MATH 1007B **Term test #1 – solution** **4 October 2012**

Total marks: 30 marks

This test is closed book. No calculators or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ______________________________ ____________

First name Last name Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

1. **Simplify** as much as possible

   a. [2] $\dfrac{\frac{a}{b}-\frac{b}{a}}{\frac{1}{b}-\frac{1}{a}} = \dfrac{\frac{a^2-b^2}{ab}}{\frac{a-b}{ab}} = \dfrac{a^2-b^2}{a-b} = a+b$

   b. [2] $\left(\dfrac{a\,b^{-1/2}}{2a^3b^{-1}}\right)^{-1} = \left[\dfrac{2a^3b^{-1}}{a\,b^{-1/2}}\right] = \dfrac{2a^2}{b^{1/2}} = \dfrac{2a^2}{\sqrt{b}}$

2. Fill in the blanks:

   a. [1] arccos 0 = $\pi/2$

   b. [1] Let $f(x)=\arctan x$. The **range** of f is $(-\pi/2, \pi/2)$

   c. [1] Let $f(x)=\log x$. The **domain** of f is $(0, +\infty)$

3. Let $f(x) = x^2$ and $g(x)=\sqrt{3x-5}$

   a. [2] What are the domains of each function, f and g?

   dom$(f) = \mathbb{R}$ (b.c. f is a polynomial)

   dom$(g) = [5/3, +\infty)$ (b.c. of condition $3x-5 \geq 0$)

   b. [2] What is the rule for $f \circ g(x)$? (*simplify as much as possible*)

   $f \circ g(x) = f(g(x)) = (g(x))^2 = \left(\sqrt{3x-5}\right)^2 = 3x-5$ (assuming $x \geq 5/3$)

   c. [3] What is the domain of $f \circ g(x)$?

   dom$(f\circ g) = \{x \in \text{dom}(g) \mid g(x) \in \text{dom}(f)\}$

   But dom$(f) = \mathbb{R}$ (no restrictions), so:

   dom$(f \circ g) = $ dom$(g) = [5/3, +\infty)$

4. [2] Prove the following identity: $\tan^2\theta - \sin^2\theta = \tan^2\theta\ \sin^2\theta$

   $\dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta} = \dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} =$

   $\ldots = \left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)(1-\cos^2\theta) = \tan^2\theta\ \sin^2\theta$

5. [2] Determine the equation of the line that is parallel to the line $x - 2y - 8 = 0$, and which passes through the point (1, –1)

   the line $x-2y-8=0$ has a slope of $1/2$: $x-2y-8=0 \iff y = \frac{1}{2}x - 4$

   $\therefore$ The line // to it has equation: $y = \frac{1}{2}x + b$, with $(1,-1)$ on the line. $\therefore$ $-1 = \frac{1}{2}(1) + b \Rightarrow b = -3/2$

   $\therefore y = \frac{1}{2}x - 3/2 \iff 2y = x - 3$

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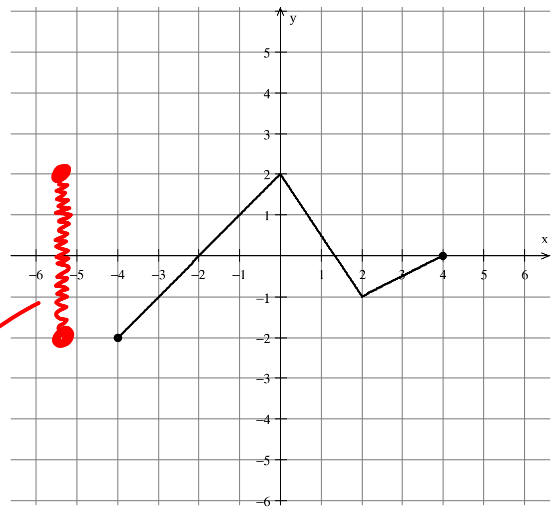
6. The graph of a function f is given.

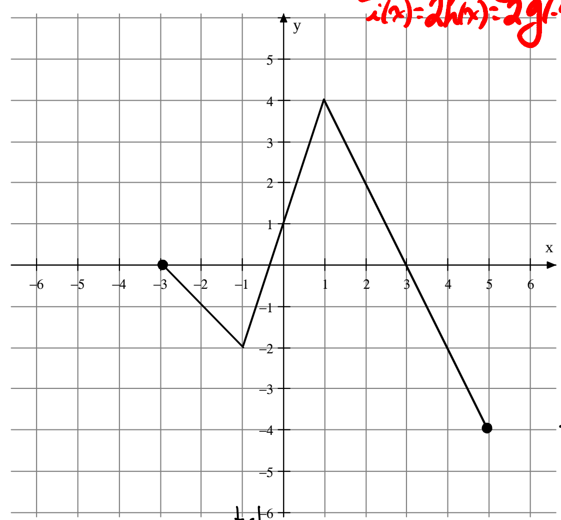
shift left $f(x)$

Stretch flip y $g(x) = f(x+1)$

$h(x) = g(-x) = f(-x+1)$

$i(x) = 2h(x) = 2g(-x) = 2f(-x+1)$





2 ways to get there...

Stretch $\times 2$ $f(x)$

flip y $g(x) = 2f(x)$

shift right $h(x) = g(-x) = 2f(-x)$

$i(x) = h(x-1) = g(-(x-1)) = 2f(-(x-1)) = 2f(-x+1)$

a. [2] Plot the curve $y = 2f(-x+1)$

b. [1] What is the range of f ? $[-2, 2]$

7. [3] On the interval $\frac{\pi}{2} \le x \le \pi$, determine $\cos(\arcsin(\frac{1}{4}))$ *(graphical OR algebraic method accepted)*

method 1

$\cos x$, if $\sin x = \frac{1}{4}$ (with $\cos x < 0$ if $\frac{\pi}{2} \le x \le \pi$)

$\cos x = -\sqrt{1-\sin^2 x} = -\sqrt{1-(\frac{1}{4})^2} = -\frac{\sqrt{15}}{4}$

$1 - \frac{1}{16}$

method 2

$\cos(\arcsin(\frac{1}{4})) = -\sqrt{1-\sin^2(\arcsin(\frac{1}{4}))} = -\sqrt{1-\frac{1}{16}}$

$= -\sqrt{15}/4$

8. [3] Solve the following for x: $\log_2 x + \log_2(x-14) = 5$ domain: $(0, +\infty)$

$x > 0$ $x > 14$

$\log_2 x + \log_2(x-14) = 5$

$\log_2[x(x-14)] = 5$

$x^2 - 14x = 2^5 = 32$

$\therefore x^2 - 14x - 32 = 0$

$(x-16)(x+2) = 0$

$\therefore x = 16$ ✓ in domain

$x = -2$ ✗ not in domain

9. [3] Determine the rule for f^{-1} if $y = f(x) = 100 \cdot 4^{-x/2}$

$y = 100 \cdot 4^{-x/2}$

$y/100 = 4^{-x/2}$

$\log_4(y/100) = -\frac{x}{2}$

$x = -2\log_4(y/100) = f^{-1}(y)$

$\therefore f^{-1}(x) = -2\log_4(x/100)$