

**MATH 151**    Instructor: R. Pyke  
**Final Examination, December, 2008**

|                     |         |
|---------------------|---------|
| Last Name:          |         |
| First Name:         |         |
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1. DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED.
2. Clearly explain your answer. No credit will be given for just writing down the answer.
3. If the answer space provided is not sufficient, write your answer on the back of the previous page.
4. Ordinary Scientific Calculators ONLY are allowed.  
NO GRAPHING CALCULATORS ALLOWED.
5. **Copying someone else's test, or deliberately exposing written papers to the view of others is forbidden and will result in a score of zero and disciplinary action.**

| Question | Score | Max |
|----------|-------|-----|
| 1        |       | 12  |
| 2        |       | 12  |
| 3        |       | 6   |
| 4        |       | 8   |
| 5        |       | 6   |
| 6        |       | 6   |
| 7        |       | 6   |
| 8        |       | 12  |

| Question | Score | Max |
|----------|-------|-----|
| 9        |       | 9   |
| 10       |       | 6   |
| 11       |       | 4   |
| 12       |       | 4   |
| 13       |       | 7   |
| 14       |       | 9   |
| 15       |       | 6   |
| Total    |       | 113 |

- (1) [Marks: 12] Use the limit laws to find the following limits or explain why they don't exist.

(a)  $\lim_{x \rightarrow 3} \frac{\sqrt{3x} - 3}{|3 - x|}$

(b)  $\lim_{x \rightarrow \infty} x \tan\left(\frac{2}{3x}\right)$

(c)  $\lim_{x \rightarrow 0^+} 2x^{\sqrt{x}}$

(d)  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \ln(\tan x)}{3 \sin(4x)}$

(2) [Marks: 12] Find the derivatives of the following functions.

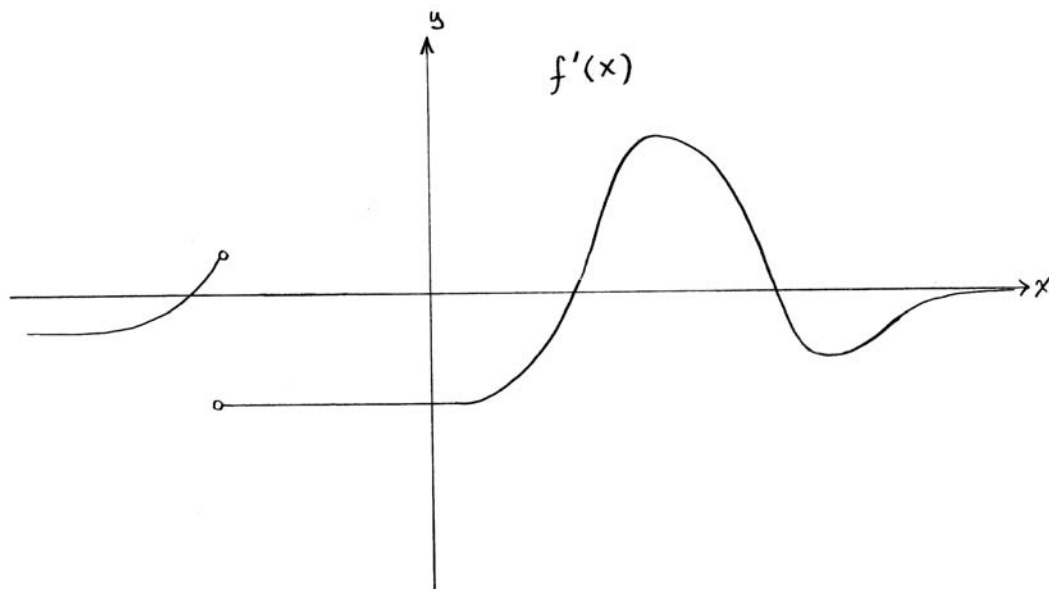
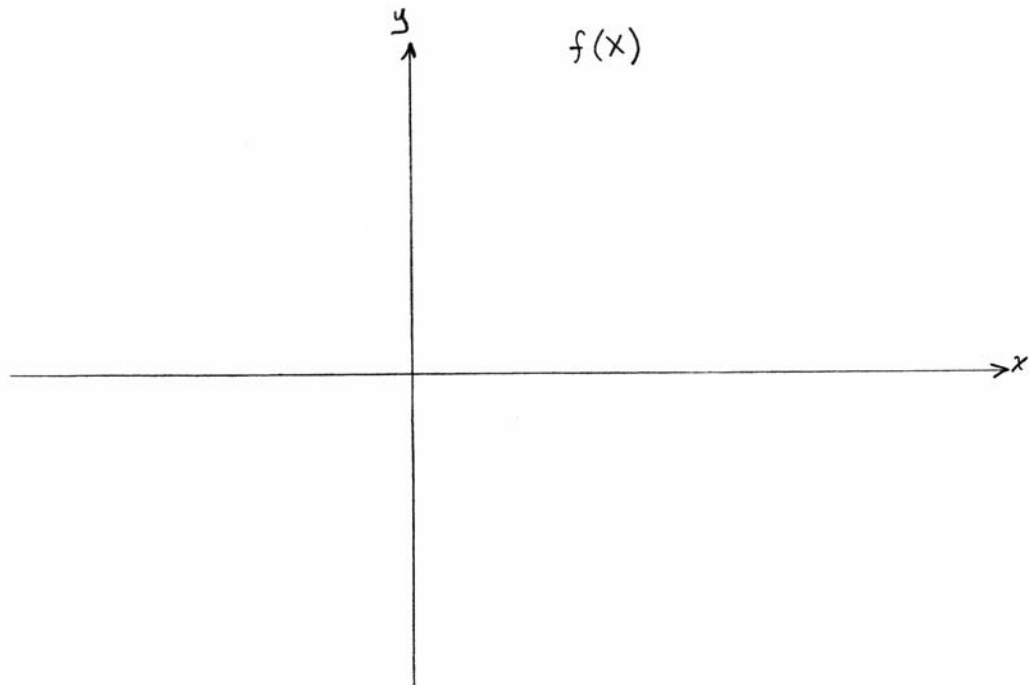
(a)  $f(x) = 1 - 3^x + x^3$

