objective function coefficient for **Corporate Returns**.

$$\frac{-4}{5} \leq \frac{-10}{c_2} \leq -\frac{1}{9}$$

$$\left. \begin{array}{l} -\frac{4}{5} \leq \frac{-10}{c_2} \\ (5c_2) \frac{4}{5} \geq \frac{10}{c_2} \times (5c_2) \\ 4c_2 \geq 50 \\ c_2 \geq 12.50 \end{array} \right\} \begin{array}{l} -\frac{10}{c_2} \leq -\frac{1}{9} \\ (9c_2) \frac{10}{c_2} \geq \frac{1}{9} \times (9c_2) \\ 90 \geq c_2 \end{array}$$

$$12.50 \leq c_2 \leq 90$$

Allow \uparrow = 70
Allow \downarrow = 7.5

Your Answer: _____

(k) (3 marks) Determine the **allowable decrease** on the **Income** constraint.

• move to intersection of D and MinDem.

D) $\left. \begin{array}{l} T + 9C = 1168 \\ T = 25 \end{array} \right\} \Rightarrow C = 127$

I) $\begin{array}{l} 4T + 5C = 952 \\ 4(25) + 5(127) = 735 \end{array}$

Allow \downarrow = 217

Your Answer: _____

- (l) (4 marks) Set up, but, DO NOT SOLVE, all the equations you would need to determine the shadow price for the **Deductions** constraint. DO NOT SOLVE.

$$\begin{array}{l} \text{D) } T + 9C = 1168 + 1 \\ \text{I) } 4T + 5C = 952 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{D) } T + 9C = 1168 + 1 \\ \text{I) } 4T + 5C = 952 \end{array}} \right\} *$$

$$\begin{array}{l} \text{and New Profit at } * \\ - \text{ Old Profit (3280)} \\ \hline = \text{Shadow Price} \end{array} \quad \left. \vphantom{\begin{array}{l} \text{and New Profit at } * \\ - \text{ Old Profit (3280)} \\ \hline = \text{Shadow Price} \end{array}} \right\}$$

- (m) (4 marks) The Save-On problem was correctly formulated and solved in Excel with the following partial sensitivity report output.

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$5	Ded Output	1168	0.9677	1168	390.6	341
\$E\$6	Inc Output	952	2.2581	952	120.353	217

Suppose Save-On can secure up to an additional 600 hours in **either** the Deductions or Income department, **but not both**. In which department, Deductions or Income, would you recommend purchasing extra time? **Circle the correct response** and provide convincing evidence.

Deductions

Income

Evidence: We can see from the graph, that both constraints have shadow prices = 0 beyond their allow \uparrow . So, must determine the TOTAL improvement to profit for each constraint.

$$\text{D) } .9677 \times 390.6 = \$377.98$$

$$\text{I) } 2.2581 \times 120.353 = \$271.77$$

Choose D, since $\$377.98 > \271.77

- (n) (3 marks) **FOR THIS QUESTION ONLY**, suppose revenue and cost savings could be realized that would change the objective function to $\text{Max } 20T + 10C$ (from the current $\text{Max } 10T + 20C$). What effect(s), if any, would this change have on the optimal solution? No calculations are required.

• new optimal solution
and

• multiple optima now exist

(these answers can be determined
by graphing the new iso profit line
 $20T + 10C$)

Please do not write below this line.

Problem 2 (43 marks)

Use the Textbook Problem 61 (Chapter 4, Steelco) to answer only the following questions:

*Hint: Please note that you are determining how to minimize the cost of producing **ONE** ton of steel.*

- (a) What are the two decisions that need to be made?
- (b) What is the objective or target for these decisions?
- (c) Set up the above problem in Excel using the style that we used in class. Make sure that your spreadsheet model is logical, well organized and easy to understand. In building formulas, use the sumproduct function where appropriate.
- (d) Solve the problem using Solver. What would be your recommendation to management in terms of the decisions that need to be made?
- (e) What will be the Total Cost of producing 5 tons of steel?
- (f) After observing the model and its solver output,
 - (1) What is the tensile strength of the finished steel?
 - (2) Which constraints are binding?
 - (3) What is the percentage of nickel in the steel produced ?
- (g) There was a requirement that it was necessary to produce 1 ton of steel. If that restriction was removed from the problem, would the total cost increase, decrease or stay the same? Explain.
- (h) You will notice that the Solver model shows a decimal answer for several of the changing cells. Briefly comment on the correct way to interpret these decimal numbers.
- (i) Place a text box on your model page which includes information regarding the optimal decision.
- (j) Print out your spreadsheet model on one page in landscape orientation and include it with your answers.
- (k) Print out the cell formulas on one page in landscape orientation showing Row and Column Headings and include it with your answers.
- (l) Formulate the above model algebraically. Be sure to clearly define your decision variables and show your complete linear programming formulation. **DO NOT SOLVE.**

Problem 3 (10 marks)

List (but do not describe) the four reasons given in the textbook for studying management science.

