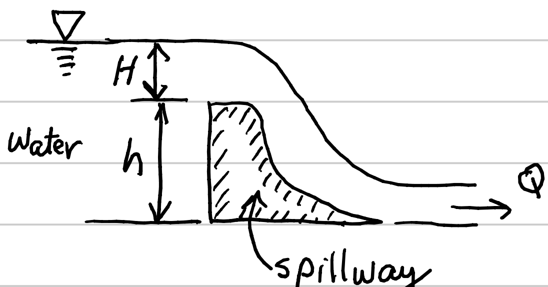
# Dynamic Similitude - Example: Froude Number Matching
(to be solved using Table 7.1 in text)

A spillway model is built to a scale of 1:25 across a channel (flume) which is 0.61m wide. The prototype is 11.4 m high and the maximum head expected is 1.52 m




(a) What height of model and what head on the model should be used?

(b) If the flow over the model at 61 mm head is 0.02 $m^3/s$, what flow per metre of prototype width may be expected?

Solution: Table 7.1 is based on $L_r = L_p/L_m$ ratios.

$\therefore$ "scale of 1:25" means $L_m = L_p/25$, i.e., model is 25 times smaller than the prototype

$\therefore L_r = L_p/L_m = 25$

(a) All heights are scaled & based on $L_r = 25 = h_r = H_r$

$\therefore \frac{h_p}{h_m} = 25$ so $h_m = h_p/25 = \frac{11.4\,m}{25} = 0.456\,m$ model height

$\therefore \frac{H_p}{H_m} = 25$ so $H_m = H_p/25 = \frac{1.52\,m}{25} = 0.0608\,m$ model head

(close to 61 mm)

(b) Spillway requires Froude number matching

Table 7.1 Discharge ratio, $Q_r = (L^{5/2} g^{1/2})_r$ (Froude # matching)

or $\frac{Q_p}{Q_m} = L_r^{5/2} g_r^{1/2}$ where $Q_m = 0.02\, m^3/s$ when $h_m = 61\,mm$ and model width $w_m = 0.61\,m$.

also note: $g_m = g_p$ so $g_p/g_m = g_r \equiv 1$.

$$\therefore Q_p = Q_m L_r^{5/2} g_r^{1/2} = (0.02 \tfrac{m^3}{s})(25^{5/2})(1^{1/2}) = 62.5\ m^3/s$$

Note: This is for scaled prototype width of

$\frac{W_p}{W_m} = L_r = 25$ or $W_p = 25 W_m = 25(0.61m) = 15.25\,m$

So the prototype flow/metre width $= \frac{62.5\ m^3/s}{15.25\,m} = 4.098\ \frac{m^3/s}{m\ \text{width}}$

flow per unit width of prototype

# Dynamic Similitude - Example: Reynolds Number Matching
(to be solved using Table 7.1 in text)

A model of a torpedo is to be tested in a fresh-water tower tank at 20°C and velocity of 24 m/s. The prototype is expected to attain a velocity of 6 m/s in fresh water at 10°C.

(a) What length ratio should be used?

(b) If the drag force on the model is 260 N, what is the expected drag force on the prototype?

__Solution__: Submerged torpedo requires Reynolds # matching

(a) Known: $V_m = 24$ m/s ; $V_p = 6$ m/s $\therefore$ $V_r = V_p/V_m = \frac{6}{24} = 0.25$

and in Table 7.1, velocity, $V_r = \left(\frac{\mu}{L\rho}\right)_r$ Reynolds # match

$$\therefore V_r = 0.25 = \frac{\mu_r}{L_r \rho_r} \quad \text{or} \quad L_r = \frac{\mu_r}{0.25\rho_r} = \frac{\mu_p/\mu_m}{0.25(\rho_p/\rho_m)}$$

__Data__: freshwater, 20°C, Table A.1, $\rho_m = 998.2$ kg/m$^3$

$\mu_m = 1.002$ E-3 N·s/m$^2$

freshwater, 10°C, Table A.4, $\rho_p = 999.7$ kg/m$^3$

$\mu_p = 1.307$ E-3 N·s/m$^2$

$$\text{Thus, } L_r = \frac{(1.307\text{E-}3/1.002\text{E-}3)}{0.25(999.7/998.2)} = 5.210$$

(model is 1/5.21 or 19.19 % of full size)

(b) Known: $D_m = 260\,N$. Find $D_p$ (drag force on prototype)

Table 7.1, Force $= D_r = D_p/D_m = (\mu^2/\rho)_r = \mu_r^2/\rho_r$ Reynolds match

$$\therefore D_p = D_m \, \mu_r^2/\rho_r = D_m \frac{(\mu_p/\mu_m)^2}{(\rho_p/\rho_m)}$$

$$D_p = (240\,N)\,\frac{\left(1.307E\text{-}3/1.002E\text{-}3\right)^2}{(999.7/998.2)} = \underline{407.7\,N}$$