CARLETON UNIVERSITY

FINAL
EXAMINATION
December 2011

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004 A, B, C, D.
Course Instructor(s): Dr. A. B. Mingarelli (Sect. A), Dr. P. K. Chan (Sect. B), Dr. Z. Montazeri (Sect. C), Mr. L. Bourbonnais (Sect. D), Dr. M. Sadeghi (Sect. E).

AUTHORIZED MEMORANDA
NON-PROGRAMMABLE CALCULATOR PERMITTED.
BLANK SHEETS OF PAPER FOR ROUGH WORK

This exam may be released to the Library and may be taken away by the student.

1. Please verify that you are in possession of a Scantron FORM
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination is out of a total of 100** and consists of 20 multiple choice questions each worth 5 marks **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.,* you get 0 marks for that question).

Return only the duly completed Scantron form, not the examination nor your work.

1. **[5 marks] Let** $f(x) = \ln(\text{Arcsin}\, x)$**. Evaluate** $f'(1/2)$**. In other words, find the derivative of** f **at** $x = 1/2$**.**
   (a) $f'(\frac{1}{2}) = \frac{4\sqrt{3}}{\pi}$ (b) $f'(\frac{1}{2}) = \frac{1}{\pi}$ (c) $f'(\frac{1}{2}) = \frac{\sqrt{3}}{4}$ (d) $f'(\frac{1}{2}) = 12$
2. **[5 marks] Let** $f(x) = |x-1| + 1$**. Calculate** $L = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$. (a) $L = 0$ (b) $L = 1$ (c) $L = -1$ (d) This limit does not exist
3. **[5 marks] Let** $f(x) = e^{x^3} \ln(x^2)$**. Evaluate** $f'(1)$**. In other words, find the derivative of** f **at** $x = 1$**.**
   (a) $f'(1) = 0$ (b) $f'(1) = 2e$ (c) $f'(1) = e^2$ (d) $f'(1) = -1$
4. **[5 marks] Let** $f(x) = |x-1| + 1$**. Calculate** $L = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$.
   (a) $L = 0$ (b) $L = 12$ (c) $L = 1$ (d) The limit does not exist.
5. **[5 marks] Two differentiable functions** f, g **are defined by** $f(x) = \sin x$ **and** $g(x) = \cos x$**. What is the value of the derivative of their composition** $g(f(x))$ **at** $x = \frac{\pi}{2}$**?**
   (a) -3 (b) -1 (c) 3.2 (d) 0
6. **[5 marks] Let** $f(x) = 3^{5\cos x}$**. Evaluate** $f'(x)$**. In other words, find the derivative of** f **at** x**.**
   (a) $f'(x) = 3^{-5\sin x}$ (b) $f'(x) = 5\ 3^{5\cos x}$ (c) $f'(x) = 5\cos x\ 3^{4\cos x - 1}$ (d) $f'(x) = -(5\ \sin x\ \ln 3\,)\, 3^{5\cos x}$
7. **[5 marks] Evaluate the limit:** $L = \lim_{x\to\infty} \left\{-2 + x^2\, 4^{-2x}\right\}$**.**
   (a) $L = 1$ (b) $L = -2$ (c) $L = 3.61$ (d) $L = 1.22$
8. **[5 marks] Evaluate** $\lim_{x\to 0^+} \frac{d}{dx} \int_1^{\sqrt{x}} \frac{\sin t^2}{t}\, dt$.
   (a) $I = 0$ (b) $I = \frac{5}{2}$ (c) This limit does not exist (d) $I = \frac{1}{2}$

9. **[5 marks] Let u be given implicitly as a differentiable function of v by $e^{uv} + u^2 v = 1$. Calculate the value of the derivative $\dfrac{du}{dv}$ at the point (u, v) where $u = 0$, $v = 1$:**

   (a) 4, (b) -1, (c) 5, (d) 0

10. **[5 marks] Find the critical points of the function f defined by $f(x) = \dfrac{\sqrt{x}}{x-2}$.**

    (a) $x = 0,\ x = 1,\ x = 2$ (b) $x = -1$ (c) $x = 0$ only (d) $x = 0$ and $x = 2$ only.

