University of Ottawa
ECO 3145 Mathematical Economics II
Fall 2012, Professor Shiell

Exam II (midterm)

Total marks: 63.

1. Consider the system of equations
   $xz^3 + y^2v^4 = 2$
   $xz + yvz^2 = 2$

   a.) (7) Does the system implicitly define v and z as functions of x and y in the neighbourhood of the point $(1, 1, 1, 1)$? Justify your answer.
   b.) (6) If yes, find an expression for $\partial v/\partial x$.

2. (5) Consider the function $f(x) = \ln(x)$. Construct the second-order Taylor series approximation of this function around the point $x = 1$.

3. Consider the manager of a toothpaste factory who wishes to minimize the cost of producing $\overline{Q}$ tubes of toothpaste. Suppose the inputs are capital, K, and labour, L, with prices P_K and P_L. Therefore the cost of production is $K \cdot P_K + L \cdot P_L$. The production function is $Q = K^{0.5}L^{0.5}$. The manager's problem is to choose K and L to minimize $K \cdot P_K + L \cdot P_L$ subject to the constraint $K^{0.5}L^{0.5} = \overline{Q}$.

   a.) (6) What are the candidates for the optimal choices of L and K?
   b.) (12) Show that the choices you have found above give a local minimum, by checking the second-order conditions (i.e. bordered Hessian).
   c.) (3) Use the envelope theorem to derive the marginal cost of production.

4. (6) Based on the concept of the "better-than set" $(S^{\geq})$, determine whether the following function is quasi-concave: $f(x_1, x_2) = 6x_1 - 9x_2$. Sketch the better-than set in your answer.

5. (18) Find the pair (x_1, x_2) that minimizes $f(x_1, x_2) = x_1^2 + x_2^2$ subject to $x_1x_2 \geq 25$ and $x_1, x_2 \geq 0$.

Some useful (and some useless) formulas and definitions

$$f(b) = f(a) + f'(a)(b-a) + \tfrac{1}{2!}f''(a)(b-a)^2 + \ldots + \tfrac{1}{n!}f^{(n)}(a)(b-a)^n + \tfrac{1}{(n+1)!}f^{(n+1)}(c_n)(b-a)^{n+1}$$

$$f\left(x_1^*(\beta_1,\ldots,\beta_m),\ldots,x_n^*(\beta_1,\ldots,\beta_m);\beta_1,\ldots,\beta_m\right) = V(\beta_1,\ldots,\beta_m)$$

$$S^{\geq} = \{(x_1,\ldots,x_n)\,|\,f(x_1,\ldots,x_n) \geq k\}$$

$$R_2(b,a) = \frac{1}{3!}f'''(c_2)(b-a)^3$$

$$(-1)^n \qquad (-1)^m$$

$$\frac{\partial V(\beta_1,\ldots,\beta_m)}{\partial \beta_i} = \frac{\partial L}{\partial \beta_i}$$

$$f(b) \geq f(a) \;\Rightarrow\; f[\theta a + (1-\theta)b] \geq f(a)$$

$$|J| \neq 0$$

$$f_i - f_n \frac{g_i}{g_n} = 0$$

$$f(b) \geq f(a) \;\Rightarrow\; \sum_{j=1}^{n} f_j(a)(b_j - a_j) \geq 0$$

$$\lambda^* = \frac{f_i(x^*)}{g_i(x^*)}$$

$$\frac{\partial V(\beta_1,\ldots,\beta_m)}{\partial \beta_i} = f_{x_1}\frac{\partial x_1^*}{\partial \beta_i} + \ldots + f_{x_n}\frac{\partial x_n^*}{\partial \beta_i} + f_{\beta_i}$$