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UNIVERSITY OF WATERLOO

MATH 137 Final Examination Calculus 1

Saturday, December 18, 2010 **9–11:30 am**

Duration of Exam: Two and a half hours **Number of Exam Pages: 10**

Circle your SECTION.

Section	Instructor	Class Times
1	I. VanderBurgh	MWF 8:30–9:20 am
2	M. Eden	MWF 9:30–10:20 am
3	D. Park	MWF 10:30–11:20 am
4	J. Lawrence	MWF 2:30–3:20 pm
5	M. Eden	MWF 12:30–1:20 pm
6	S. Speziale	MWF 2:30–3:20 pm
7	N. Spronk	MWF 2:30–3:20 pm
9	B. Charbonneau	MWF 10:30–11:20 am
10	S. Speziale	MWF 8:30–9:20 am

Instructions

1. You can tear out page 10 at the end and use it as a scrap paper.
2. If you are in Prof. Vrscay's Section 8, then you are writing the wrong exam!
3. Put your name, signature, and ID number at the top of this page. To prevent loss of your exam, circle your section number.
4. NO ELECTRONIC DEVICES other than your "Pink Tie" Faculty Approved calculators are allowed at your examination desk.
5. Use the backs of pages for overflow or rough work.
6. Show all your work required to obtain answers.

Question	Mark
1	/ 9
2	/ 10
3	/ 10
4	/ 12
5	/ 9
6	/10
7	/ 12
8	/ 12

[3] 1. (a) If $f(x) = \sin\left(xe^x\right)$, calculate $f'(0)$.

[4] (b) Find the equation of the tangent line to the curve defined by the equation $4x^2 + 9y^2 = 72$ at the point $(-3, 2)$.

[2] (c) Prove that if f is differentiable and $f(-x) = -f(x)$ for all x, then $f'(x)$ is an even function. (Hint: Use the Chain Rule.)

[3] 2. (a) Calculate $\dfrac{d}{dx}(x^{\ln x})$ when $x > 0$.

[3] (b) 10 milligrams of a radioactive element with a half-life of 140 days is placed in a lead container. How many milligrams will be present after 70 days?

[4] (c) An airport has a light beacon which rotates at 3 radians/second. This beacon sits 3 km from a straight highway. What speed is the beam from the beacon traveling along the highway at a point which is 5 km from the beacon itself?

[4] 3. (a) Find the absolute maximum and the absolute minimum values of $f(x) = 2\sin x + \sin 2x$ on the interval $[0, \frac{\pi}{2}]$. (Hint: After finding $f'(x)$, use a double angle formula.)

[3] (b) Evaluate $\lim_{x\to 0} \dfrac{\arcsin x}{e^x - 1}$. (Here, $\arcsin x = \sin^{-1} x$ is the inverse sine function.)

[3] (c) Evaluate $\lim_{x\to\infty} x^{1/x}$.

4. Consider the function $f(x) = (x + 1)e^{-x}$.

[2] (a) Find all the intercepts of $y = f(x)$.

[2] (b) Find all horizontal asymptotes of $y = f(x)$.

[3] (c) Find all critical numbers of f, and determine the intervals on which f is increasing/decreasing.

[3] (d) Find $f''(x)$, and use it to determine the concavity of $y = f(x)$ as well as any inflection points.

[2] (e) Sketch $y = f(x)$, indicating all intercepts, local maximum/minimum and inflection points.

[5] 5. (a) Prove the Increasing Test: If $f'(x) > 0$ for all x in the interval (a, b), then f is increasing on the interval (a, b).

[4] (b) Using the Mean Value Theorem, prove that if $a < b$, then

$$|\sin b - \sin a| \leq b - a.$$

[3] 6. (a) Show that the equation $f(x) = x^5 + x - 1 = 0$ cannot have more than one real solution.

[2] (b) Use Newton's method with the initial approximation $x_1 = 1$ to find the second approximation x_2 to the solution of the equation $f(x) = x^5 + x - 1 = 0$.

[5] (c) A right circular cylinder is inscribed in a cone with height 4 and base radius 3. Find the largest possible volume for such a cylinder.

[4] 7. (a) Express the following limit as a definite integral and then evaluate the definite integral using the Fundamental Theorem of Calculus:

$$\lim_{n\to\infty} \frac{\pi}{n} \sum_{i=1}^{n} \sin \frac{i\pi}{n}.$$

[3] (b) Evaluate $\displaystyle\int_{-1}^{2} |x|\, dx$.

[2] (c) If f is a continuous even function with $\displaystyle\int_0^1 f(x)\, dx = A$ and g is a continuous odd function with $\displaystyle\int_0^1 g(x)\, dx = B$, then express $\displaystyle\int_{-1}^{1} (f(x) + g(x))\, dx$ in terms of A and B.

[3] (d) If $h(x) = \displaystyle\int_{x^2}^{0} \frac{t}{\sqrt{1+t^4}}\, dt$, compute $h'(x) = \dfrac{d}{dx} h(x)$.

[3] 8. (a) Evaluate $\displaystyle\int \left(\frac{1}{1+x^2} + e^{\tan x} \sec^2 x \right) dx.$

[4] (b) Evaluate $\displaystyle\int_0^1 \frac{x}{\sqrt{4-x^2}}\, dx.$

[5] (c) Find the area of the region bounded by $y = 12 - x^2$ and $y = x^2 - 6$.

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