CARLETON UNIVERSITY
Department of Economics

ECON 4301A
Market Structure and Firm Behaviour

2012 Fall

ASSIGNMENT # 2 - SOLUTIONS

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1. (a) To derive the best-response functions for each firm, maximize the firm's profits given the amount the other firm is producing.

Firm 1:

$$\underset{\{q_1\}}{\text{Max}}\, \pi_1 = (100 - q_1 - q_2)(q_1) - 20 \cdot q_1$$

The first-order condition is given by:

$$\frac{\partial \pi_1}{\partial q_1} = 100 - 2q_1 - q_2 - 20 = 0$$

2 marks Thus, Firm 1's best-response function is:

$$\boxed{q_1 = \frac{80 - q_2}{2}} \tag{1}$$

Firm 2:

$$\underset{\{q_2\}}{\text{Max}}\, \pi_2 = (100 - q_1 - q_2)(q_2) - 30 \cdot q_2$$

The first-order condition is given by:

$$\frac{\partial \pi_2}{\partial q_2} = 100 - q_1 - 2q_2 - 30 = 0$$

2 marks Thus, Firm 1's best-response function is:

$$\boxed{q_2 = \frac{70 - q_1}{2}} \tag{2}$$

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2 marks

(b) Graphing equations (1) and (2) yields:

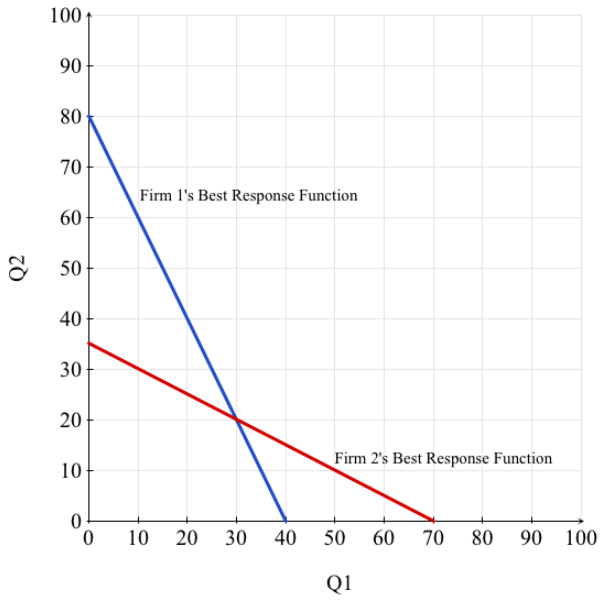

Figure 1: Best Response Functions

(c) The Nash Equilibrium occurs where both firms are doing the best they can given the level of output produced by the other firm (i.e. neither firm has an incentive to change their output level given the amount produced by the other firm). This occurs where the two best-response functions are equal to each other. Substituting equation (2) into (1):

$$q_1 = 40 - \left(\frac{1}{2}\right)\left(\frac{70 - q_1}{2}\right)$$

Simplifying:

$$\begin{aligned} q_1 &= \frac{160 - 70 + q_1}{4} \\ 4q_1 - q_1 &= 90 \\ 3q_1 &= 90 \end{aligned}$$

2 marks

$$\boxed{q_1 = 30} \tag{3}$$

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Substituting equation (3) into equation (2):

$$q_2 = \frac{70 - 30}{2}$$

2 marks

$$\boxed{q_2 = 20} \tag{4}$$

Substituting equations (3) and (4) into the demand function yields:

$$P = 100 - 30 - 20$$

2 marks

$$\boxed{P = 50} \tag{5}$$

(d) If the two firms collude and act as a monopolist, then the industry output would fall and the price would increase.

Note that if the two firms are colluding then they are better off using the low cost firm's production technology. Thus, the marginal cost in the monopoly case is 20. There is no issue with this transformation because there are no capacity constraints and at no level of output are the two firms better off to switch facilities. (Note: think whether this would still be the case if $MC_2 = q_2^2$).

