1. An organ pipe open at both ends is placed near a loudspeaker fed by an audio oscillator of variable frequency. You observe that the pipe resonates only at frequencies 1342 Hz, 1789 Hz, and 2236 Hz as the frequency of the audio oscillator is varied from 1000 Hz to 2500 Hz. The speed of sound is 340 m/s.
a) What is the fundamental frequency of the organ pipe?
b) What is the length of the pipe?
c) Sketch the standing wave corresponding to the frequency 1342 Hz.
d) A 38 cm long violin string has linear density 0.45g/m. What is the tension in the string if its second harmonic is in tune with the fourth harmonic of the organ pipe?

a) $f = nf_1 = \frac{nv}{2L}$ for an open pipe

3

n is increasing by 1 for each resonance

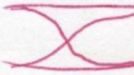
$\Delta f = f_1$ $\quad 1789 - 1342 = \boxed{447 \text{ Hz} = f_1}$

✓ $2236 - 1789 = 447$ Hz

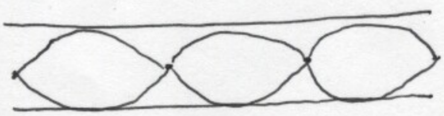
b) $2L = \frac{v}{f_1}$ $\quad L = \frac{v}{2f_1} = \frac{340}{2 \cdot 447} \Rightarrow L = 38$ cm

2

c) $\frac{1342}{447} = n$ or $\boxed{n = 3}$

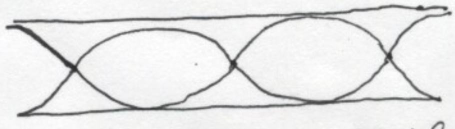

$\leftarrow f_1 = 1342$ Hz

2



displacement wave

or



pressure wave

d) $2f_{violin} = 4f_1$ $\quad f_{violin} = 2f_1 = 894$ Hz

3

now $\boxed{f_{violin} = \frac{v}{2L}} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$T = \mu(2Lf_{violin})^2 = .00045(2 \cdot .38 \cdot 894)^2$

$\boxed{T = 207.7 \text{ N}}$

1 mark for correct equations

full marks if correct answer based on previous answers

$V = \sqrt{\frac{T}{\mu}}$

2. A 9 kg baby is suspended in a baby bouncer, a soft secure seat suspended from elastic cords such that the baby's feet can just touch the floor at the equilibrium position. The baby will bounce up and down at maximum amplitude when she pushes off of the floor 2 times every 3 seconds.
a) What is the spring constant $k$ of the elastic cords?
b) If the amplitude of the baby's oscillations is 5.5 cm, what is the maximum acceleration the baby feels?
c) If the baby stops pushing off of the floor, her amplitude decreases by 30% over one period. What is the damping constant $b$?

a) $\omega_d = 2\pi\left(\frac{2}{3}\right) = 4.189 \text{ rad/s}$

3

$\omega_0 = \omega_d$ at resonance so $\sqrt{\frac{k}{m}} = \omega_d$ $\quad k = m\omega_d^2$

or $\boxed{k = 157.9 \text{ N/m}}$

b) $a_{max} = \omega^2 A = .965 \text{ m/s}^2$

2

c) $A = A_0 e^{-\frac{bT}{2m}}$ and $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \frac{2\pi}{T}$

or $T = \frac{2\pi}{\sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}}$

so

5

$.7 = e^{-\frac{bT}{2m}}$

$\ln(.7) = \frac{b}{2m} \cdot \frac{2\pi}{\sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}}$ $\qquad \left(\frac{\ln .7}{2\pi}\right)^2 = \frac{\left(b/2m\right)^2}{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$