(b)  $g(z) = \sqrt{\arctan(\sqrt{z})}$

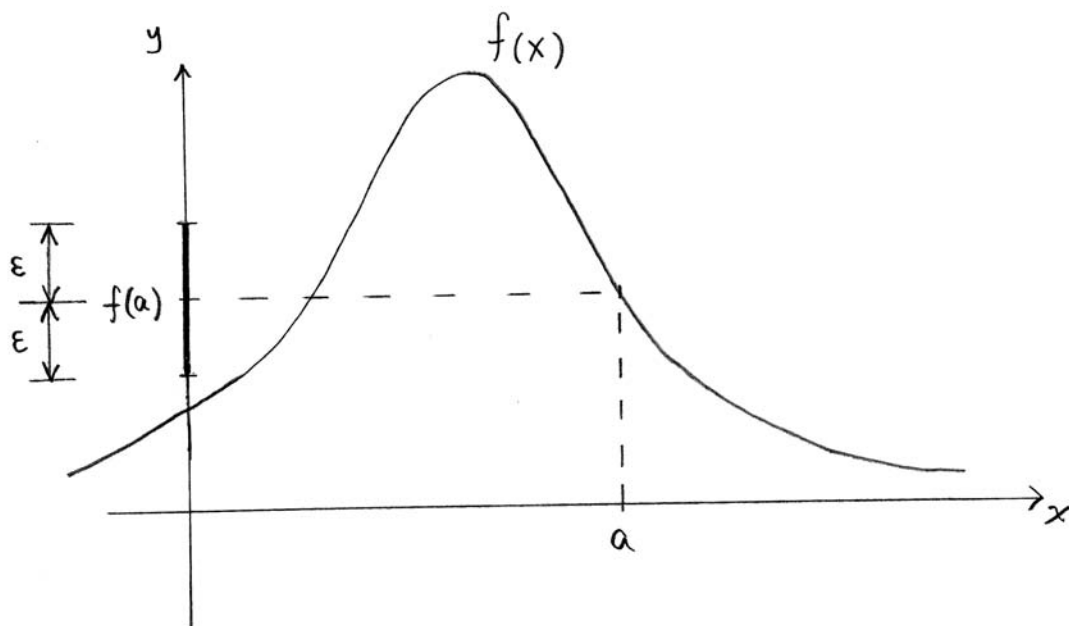
(c)  $y = x - \log_2(\sqrt{x^2 + 2}) \cos(2x)$

(d)  $h(t) = \sqrt{\frac{2t^2 + 1}{t^2 - 2}}$

- (3) [Marks: 6] Below is the graph of  $f'(x)$ , the derivative of a function  $f(x)$ . Above it sketch the graph of  $f(x)$ . (Did you read this question carefully?)