11. **[5 marks] Determine where the function f whose values are given by $f(x) = x^4 - 2x^3 + 6$ is concave up.**

    (a) $\{x : x < 0\}$ (b) $\{x : 1 < x\}$ (c) $\{x : 0 < x < 1\}$ (d) $\{x : 2 < x < \infty\}$

12. **[5 marks] Find all the asymptotes of the function defined by $p(x) = \dfrac{x+1}{(x-1)(x-2)}$.**

    (a) Vertical asymptotes at $x = -1, 1, 2$ only. No horizontal asymptotes. (b) Vertical asymptotes at $x = 1$ and $x = 2$. Horizontal asymptote given by $y = 0$. (c) Vertical asymptotes at $x = -1$ and $x = 1$ only. Horizontal asymptote given by $y = 2$. (d) Vertical asymptotes at $x = 1$ and $x = 2$. Horizontal asymptote at $y = -1$.

13. **[5 marks] An antiderivative of $f(x) = \tan(x+1)$ is given by**

    (a) $\ln(\sec^2(x+1)) - 1$ (b) $\ln|\cos(x+1)| + 2$ (c) $\ln(\sin x)$ (d) $-\ln|\cos(x+1)|$.

14. **[5 marks] Which of the following functions has a point of inflection at $x = 0$?**

    (a) $f(x) = x^3 + x - 2$ (b) $f(x) = -x^2 - 4x + 8$ (c) $f(x) = x^4 + x + 4$ (d) $f(x) = x^2 + x + 1$

15. **[5 marks] Evaluate** $\displaystyle\int_0^3 x\,\sqrt{x+1}\,dx$

    (a) $\dfrac{116}{15}$ (b) $\dfrac{124}{15}$ (c) $\dfrac{2428}{25}$ (d) $\dfrac{298}{15}$

16. **[5 marks] The improper integral** $\displaystyle\int_0^\infty x\,3^{-x^2}\,dx$ **is given by**

    (a) $\dfrac{1}{2\ln 3}$ (b) 1 (c) $\dfrac{1}{9}$ (d) $\dfrac{1}{3}\ln 3$

17. **[5 marks] The value of** $\displaystyle\int_{1/3}^1 2t^3\,\ln 3t\,dt$ **is given by**

    (a) $2\ln 3$ (b) $\dfrac{61}{162}$ (c) $\dfrac{1}{3}\ln 3 - \dfrac{8}{27}$ (d) $\dfrac{1}{2}\ln 3 - \dfrac{10}{81}$

18. **[5 marks] Evaluate and simplify the indefinite integral:** $\displaystyle\int (x^2+1)\,e^{-3x}\,dx$.

    (a) $\dfrac{e^{-3x}}{9}\left(6x^2+9x+2\right) + C$ (b) $-\dfrac{e^{-3x}}{27}\left(x^2+6x+1\right) + C$ (c) $-\dfrac{e^{-3x}}{27}\left(9x^2+6x+11\right) + C$ (d) $\dfrac{e^{-3x}}{27}\left(9x^2-6x+2\right) + C$

19. **[5 marks] Find an expression for the area between the curves defined by $y = 2$ and $y = 2x^2$ in the first quadrant.**

    (a) $2\displaystyle\int_0^1 (1-x^2)\,dx$ (b) $\displaystyle\int_0^1 (x - \sqrt{x})\,dx$ (c) $\displaystyle\int_0^1 \left(x - 2x^2\right)\,dx$ (d) $\displaystyle\int_0^1 (x - x^2)\,dx$

20. **[5 marks] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \cos x$ between $x = 0$ and $x = \pi/2$ about the $y-$axis.**

    (a) $\pi\displaystyle\int_0^{\pi/2} x^2\cos x\,dx$ (b) $\displaystyle\int_0^{\pi/2} \cos x\,dx$ (c) $\displaystyle\int_0^{\pi/2} x\,\sin x\,dx$ (d) $\pi\displaystyle\int_0^{\pi/2} x\,\cos x\,dx$