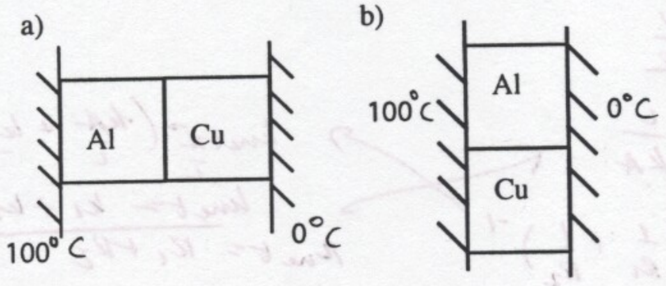
$\left(\frac{\ln .7}{2\pi}\right)^2 \omega_0^2 = \left(\frac{b}{2m}\right)^2 \left(1 + \left(\frac{\ln .7}{2\pi}\right)^2\right)$

note: if take
$T = 1.5 \text{ sec}$ X
$b = -\frac{2m}{T} \ln .7$
$= 4.28 \text{ kg/s}$

so $\frac{b}{2m} = \frac{\frac{\ln .7}{2\pi}\,\omega_0}{\sqrt{1 + \left(\frac{\ln .7}{2\pi}\right)^2}} = 0.056675$

so $b = 2m \frac{\ln .7 \cancel{/2\pi}\,\omega_0}{\sqrt{1 + \left(\frac{\ln .7}{2\pi}\right)^2}} = \boxed{\begin{matrix} 4.28 \\ 1.02 \text{ kg/sec} \end{matrix}}$

3. Two cubes of equal side length, one of aluminum and the other of copper, are arranged between heat reservoirs at 100° C and 0° C as shown in diagram a. The heat reservoir at 0° C consists of a mixture of water and ice. The heat conducted by the cubes melts 1250 g of ice every 10 minutes.
a) What is the length of the side of a cube?
b) What is the temperature at the junction between the aluminum and copper cubes?
c) How much ice would melt in 10 minutes if the cubes were instead arranged as in diagram b?




$$\frac{\Delta Q}{\Delta t} = \frac{mL}{T} = \frac{1.25 \times 334\times10^3}{10\times 60} = 696\ \mathrm{J/s}$$

$$\frac{\Delta Q}{\Delta t} = K_{Al}\cdot L^2\frac{\Delta T_1}{L} = K_{Cu}L^2\frac{\Delta T_2}{L} \qquad \Delta T_1 = 100 - T$$

$$\Delta T_2 = T$$

thus $K_{Al}(100-T) = K_{Cu}T$

3 b) $\boxed{T = \frac{K_{Al}\,100}{K_{AL}+K_{Cu}} = 34.7\,°C}$

4 a) $\boxed{L = \frac{\frac{\Delta Q}{\Delta t}}{K_{Cu}T} = .052\ \mathrm{m}}$ or 5.2 cm

c) $$\frac{\Delta Q}{\Delta t} = (K_{Al}+K_{Cu})\cdot L\cdot 100 = \frac{mL_F}{600}$$

3

$$m = \frac{(K_{Al}+K_{Cu})L\cdot 100\cdot 600}{L_F} = \boxed{5.51\ \mathrm{kg}}$$

(a) $\phi = -1.374$ rad $(+1.78$ rad$)$

(b) $0.298$ & $0.177$ sec. (c) $x(0) = 2.346$ cm

4. A spring with spring constant $k = 250\,\mathrm{N/m}$ vibrates with an amplitude $A = 12.0\,\mathrm{cm}$ when a mass of $0.380kg$ hangs from it. The mass passes through the equilibrium point moving upwards at $t = 0.110s$.

(d) $F = 5.866$
(e) $v = 3.078$ m
$t_M = 0.115$ s

Don's soln

3 ← a) What is the equation describing this motion? (compute the phase constant $\phi$.)

2 ← b) At what times after $t = 0.110s$ will the spring first have its maximum and minimum lengths?

1 ← c) What is the displacement at $t = 0$?

2 ← d) What is the force exerted by the spring at $t = 0$?