The maximization problem now becomes:

$$\underset{\{Q\}}{\text{Max}}\, \pi = (100 - Q) \cdot Q - 20 \cdot Q$$

The first-order condition is given by:

$$\frac{\partial \pi}{\partial Q} = 100 - 2Q - 20 = 0$$

2 marks

Thus, the equilibrium quantity is:

$$\boxed{Q = 40} \tag{6}$$

Substituting equation (6) into the demand function yields the following equilibrium price:

$$P = 100 - 40$$

2 marks

$$\boxed{P = 60} \tag{7}$$

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To determine why the collusive outcome is not a Nash equilibrium, we need to look at the payoff matrix with the following scenarios:

- Both firms collude
- Firm 1 cheats
- Firm 2 cheats
- Both firms cheat and compete over quantity (i.e. Cournot competition)

i. *Both firms collude*
In this situation, both firms split the monopoly profits

$$\begin{aligned} \pi &= (P - MC) \cdot Q \\ \pi &= (60 - 20) \cdot 40 \end{aligned}$$

2 marks

$$\boxed{\pi_1 = 800} \qquad \boxed{\pi_2 = 800}$$

ii. *Firm 1 cheats*
If Firm 1 cheats, then Firm 2 produces the collusive amount – $q_2 = 20$. Then use Firm 1's best-response function given by (1) to determine how much Firm 1 produces.

$$\begin{aligned} q_1 &= \frac{80 - q_2}{2} \\ q_1 &= \frac{80 - 20}{2} \\ q_1 &= 30 \end{aligned}$$

2 marks

Note that Firm 2 uses Firm 1's technology because they are colluding – $MC_2 = 20$.

$$\begin{aligned} \pi_1 &= (P - MC_1) \cdot q_1 \\ \pi_1 &= (50 - 20) \cdot 30 \end{aligned} \qquad \begin{aligned} \pi_2 &= (P - MC_1) \cdot q_2 \\ \pi_2 &= (50 - 20) \cdot 20 \end{aligned}$$

$$\boxed{\pi_1 = 900} \qquad \boxed{\pi_1 = 600}$$

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iii. *Firm 2 cheats*

If Firm 2 cheats then they have to use their higher cost production technology. Firm 1 will produce $q_1 = 20$. Substituting into Firm 2's best-response function (2) yields:

$$\begin{aligned} q_2 &= \frac{70 - q_1}{2} \\ q_2 &= \frac{70 - 20}{2} \\ q_2 &= 25 \end{aligned}$$

Substituting q_1 and q_2 into the demand function yields:

$$P = 100 - 20 - 25$$

$$P = 55$$

2 marks

Note that Firm 2 uses Firm 1's technology because they are colluding – $MC_2 = 20$.

$$\begin{aligned} \pi_1 &= (P - MC_1) \cdot q_1 \\ \pi_1 &= (55 - 20) \cdot 20 \end{aligned} \qquad \begin{aligned} \pi_2 &= (P - MC_2) \cdot q_2 \\ \pi_2 &= (55 - 30) \cdot 25 \end{aligned}$$

$$\boxed{\pi_1 = 700} \qquad \boxed{\pi_2 = 625}$$

iv. *Both firms cheat*

2 marks

Using the results from part (c) we have:

$$\begin{aligned} \pi_1 &= (P - MC_1) \cdot q_1 \\ \pi_1 &= (50 - 20) \cdot 30 \end{aligned} \qquad \begin{aligned} \pi_2 &= (P - MC_2) \cdot q_2 \\ \pi_2 &= (50 - 30) \cdot 20 \end{aligned}$$

$$\boxed{\pi_1 = 900} \qquad \boxed{\pi_2 = 400}$$

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2 marks

The payoff matrix is thus given by:

Table 1: Payoff Matrix for Collusive Game

		Firm 2	
		Collude	Cheat
Firm 1	Collude	(800,800)	(700,625)*
	Cheat	*(900,600)*	*(900,400)

4 marks

Therefore, the Nash Equilibrium is (CHEAT,COLLUDE). The collusive outcome is not a Nash equilibrium because Firm 1 can always do better by expanding their output. However, notice now that Firm 2 always wants to collude. The reason is that it gives them access to a technology that allows them to produce at a lower cost. This benefit outweighs the loss to Firm 2 of Firm 1 cheating on the collusive agreement and increasing the amount they produce.