Question 2 (43 marks)

Steelco, Question 61, Chapter 4

(a) The 2 decisions facing Steelco are:

- How many tons of Alloy 1 to use in the production of 1 ton of steel*
- How many tons of Alloy 2 to use in the production of 1 ton of steel*

} 2 marks

(b) The target is **Minimize Total Cost** (Total Cost)

} 1 mark

c) See attached Model2

} 10 marks

(d) The 1 ton of steel should consist of

- 62.5% of Alloy 1 and 37.5% of Alloy2**
- or**
- 625 pounds of Alloy1 and 375 pounds of Alloy2**

} 2 marks

If students assumed that the tons listed in the problem were metric tons, then pounds would be replaced with kilograms

(e) **\$968.75 (\$193.75 x 5)**

} 1 mark

(f)

- (1) 45,000 psi
- (2) There are 2 Binding constraints: Tensile Strength and Amount produced must equal 1 ton
- (3) 1.1875%

} 1 mark
1 mark
1 mark

(g) Total cost would **DECREASE to \$0.00**

because this constraint is binding

and removing it makes the constraints easier to meet, therefor less cost

(In this case, you would produce no steel to achieve min cost)

} 2 marks
1 mark
1 mark

(h) Decimal numbers are OK because # tons needed can be fractional

There is no practical reason to restrict the changing cells to be integers

} 2 marks

(i) See Model2 for textbox

} 1 mark

(j) See Model2

} 2 marks

(k) See Formulas2

} 3 marks

(l) See Sheet: Algebraic Formulation

} 12 marks; see mark breakdown on alg formulation sheet

Problem #2
Steelco (SMA Chapter 4, Question 61)

Input Data

	Alloy 1	Alloy 2	Total Cost	Minimum Model Req	Model Output	Max Model Req	
Cost per ton	\$190	\$200	\$ 193.75	0.018	0.021875	0.025	
Percent silicon	2.0%	2.5%		0.009	0.011875	0.012	percentage
Percent nickel	1.0%	1.5%		0.032	0.03375	0.035	percentage
Percent carbon	3.0%	4.0%		45	45		psi
Tensile strength (1000's psi)	42	50					

	Min Model Requirement	Max Model Requirement	
Percent silicon	0.018	0.025	percentage
Percent nickel	0.009	0.012	percentage
Percent carbon	0.032	0.035	percentage
Tensile strength (1000's psi)	45		psi

Action Plan

Inputs to 1 ton of Steel	Alloy 1	Alloy 2	Total
	0.625	0.375	1

You must produce 1 ton of steel

Marking Guide:

- * This sheet is worth 10 marks for the Excel model (part c) + 2 marks for just printing the page in landscape (part j) - deduct 1 mark if it is printed in Portrait)
- * Deduct a maximum of 1 mark if students do not use some formatting (bold, shading ,borders etc)
- * Do NOT deduct any marks if their model layout is well organized, easily understood by you the marker, and they show at least one formatting activity.
- * 1 mark should be allocated to the 1 textbox (part i)
- * 1 mark should be allocated to titles (Model requirement, % silicon, Alloy1, etc)

Part (i)

Optimal Decision:

To produce 1 ton of steel you need to mix 62.5% of Alloy 1 and 37.5% of Alloy 2.

OR

to produce 1 ton of steel you need to mix 625 pounds of Alloy 1 and 375 pounds of Alloy 2

	A	B	C	D	E	F	G	H	I	J
1	Problem #2									
2	Steelco (SMA Chapt)									
3										
4	Input Data									
5										
6										
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22	Action Plan									
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Total Cost = $\text{SUMPRODUCT}(B7:C7, \$B\$24:\$C\$24)$
 Minimum Model Req = $=B16*\$F\24
 Maximum Model Req = $=C16*\$F\24
 Model Output = $\text{SUMPRODUCT}(B8:C8, \$B\$24:\$C\$24)$
 Model Output = $\text{SUMPRODUCT}(B9:C9, \$B\$24:\$C\$24)$
 Model Output = $\text{SUMPRODUCT}(B10:C10, \$B\$24:\$C\$24)$
 Model Output = $\text{SUMPRODUCT}(B11:C11, \$B\$24:\$C\$24)$

Alloy 1 Alloy 2 Total
 0.625 0.375 = 1 tons

Marking Guide: Part k
 * This printing of this formula page is worth 3 marks in Total
 * deduct a maximum of 2 marks if the formulas are cutoff: ie: $\text{SUMPRODUCT}(B8:C8, \$B..$
 * Formulas other than sumproduct are OK; Do not deduct.
 * deduct 1 mark if sheet is not printed in Landscape

Algebraic Formulation (SMA Question 61, Chapter 4, Steelco)

Let A_1 = # tons of Alloy1 to put in 1 ton of steel
 A_2 = # tons of Alloy2 to put in 1 ton of steel

OR

A_1 = % of Alloy1 in 1 ton of steel
 A_2 = % of Alloy2 in 1 ton of steel

} 2 marks

Minimize $190A_1 + 200A_2$
 subject to:

} 1 mark

$$.02A_1 + .025A_2 \geq .018$$

$$.01A_1 + .015A_2 \geq .009$$

$$.03A_1 + .04A_2 \geq .032$$

$$42A_1 + .50A_2 \geq 45$$

$$.02A_1 + .025A_2 \leq .025$$

$$.01A_1 + .015A_2 \leq .012$$

$$.03A_1 + .04A_2 \leq .035$$

$$A_1 + A_2 = 1$$

$$A_1, A_2 \geq 0$$

} 9 marks (1 marks each)

Question 3 (10 marks)

List (but do NOT describe) the four reasons given in the textbook for studying management science.

Answer:

1. Development of Logical Modeling Skills
2. Development of Quantitative Skills
3. Development of Spreadsheet Skills
4. Development of Intuition

Source: Page 15-16 Custom Publication for Commerce 290

Marking key:

10 marks

Although we would expect that students would understand the details under each of these steps, this question asks them only to "list" the 4 reasons, not provide explanations, descriptions or details.

If they do provide correct details or explanations, do NOT deduct any marks.