2 ← e) What is the maximum speed of the mass and when is it first reached after $t = 0$?

a) $y = A\cos(\omega t + \phi)$  $A = 12.0$ cm (given)

$\omega = \sqrt{\frac{k}{m}} = \boxed{25.65 \text{ rad/s}}$

$x = 12 \cos(\omega t + 1.89$ rad$)$

upward: +

$x = 0.12$ m $\cos(25.65t - 1.25$ rad$)$

$71.6°$

down: +

$y_E = 1.49$ cm

(c) $x(0) = 3.75$ cm

$x_{max} = 12$

$y = 0$ at $t = .110$ s

$\cos(\omega t + \phi) = 0$  $\omega t + \phi = \cancel{\frac{\pi}{2}}, \frac{3\pi}{2}, \cancel{\frac{5\pi}{2}} \ldots$

$\cos(\frac{\pi}{2} + \varepsilon) < 0$ so $\pi/2$ $5\pi/2$ etc correspond to the downward movement (3)

$\phi = \frac{3\pi}{2} - \omega t = 3\pi/2 - .282 = \boxed{1.89 \text{ rad}}$

$$\boxed{y = 12.0 \text{ cm} \cos(25.65 \text{ rad/s}\ t + 1.89 \text{ rad})}$$

b) $\omega t + \phi = 0, \pi, 2\pi, 3\pi$  $t = -\phi/\omega, \pi/\omega - \phi/\omega$, etc

note $-\phi/\omega = .07368$

$\boxed{t = .1713 \text{ and } .2938 \text{ s}}$ (2)
minimum  maximum

$\pi/\omega = .12248$

as it is heading to minimum length immediately after .110 s

c) $y = 12 \cos 1.89$ rad $= \boxed{-3.766 \text{ cm}}$ (1)

d) equilibrium position is $d = \frac{mg}{k} = .0149$ m below the unstretched length thus the spring is stretched $.0149 + .0377 = .0526$ m (2)

$F = k(.0526 \text{ m}) = \boxed{13.15 \text{ N}}$ 1

e) $v_{max} = \omega A = \boxed{3.078 \text{ m/s}}$ and occurs at $t = .110$ sec (2)

5. A 0.200 kg aluminum calorimeter can contains 0.500 kg of water at 30°$C$. A 0.100 kg piece of ice cooled to $-20°C$ is placed in the can. Assume no heat losses to the outside $c_{water} = 4190\,\mathrm{J/kg\,K}$, $c_{ice} = 2100\,\mathrm{J/kg\,K}$, $L_{f(water)} = 334 \times 10^3 J/kg$, $c_{Al} = 910\,\mathrm{J/kg\,K}$.

a) What is the final temperature of the system?

b) A second piece of ice at $-20°C$ is then placed into the calorimeter can. What is the final temperature of the system once it reaches a new equilibrium?

c) If not all of the ice melts how much of the ice will remain in the can?

d) Would your answer for part b) be different if both pieces of ice were added at the same time? Be sure to explain your answer.

a) $\Delta Q = 0 = m_{Al} c_{Al} (T-30) + m_w c_w (T-30)$

assuming all ice melts $\quad + m_i c_i 20 + m_i L + m_i c_w T = 0$

4

$$\left[m_{Al} c_{Al} + (m_w + m_i) c_w\right] T = (m_{Al} c_{Al} + m_w c_w) 30 - m_i c_i 20 - m_i L$$

$$[.2 c_{Al} + .6 c_w] T = (.2 c_{Al} + .5 c_w) \cdot 30$$

$$182 + 2514 \qquad - .1 \cdot 2100 \cdot 20 - .1 \cdot 334 \times 10^3$$

$$2696\, T = 68{,}310 - 4200 - 33{,}400$$

$$= 30{,}710$$

$$\boxed{T = 11.4°C}$$

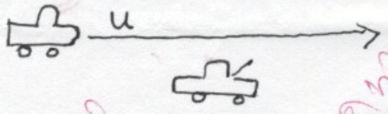
b) $\overset{37{,}600}{4200 + 33{,}400} > 30{,}710$ so not all of the new ice cube will melt and $\boxed{T = 0}$