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2. (a) In order to determine the critical discount factor we need to determine the payoff matrix with the following scenarios:

   - Both firms collude
   - Firm 1 cheats
   - Firm 2 cheats
   - Both firms cheat and compete over prices (i.e. Bertrand competition)

   i. *Both firms collude*
   In this situation, both firms split the monopoly profits. The maximization problem now becomes:

$$\underset{\{Q\}}{\text{Max}}\, \pi = (130 - Q) \cdot Q - 10 \cdot Q$$

   The first-order condition is given by:

$$\frac{\partial \pi}{\partial Q} = 130 - 2Q - 10 = 0$$

   Thus, the equilibrium quantity is:

$$\boxed{Q = 60} \tag{8}$$

   Substituting equation (8) into the demand function yields the following equilibrium price:

$$P = 130 - 60$$

$$\boxed{P = 70} \tag{9}$$

   The monopoly profits are:

$$\begin{aligned} \pi &= (P - MC) \cdot Q \\ \pi &= (70 - 10) \cdot 60 \\ \pi &= 3600 \end{aligned}$$

   Thus, the profits for each firm if they split the monopoly profits are:

1 mark

$$\boxed{\pi_1 = 1800} \quad \boxed{\pi_2 = 1800}$$

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ii. *Firm 1 cheats*
In this situation, Firm 2 sets a price of $P_2 = 70$. Firm 1 can undercut Firm 2 and charge a price of $P_1 = 70 - \varepsilon$ and serve the entire market. Thus, Firm 1 receives approximately the monopoly profits and Firm 2 earns zero profits:

2 marks

$$\boxed{\pi_1 \approx 3600} \quad \boxed{\pi_2 = 0}$$

iii. *Firm 2 cheats*
In this situation, Firm 1 sets a price of $P_1 = 70$. Firm 2 can undercut Firm 1 and charge a price of $P_2 = 70 - \varepsilon$ and serve the entire market. Thus, Firm 2 receives approximately the monopoly profits and Firm 1 earns zero profits:

2 marks

$$\boxed{\pi_1 = 0} \qquad \boxed{\pi_2 \approx 3600}$$

iv. *Both firms cheat*
In this situation each firm will continue to undercut the other firm's price until price equals marginal cost. Thus, $P_1 = P_2 = 10$. Since price equals marginal cost, each firm's profits will be equal to zero.

2 marks

$$\boxed{\pi_1 = 0} \qquad \boxed{\pi_2 = 0}$$

2 marks The payoff matrix is thus given by:

Table 2: Payoff Matrix for Collusive Game

		Firm 2	
		Collude	Cheat
Firm 1	Collude	(1800,1800)	(0,3600)
	Cheat	(3600,0)	(0,0)

The critical discount factor that sustains collusion as a Nash Equilibrium to an infinitely repeated game is determined by comparing the payoff a firm receives colluding and splitting the monopoly profit (i.e. V^M), with the payoff the firm receives from cheating (i.e. V^C).

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The payoff a firm receives from always colluding is:

$$V^M = \sum_{t=1}^{\infty} \delta^{t-1} \cdot 1800$$

$$V^M = \frac{1800}{1-\delta}$$

The payoff a firm receives from cheating is:

$$V^C = 3600 + \sum_{t=2}^{\infty} \delta^{t-1} \cdot 0$$

$$V^C = 3600$$

1 mark

Collusion can be sustained if:

$$\begin{aligned} V^M &\geq V^C \\ \frac{1800}{1-\delta} &\geq 3600 \\ 1800 &\geq 3600 - 3600 \cdot \delta \\ 3600\delta &\geq 1800 \end{aligned}$$

$$\boxed{\delta \geq 0.50}$$

2 marks

(b) Bertrand competition is more likely to sustain collusion because the punishment is harsher ($0 < 1600$), and is also more than sufficient to make up for the increased incentive to deviate under Bertrand competition (i.e. if the firm cheats they receive monopoly profits).