- (4) [Marks: 8] (a) Here is a graph of a function  $f(x)$  along with an interval of length  $2\varepsilon$  centred at the point  $f(a) = L$ . Sketch an interval  $I$  around  $a$  such that if  $x \in I$  then  $|f(x) - L| < \varepsilon$ .



- (b) Use the precise  $\varepsilon - \delta$  method to prove that

$$\lim_{x \rightarrow 1} \frac{x^2 + 1}{2x} = 1$$

- (5) [Marks: 6] Find values of the constants  $a$  and  $b$  so that the following function is differentiable everywhere.

$$f(x) = \begin{cases} ax^2 + bx & x < 1 \\ (x - 2)^2 + 3 & x \geq 1 \end{cases}$$

Illustrate your answer by sketching the resulting  $f(x)$ .

(6) [Marks: 6] (a) State the Mean Value Theorem. Make a sketch illustrating the theorem.

(b) Use the Mean Value Theorem to prove that the equation  $2x^4 + x = 3$  has at most two solutions.

- (7) [Marks: 6] Find the slant asymptote of the function  $f(x) = 5x + 2 - \sqrt{9x^2 + x + 1}$  as  $x \rightarrow +\infty$  (only this one limit).

- (8) [Marks: 12] Make a sketch of the graph of  $f(x) = \frac{2x^2 - 3x}{(x + 5)^2}$  by following the steps below. Here are the first and second derivatives of  $f(x)$ ;

$$f'(x) = \frac{23x - 15}{(x + 5)^3}, \quad f''(x) = \frac{-46x + 160}{(x + 5)^4}$$

(a) What is the domain of  $f(x)$ ?

(a) Find the critical points of  $f(x)$  and classify them using the first derivative test.

(b) Find regions where  $f(x)$  is increasing or decreasing.

(c) Find any asymptotes. Be sure to determine how the graph of  $f(x)$  approaches the asymptote.

(d) Find any points of inflection and determine regions where the graph is concave up or concave down.

(e) Use this information to make a sketch of  $f(x)$  below.

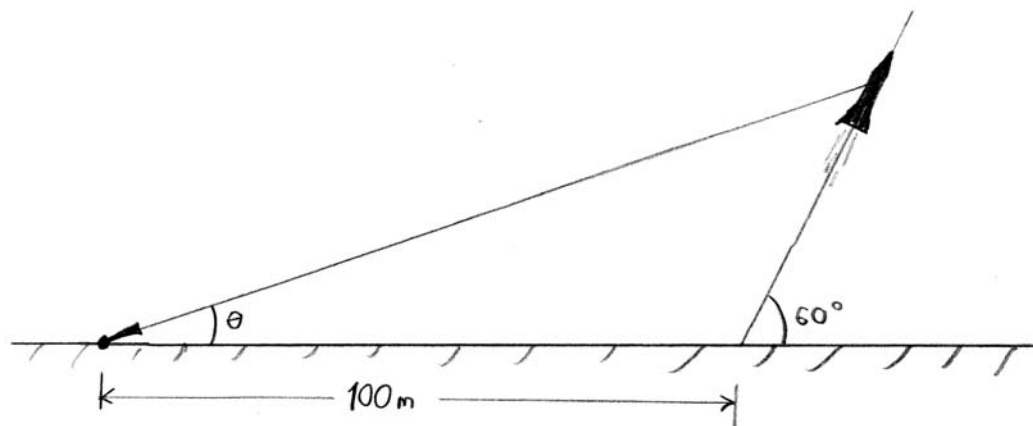
(9) [Marks: 9] Consider the curve defined implicitly by the equation  $y^2 = x^3 + 3x^2$ .

(a) Find two points on the curve where the tangent line is horizontal.

(b) Find a point on the curve, but not the origin  $(0,0)$ , where the tangent line is vertical.

(b) Is the curve concave up or concave down at the point  $(1,2)$ ?

- (10) [Marks: 6] A video camera on the ground is following a rocket that has been launched 100 metres away. The rocket is following a straight line that is inclined  $60^\circ$  from the horizontal at a speed of 120 metres per second (see diagram below).