4

c) $\boxed{m = \frac{6890}{L_f} = .0206 \text{ kg of ice are left}}$

2 d) No - $\Delta Q = 0$ can be computed by combining terms in any way you choose

6. You are standing by the roadside next to your broken car watching a truck approach and hear its horn at a frequency of 1100 Hz. Immediately after the truck passes you the frequency you detect becomes 950 Hz. Assume the speed of sound in air is 340 m/s.
a) What is the speed of the truck?
b) What frequency does the driver of the truck hear reflected from the car as he approaches the car?
c) What is the beat frequency of the two signals heard by the truck driver? Is this audible?




a) $$f_{approach} = \frac{f_o}{1 - u/v} \qquad f_{away} = \frac{f_o}{1 + u/v}$$

thus $$f_{away} = \frac{f_{approach}(1 - u/v)}{1 + u/v}$$

$$\frac{f_{away}}{f_{approach}} = \frac{(1 - u/v)}{(1 + u/v)} = .864 \qquad (1 - u/v) = .864(1 + u/v)$$

$$(1 + .864)\,u/v = 1 - .864$$

or $$u = v\left(\frac{1 - .864}{1 + .864}\right) = .07296v$$

$$\boxed{u = 24.8 \text{ m/s}}$$

b) $$\boxed{f_r = f_{approach}(1 + u/v) = 1180 \text{ Hz}}$$

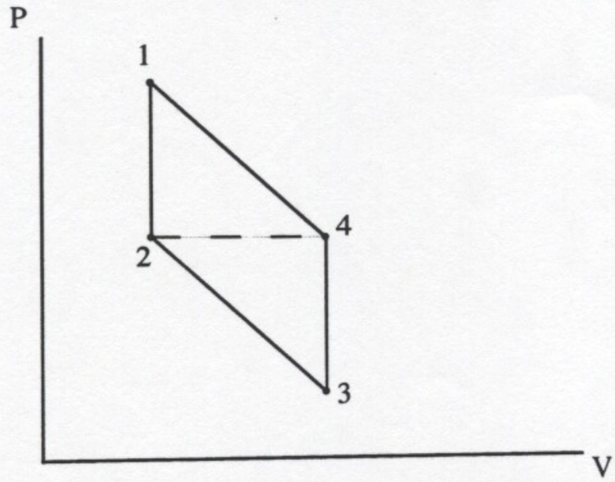
c) now $$f_o = f_{approach}(1 - u/v) = 1019.8 \text{ Hz}$$

so $$f_{beat} = 1180 - 1019.8 = 160.2 \text{ Hz}$$

No - the two tones are too far apart

Assume the gas is monoatomic

7. A heat engine has the PV-curve shown below. Note the curves are made of two right triangles as indicated by the dashed line. $P_1 = 3 \times 10^5\,\text{Pa}$, $P_2 = P_4 = 2 \times 10^5\,\text{Pa}$ and $P_4 = 1 \times 10^5\,\text{Pa}$. $V_1 = V_2 = 1 \times 10^{-3}\,\text{m}^3$ and $V_3 = V_4 = 2 \times 10^{-3}\,\text{m}^3$. The temperature at $T_1 = 350\text{K}$. Find the efficiency of the heat engine.