* * *

QUESTION # 3 : 15 MARKS

(a) The demand functions for the two firms are:

$$q_1 = 24 - 5p_1 + 2p_2 \Rightarrow MR_1 = 24 - 10p_1 + 2p_2$$
$$q_2 = 24 - 5p_2 + 2p_1 \Rightarrow MR_2 = 24 - 10p_2 + 2p_1$$

- Firm 1 maximizes profit by setting:

$$MR_1 = MC_1$$

(2) $$24 - 10p_1 + 2p_2 = 0$$
$$24 + 2p_2 = 10p_1$$

$$\therefore \boxed{p_1 = \frac{24 + 2p_2}{10}}$$

- Firm 2 maximizes profit by setting:

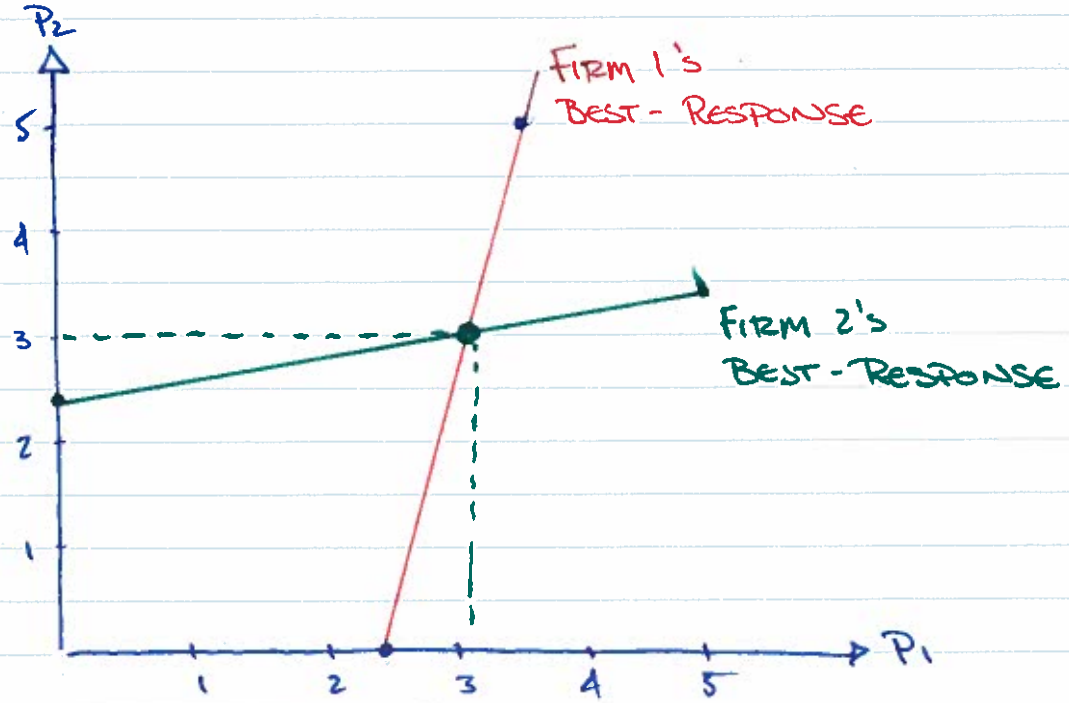
$$MR_2 = MC_2$$

(2) $$24 - 10p_2 + 2p_1 = 0$$
$$24 + 2p_1 = 10p_2$$

$$\therefore \boxed{p_2 = \frac{24 + 2p_1}{10}}$$

(b)

(3)



