# CARLETON UNIVERSITY

**FINAL**
**EXAMINATION**
December 2011

**DURATION: 3 HOURS**

**Department Name and Course Number:** School of Mathematics and Statistics, MATH 1004 A, B, C, D.
**Course Instructor(s):** Dr. A. B. Mingarelli (Sect. A), Dr. P. K. Chan (Sect. B), Dr. Z. Montazeri (Sect. C), Mr. L. Bourbonnais (Sect. D), Dr. M. Sadeghi (Sect. E).

AUTHORIZED MEMORANDA
NON-PROGRAMMABLE CALCULATOR PERMITTED.
BLANK SHEETS OF PAPER FOR ROUGH WORK

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2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination is out of a total of 100** and consists of 20 multiple choice questions each worth 5 marks **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.,* you get 0 marks for that question).

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1. **[5 marks] Let** $f(x) = \ln(\text{Arcsin}\, x)$**. Evaluate** $f'(1/2)$**. In other words, find the derivative of** $f$ **at** $x = 1/2$**.**
   (a) $f'(\frac{1}{2}) = \frac{4\sqrt{3}}{\pi}$ (b) $f'(\frac{1}{2}) = \frac{1}{\pi}$ (c) $f'(\frac{1}{2}) = \frac{\sqrt{3}}{4}$ (d) $f'(\frac{1}{2}) = 12$
2. **[5 marks] Let** $f(x) = |x-1| + 1$**. Calculate** $L = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$. (a) $L = 0$ (b) $L = 1$ (c) $L = -1$ (d) This limit does not exist
3. **[5 marks] Let** $f(x) = e^{x^3} \ln(x^2)$**. Evaluate** $f'(1)$**. In other words, find the derivative of** $f$ **at** $x = 1$**.**
   (a) $f'(1) = 0$ (b) $f'(1) = 2e$ (c) $f'(1) = e^2$ (d) $f'(1) = -1$
4. **[5 marks] Let** $f(x) = |x-1| + 1$**. Calculate** $L = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$.
   (a) $L = 0$ (b) $L = 12$ (c) $L = 1$ (d) The limit does not exist.
5. **[5 marks] Two differentiable functions** $f, g$ **are defined by** $f(x) = \sin x$ **and** $g(x) = \cos x$**. What is the value of the derivative of their composition** $g(f(x))$ **at** $x = \frac{\pi}{2}$**?**
   (a) $-3$ (b) $-1$ (c) 3.2 (d) 0
6. **[5 marks] Let** $f(x) = 3^{5\cos x}$**. Evaluate** $f'(x)$**. In other words, find the derivative of** $f$ **at** $x$**.**
   (a) $f'(x) = 3^{-5\sin x}$ (b) $f'(x) = 5\ 3^{5\cos x}$ (c) $f'(x) = 5\cos x\ 3^{4\cos x - 1}$ (d) $f'(x) = -(5\ \sin x\ \ln 3\,)\, 3^{5\cos x}$
7. **[5 marks] Evaluate the limit:** $L = \lim_{x\to\infty} \left\{-2 + x^2\, 4^{-2x}\right\}$**.**
   (a) $L = 1$ (b) $L = -2$ (c) $L = 3.61$ (d) $L = 1.22$
8. **[5 marks] Evaluate** $\lim_{x\to 0^+} \frac{d}{dx} \int_1^{\sqrt{x}} \frac{\sin t^2}{t}\, dt$.
   (a) $I = 0$ (b) $I = \frac{5}{2}$ (c) This limit does not exist (d) $I = \frac{1}{2}$

9. **[5 marks] Let $u$ be given implicitly as a differentiable function of $v$ by $e^{uv} + u^2 v = 1$. Calculate the value of the derivative $\dfrac{du}{dv}$ at the point $(u, v)$ where $u = 0$, $v = 1$:**

   (a) 4, (b) $-1$, (c) 5, (d) 0

10. **[5 marks] Find the critical points of the function $f$ defined by $f(x) = \dfrac{\sqrt{x}}{x-2}$.**

    (a) $x = 0,\ x = 1,\ x = 2$ (b) $x = -1$ (c) $x = 0$ only (d) $x = 0$ and $x = 2$ only.