Find an expression that would tell you how quickly (in radians per second) the angle  $\theta$  of elevation of the camera is changing 2 minutes after launch. Do not calculate this number: only provide the formula that would allow you to do so. You must also show how all quantities appearing in your formula would be calculated. You may need (some of) these identities;

$$\sin^2 x + \cos^2 x = 1, \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}, \quad a^2 = b^2 + c^2 - 2bc \cos A$$

(the next page is blank if you need it for your solution)

Problem (10), continued

- (11) [Marks: 4] Find the absolute maximum and absolute minimum values of  $f(x) = x^2 - \ln x$  on the interval  $[\frac{1}{2}, 2]$ .

- (12) [Marks: 4] A closed box with a square base is to be built. The cost of the material for the 4 sides is \$20 per square metre and the cost of the material for the top and bottom is \$50 per square metre.

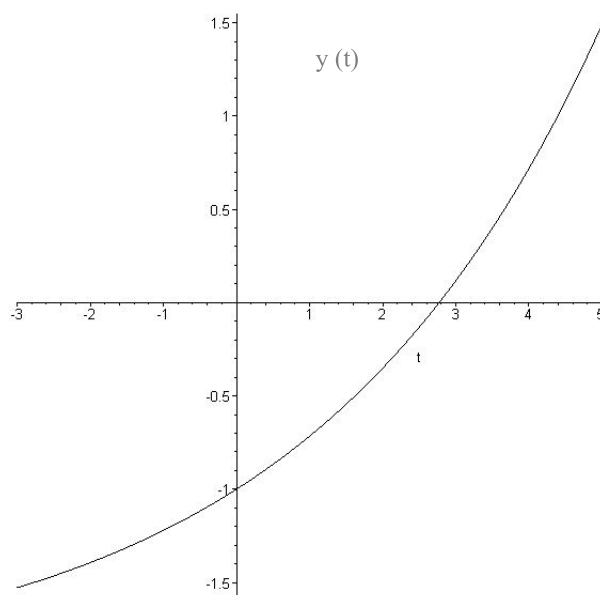
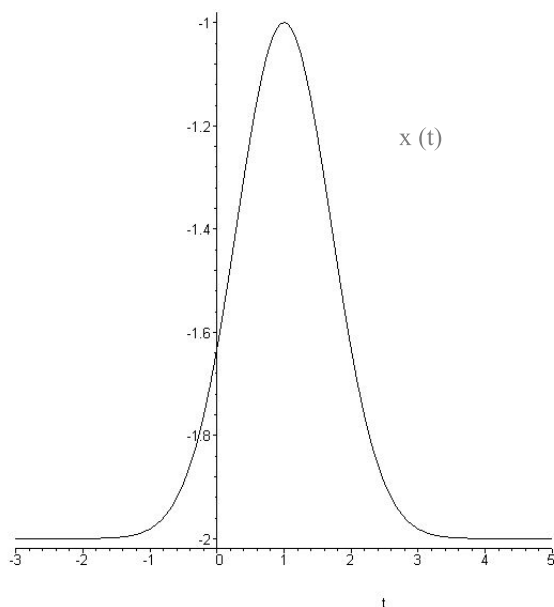
If you have \$100 to spend for material to build the box, what is the maximum volume of box that you can build?

- (13) [Marks: 7] Make a sketch that shows that the graphs of  $\tan x$  and  $x^3 + 1$  intersect at some positive  $x$ -value. Then find an approximation to this value by the following methods;

(a) Use the bisection method beginning with the interval  $[0, 1.5]$  to find an interval of length less than 0.5 that contains a solution.

(b) Apply Newton's method for one iteration starting with your approximation in part (a). (If you didn't get (a), use 1 as a starting point.)

(14) [Marks: 9] Consider functions  $x(t)$  and  $y(t)$  given below;



(a) Make a sketch of the parametric curve  $(x(t), y(t))$ .

(b) Using the graphs of  $x(t)$  and  $y(t)$  (not any formula for  $x(t), y(t)$  you may guess at), find a point on the parametric curve where the tangent line has slope 2. (Hint: Remember the formula for  $dy/dx$ .) Explain your reasoning by referring to the graphs.

- (15) [Marks: 6] (a) Make a sketch of the polar curve  $r = \cos\left(2t + \frac{\pi}{2}\right) - 1$ . Be sure to indicate directions which determine special features of the graph, the size of the graph, and any points where the curve crosses the  $x$  and  $y$  axis.