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(c) Substituting P_1 into P_2 yields:

$$P_2 = \frac{24}{10} + \frac{2}{10}\left[\frac{24}{10} + \frac{2}{10}P_2\right]$$

$$P_2 = \frac{240}{100} + \frac{48}{100} + \frac{4}{100}P_2$$

(2) $$\frac{96}{100}P_2 = \frac{288}{100} \quad \therefore \boxed{P_2^* = 3}$$

$$\Rightarrow P_1 = \frac{24 + 2P_2}{10}$$

(2) $$P_1 = \frac{24 + 2(3)}{10} \quad \therefore \boxed{P_1^* = 3}$$

$$\Rightarrow q_1 = 24 - 5P_1^* + 2P_2^*$$
$$q_1 = 24 - 5(3) + 2(3) \quad \therefore q_1^* = 15$$

$$\Rightarrow q_2 = 24 - 5P_2^* + 2P_1^*$$
$$q_2 = 24 - 5(3) + 2(3) \quad \therefore q_2^* = 15$$

- The profits for firm 1 are:

$$\pi_1 = P_1 \cdot q_1 - MC_1 \cdot q_1$$
$$\pi_1 = (3)(15) - (0)(15)$$

(2) $$\pi_1 = 45 - 0 \quad \therefore \boxed{\pi_1^* = 45}$$

- The profits for firm 2 are:

$$\pi_2 = P_2 \cdot q_2 - MC_2 \cdot q_2$$
$$\pi_2 = (3)(15) - (0)(15)$$

(2) $$\pi_2 = 45 - 0 \quad \therefore \boxed{\pi_2^* = 45}$$

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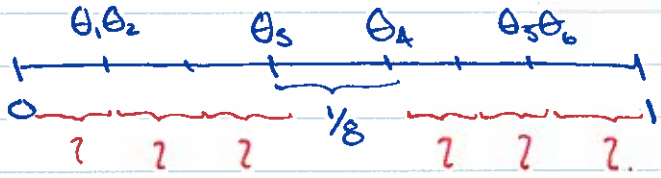
QUESTION # 4 : 15 MARKS

(a) We want to determine the equilibrium location of the 6 firms when the distance between the two most interior firms is fixed at 1/8.

- For a Nash equilibrium in locations, we must have:
  - (i) No firm's market length is less than any other firm's half-market length.
  - (ii) The two peripheral firms must be paired, to an interior firm.

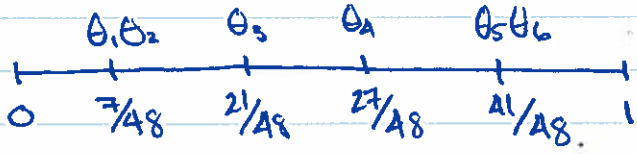


- In order to satisfy condition (i) all of the lengths ℓ must be equal. This implies that:

$$6\ell + 1/8 = 1$$
$$6\ell = 7/8 \quad \therefore \boxed{\ell^* = 7/48}$$

- Therefore, the equilibrium locations are:

(5)



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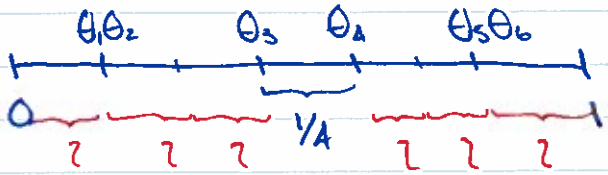
(b) Now, the distance is equal to 1/4:

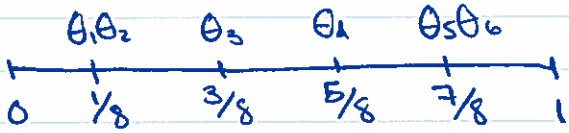


- This implies that:

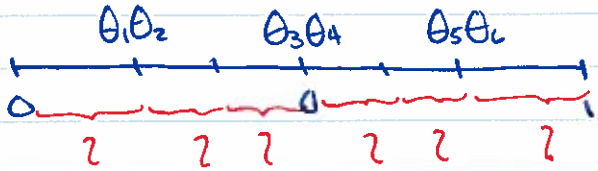
$$6\ell + 1/4 = 1$$
$$6\ell = 3/4 \quad \therefore \boxed{\ell^* = 1/8}$$

- Therefore the equilibrium locations are:

(5)



(c) Now, the distance between firms 3 and 4 is 0:

