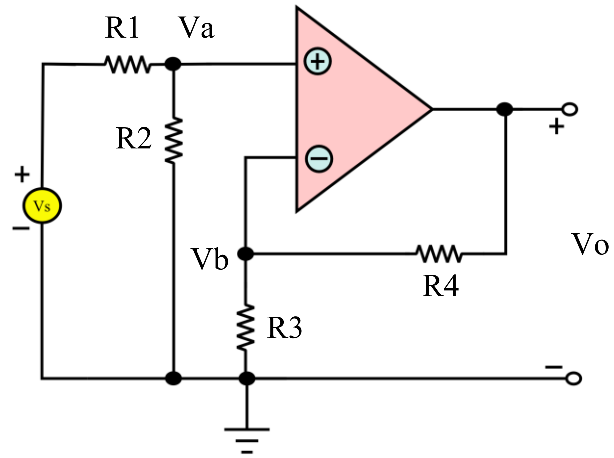
Name:		ID:	
Answer all Questions. Time 1.5 Hours			
Q1	Q2	Q3	Total

Question: 1: For the following circuit and stated conditions, find the output voltage, Vo.



The resistances are: R1 = 100 Ohms, R2 = 200 Ohms, R3 = 300 Ohms, R4 = 400 Ohms. Also, Vs is 5 volts DC.

Solution for the question:

V1 = Vs * R2/(R1+R2) (R1 and R2 are a voltage divider b/c of Golden Rule #2: no current into the opamp.

V2 = Vo * R3/(R3+R4) (R4 and R3 are also a voltage divider b/w Vo and ground)
(Can also be expressed as Vo = V2*(1+R4/R3)

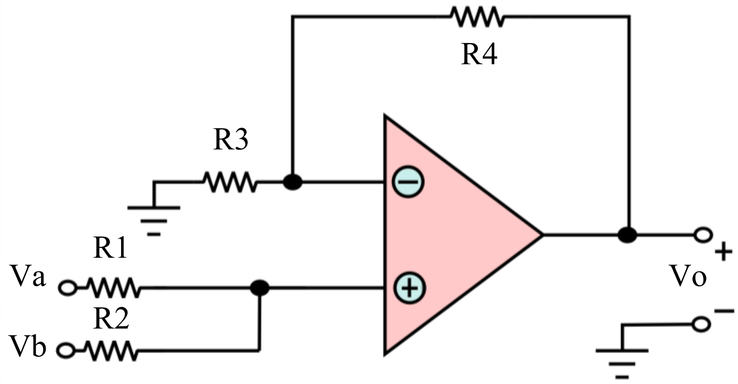
V1 = V2 because of Golden Rule #1 (output makes the inverting and non-inverting input voltages the same). Note that V1 will be found to be equal to 3.33 volts.

Therefore, Vs * R2/(R1+R2) = Vo * R3/(R3+R4).

So Vo=Vs*(R2/(R1+R2)) * (R3+R4)/R3

And then Vo = Vs * (200/300)*(700/300) = 5*14/9 = 7.78 volts.

Question: 2 For the following circuit, find the output voltage, Vo, given the stated conditions.



Resistances: R1=100 Ohms, R2=200 Ohms, R3=300 Ohms and R4=400 Ohms.
The voltage Va is 1 Volt and the voltage Vb is 5 Volts.

Solution

At the non-inverting input node, we have a voltage V2:

$(V2 - Va)/R1 + (V2 - Vb)/R2 = 0$,

yielding

$V2 = (Va/R1 + Vb/R2)/(1/R1 + 1/R2)$

At the inverting input note, we have a voltage V1:

$(0-V1)/R3 = (V1-Vo)/R4$

yielding

$V1 = Vo/(1+R4/R3)$

Because of Golden Rule #1 (Feedback makes the two opamp input voltages identical), V1=V2. Therefore:

$Vo/(1+R4/R3) = (R2*Va + R1*Vb)/(R1+R2)$

So

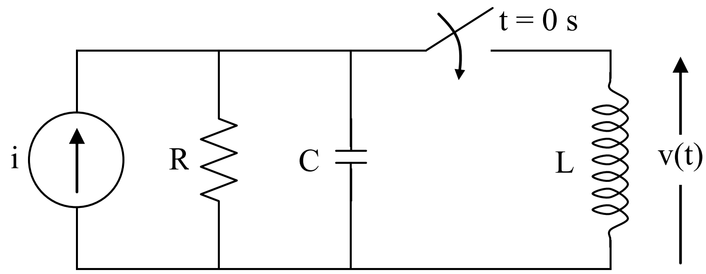
$Vo = (R2*Va + R1*Vb)*(R3+R4)/(R3*(R1+R2))$

Substituting the values above:

$Vo = (200*1 + 100*5)*(300+400)/(300*(100+200)) = 700*700/(300*300) = 5.4$ volts.

Question: 3 RLC Circuits:

Consider the circuit shown below. R = 1.0 kΩ; L = 1.6 H; C = 10 μF; i = 0.02 A



Answer the following:

a) $i(t) = i_R(t) + i_L(t) + i_C(t)$

$= v/R + i0 + (1/L)\int v.dt + C.dv/dt$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v(t) = \frac{1}{C}\frac{di}{dt} \quad \text{(ODE)}$$ (5)

---

b) Write a characteristic equation and its natural solution.

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0 \quad \rightarrow \quad S^2 + 2\,.\,(50)\,S + 250^2 = 0$$ (2)

$$S1, S2 = -\frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}} = -50 \pm j\,244.95 \text{ rads / sec}$$

natural solution: $vn(t) = A\,.\,e^{-50t}\,.\,\cos(244.95\,t + \phi)$ volts (3)

---

c) determine initial conditions for the circuit

Voltage across the cap: v0 = i * R = 20 volts; (2)

Current through the ind: i0 = 0 amps (1)

i = v0/R + C.dv/dt + i0; thus: (i – v0/R)/C = dv/dt = 0 v/s; (2)

---

d) find the forced solution and complete solution: v(t), $t \geq 0$

vf = 0 volts as the inductor will act as a short circuit. (1)

Hence: the total solution: v = vn + vf;

$v = A\,.\,e^{-50t}\,.\,\cos(244.95\,t + \phi)$ volts

From IC: $v(0) = 20 = A.\cos(\phi)$; $dv/dt = 0 = -50\,A\cos(\phi) - 244.95\,.\,A\sin(\phi)$.

OR: $0 = 50 \times 20 + 244.95\,A\sin(\phi)$. OR: $-50 \times 20 / 244.95 = A\sin(\phi) = -4.0825$

Hence: ϕ = -0.2014 rads = -11.5369 degrees; A = 20 / cos (ϕ) = 20.4124.

Hence: $v = 20.4124\,.\,e^{-50t}\,.\,\cos(244.85\,t - 0.2014)$ volts (4)

Hence: $v = 20.\,e^{-50t}\,.\,\cos(244.85\,t) + 4.0824.\,e^{-50t}\,.\,\sin(244.85\,t)$ volts