$\varepsilon = \dfrac{W}{Q_H}$ (1) $\qquad W$ = area inside of the PV curve

$$= \frac{1}{2}(P_1 - P_2)(V_4 - V_2) + \frac{1}{2}(V_4 - V_2)(P_4 - P_3)$$

as the area is 2 right triangles

$$W = \frac{1}{2}\left(10^5 \times 10^{-3}\right) + \frac{1}{2}\left(10^{-3}\right)\left(10^5\right)$$

$$\boxed{W = 100\,\text{J}}$$ (3)

$$Q_H = Q_{21} + Q_{14}$$

now $\quad nRT_1 = P_1V_1 \qquad nRT_2 = P_2V_2 \qquad nRT_3 = P_3V_3 \qquad nRT_4 = P_4V_4$

so $\quad \dfrac{T_2}{T_1} = \dfrac{P_2V_2}{P_1V_1} = \dfrac{2}{3} \qquad \dfrac{T_3}{T_1} = \dfrac{P_3V_3}{P_1V_1} = \dfrac{2}{3} \qquad \dfrac{T_4}{T_1} = \dfrac{P_4V_4}{P_1V_1} = \dfrac{4}{3}$ (2)

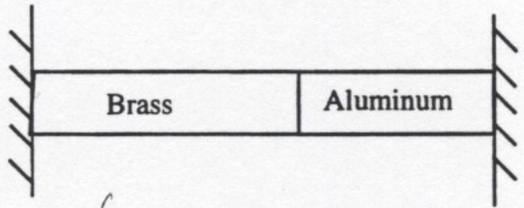
so $\quad Q_{21} = nC_v(T_1 - T_2) = \dfrac{1}{3}nC_vT_1$

$\quad Q_{14} = nC_v(T_4 - T_1) + W_{41} = \dfrac{1}{3}nC_vT_1 + 250\,\text{J}$

$\quad W_{41} = 50\,\text{J} + (1 \times 10^{-3})\, 2 \times 10^5\,\text{Pa} = 50 + 200 = 250\,\text{J}$

$Q_H = 2/_3\, nC_vT_1 + 250\,\text{J} = \boxed{550\,\text{J}}$ (4) $\qquad \dfrac{W}{Q_H} = \dfrac{100}{550} = .18$

8. A brass bar 2.0 m long, $\alpha_{Brass} = 2.0 \times 10^{-5} K^{-1}$, $Y_{Brass} = 9.0 \times 10^{10} N/m^2$ and an aluminum bar 1.0 m long, $\alpha_{Al} = 2.4 \times 10^{-5} K^{-1}$, $Y_{Al} = 7.0 \times 10^{10} N/m^2$ are attached as shown at 22° C. At 22° C the rods just touch. If the breaking stress of aluminum is $2.2 \times 10^8 N/m^2$ and brass is $4.7 \times 10^8 N/m^2$, what is the maximum temperature you can reach before breaking one of the bars? Which one breaks first?




$$\left(\frac{F}{A}\right) = Y\frac{\Delta L}{L}$$

$$\Delta L = \alpha L \Delta T$$

7pt

$$\left(\frac{F}{A}\right)_{Brass} + \left(\frac{F}{A}\right)_{Al} = Y_{Brass}\left(\frac{\Delta L}{L}\right)_{Brass} + Y_{Al}\left(\frac{\Delta L}{L}\right)_{AL}$$

$$\left(\frac{F}{A}\right)_{Brass} + \left(\frac{F}{A}\right)_{Al} = \left(Y_{Brass}\,\alpha_{Brass} + Y_{Al}\,\alpha_{Al}\right)\Delta T$$

1pt

Al breaks first because

$$2.2 \times 10^8 \frac{N}{m^2} < 4.7 \times 10^8 N/m^2$$

$$2.2 \times 10^8 \frac{N}{m^2} = \left[\left(9.0 \times 10^{10} \frac{N}{m^2}\right)\left(2.0 \times 10^{-5} K^{-1}\right) + \left(7.0 \times 10^{10} \frac{N}{m^2}\right)\left(2.4 \times 10^{-5} K^{-1}\right)\right]\Delta T$$

1 pt

$$\Delta T = 63.2°C$$

$$\Delta T = T_f - 22°C$$

1pt

$$T_f = 85.2°C$$