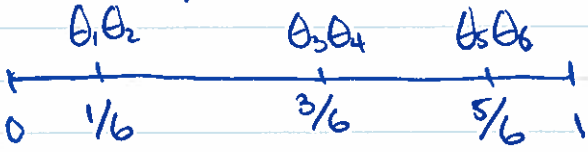


- This implies that:

$$6\ell = 1 \quad \therefore \boxed{\ell^* = 1/6}$$

- Therefore, the equilibrium locations are:

(5)



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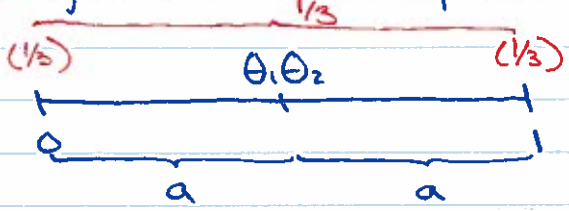
QUESTION # 5: 10 MARKS

(a) The Nash equilibrium to the location game requires that no firm has an incentive to move given the location of the other firms. This requires that two conditions are satisfied: (2)

(i) No firm's market length is less than any other firm's half-market length.

(ii) The two peripheral firms must be adjacent to an interior firm.

(b) Since the firms must be paired we have:



- We know that the two profits have to be equal and that the sum of the length have to equal 1.

$\Pi_1 = M \cdot a + 1/3$ but $M = 1/3$ $\therefore \Pi_1 = 1/3(a) + 1/3$

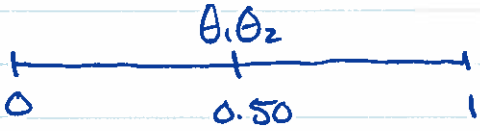
$\Pi_2 = M \cdot a + 1/3$ $\therefore \Pi_2 = 1/3(a) + 1/3$

- The two conditions that need to be satisfied are:

(i) $\Pi_1 = \Pi_2$

(ii) $2a = 1$

- Using condition (ii) we can see that $a^* = 1/2$. Therefore, the equilibrium locations are: (2)



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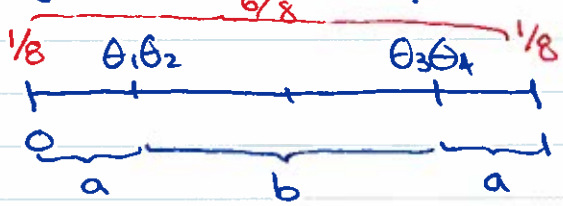
(c) Since both firms are located in the middle we should expect that the two drinks taste exactly the same.
(2) As a result, the market does not do a very good job of providing a variety of products.

(d) Since the majority of consumers are located at the ends of the product space we should expect
(2) that the socially optimal locations would have the firms farther apart and closer to the end points.

(e) Since the firms must be paired we have:



- We know that the ~~four~~ profits must be equal and that the length must equal 1.

$\pi_1 = 1/8 + m \cdot a$ but $m = 6/8$ $\therefore \pi_1 = 1/8 + 6/8\,a$

$\pi_2 = m \cdot (b/2)$ $\therefore \pi_2 = (6/8b) \cdot b$

- The two conditions that must be satisfied are:
  (i) $(1/8) + (6/8)a = (6/16)b$
  (ii) $2a + b = 1$

- (ii) states that $b = 1 - 2a$ which we can substitute into (i):

$(1/8) + (6/8)a = (6/16) - (12/16)a$

$(24/16)a = 4/16$

$\therefore \boxed{a^* = 1/6}$

$\Rightarrow b = 1 - 2a$

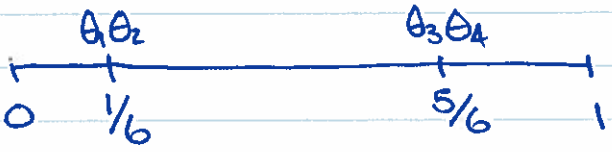
$b = 1 - 2(1/6)$

$b = 1 - 1/3$

$\therefore \boxed{b^* = 4/6}$

- Therefore, the symmetric Nash equilibrium in locations is:

(2)



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