Question 1 (7 marks)
Find the volume of the region between the paraboloids $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$.

Question 2 (8 marks)
Exactly one of the following 2-dimensional vector fields is a gradient field. Determine which one it is and explain why the others are not gradient vector fields. What is the potential function for the gradient vector field ?
$\vec{F}(x,y) = xe^y\,\hat{\imath} + xe^y\,\hat{\jmath}$
$\vec{G}(x,y) = e^y\,\hat{\imath} + xe^y\,\hat{\jmath}$
$\vec{H}(x,y) = x\,\hat{\imath} + xe^y\,\hat{\jmath}$

Question 3 (8 marks)
Let S be the part of the cone $z = \sqrt{x^2 + y^2}$ where $1 \leq z \leq 4$, oriented downward. Find the flux of $\vec{F}(x, y, z) = 2xz\,\hat{\imath} - yz\,\hat{\jmath} + (z - 3y^2)\,\hat{k}$ through S.

Question 4 (8 marks)
Find the flux of $\vec{F}(\vec{r}) = \vec{r}$ through the sphere of radius R centred at the origin and oriented outwards in two ways:
a) directly and b) use the Divergence Theorem.

Question 5 (8 marks)
Let $\vec{F}(x, y, z) = (\pi y z^4 - 2y)\,\hat{\imath} + (\pi x z^4 + 2x)\,\hat{\jmath} + (4\pi x y z^3 - z\cos(z))\,\hat{k}$ and let C be the circle of radius 4 in the xy plane, centred at the origin and oriented counterclockwise when viewed from above. Calculate $\oint_C \vec{F} \cdot d\vec{r}$ in two ways:
a) directly and b) use Stokes' Theorem.

Question 6 (7 marks)
Find the flux of $\vec{F}(x, y, z) = 2y^2 z\,\hat{\imath} - xyz\,\hat{\jmath} + x^2yz^3\,\hat{k}$ through the negative x part of the cylinder $x^2 + y^2 = 9\,,\ 0 \leq z \leq 1$, oriented outward.

Question 7 (8 marks)
Consider the vector field $\vec{F}(x,y,z) = e^x yz\,\hat{\imath} - \frac{3}{2}e^x y^2 z\,\hat{\jmath} + e^x yz^2\,\hat{k}$.
a) Find the divergence of $\vec{F}$. What would the Divergence Test tell you about $\vec{F}$?
b) Find the curl of $\vec{F}$. What would the Curl Test tell you about $\vec{F}$?

Question 8 (6 marks)
Calculate the line integral of $\vec{F}(x,y) = (1+2x)\,\hat{\imath}+2y\,\hat{\jmath}$ along the path C, where C consists of the straight line from $(0,0)$ to $(-3,0)$ and then a circular arc (part of the circle $x^2+y^2 = 9$) from $(-3,0)$ to $(0,-3)$.