11. **[5 marks] Determine where the function $f$ whose values are given by $f(x) = x^4 - 2x^3 + 6$ is concave up.**

    (a) $\{x : x < 0\}$ (b) $\{x : 1 < x\}$ (c) $\{x : 0 < x < 1\}$ (d) $\{x : 2 < x < \infty\}$

12. **[5 marks] Find all the asymptotes of the function defined by $p(x) = \dfrac{x+1}{(x-1)(x-2)}$.**

    (a) Vertical asymptotes at $x = -1, 1, 2$ only. No horizontal asymptotes. (b) Vertical asymptotes at $x = 1$ and $x = 2$. Horizontal asymptote given by $y = 0$. (c) Vertical asymptotes at $x = -1$ and $x = 1$ only. Horizontal asymptote given by $y = 2$. (d) Vertical asymptotes at $x = 1$ and $x = 2$. Horizontal asymptote at $y = -1$.

13. **[5 marks] An antiderivative of $f(x) = \tan(x+1)$ is given by**

    (a) $\ln(\sec^2(x+1)) - 1$ (b) $\ln|\cos(x+1)| + 2$ (c) $\ln(\sin x)$ (d) $-\ln|\cos(x+1)|$.

14. **[5 marks] Which of the following functions has a point of inflection at $x = 0$?**

    (a) $f(x) = x^3 + x - 2$ (b) $f(x) = -x^2 - 4x + 8$ (c) $f(x) = x^4 + x + 4$ (d) $f(x) = x^2 + x + 1$

15. **[5 marks] Evaluate** $\displaystyle\int_0^3 x\,\sqrt{x+1}\,dx$

    (a) $\dfrac{116}{15}$ (b) $\dfrac{124}{15}$ (c) $\dfrac{2428}{25}$ (d) $\dfrac{298}{15}$

16. **[5 marks] The improper integral** $\displaystyle\int_0^\infty x\,3^{-x^2}\,dx$ **is given by**

    (a) $\dfrac{1}{2\ln 3}$ (b) 1 (c) $\dfrac{1}{9}$ (d) $\dfrac{1}{3}\ln 3$

17. **[5 marks] The value of** $\displaystyle\int_{1/3}^1 2t^3\,\ln 3t\,dt$ **is given by**

    (a) $2\ln 3$ (b) $\dfrac{61}{162}$ (c) $\dfrac{1}{3}\ln 3 - \dfrac{8}{27}$ (d) $\dfrac{1}{2}\ln 3 - \dfrac{10}{81}$

18. **[5 marks] Evaluate and simplify the indefinite integral:** $\displaystyle\int (x^2+1)\,e^{-3x}\,dx$.

    (a) $\dfrac{e^{-3x}}{9}\left(6x^2+9x+2\right) + C$ (b) $-\dfrac{e^{-3x}}{27}\left(x^2+6x+1\right) + C$ (c) $-\dfrac{e^{-3x}}{27}\left(9x^2+6x+11\right) + C$ (d) $\dfrac{e^{-3x}}{27}\left(9x^2-6x+2\right) + C$

19. **[5 marks] Find an expression for the area between the curves defined by $y = 2$ and $y = 2x^2$ in the first quadrant.**

    (a) $2\displaystyle\int_0^1 (1-x^2)\,dx$ (b) $\displaystyle\int_0^1 (x - \sqrt{x})\,dx$ (c) $\displaystyle\int_0^1 \left(x - 2x^2\right)\,dx$ (d) $\displaystyle\int_0^1 (x - x^2)\,dx$

20. **[5 marks] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \cos x$ between $x = 0$ and $x = \pi/2$ about the $y-$axis.**

    (a) $\pi\displaystyle\int_0^{\pi/2} x^2\cos x\,dx$ (b) $\displaystyle\int_0^{\pi/2} \cos x\,dx$ (c) $\displaystyle\int_0^{\pi/2} x\,\sin x\,dx$ (d) $\pi\displaystyle\int_0^{\pi/2} x\,\cos x\,dx$