1. Given $A^{-1} = \begin{bmatrix} -1 & 3 \\ 4 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

What is the solution of the equation $A\mathbf{x} = \mathbf{b}$? $X = A^{-1} b$

(a) $\begin{bmatrix} 5 \\ 9 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 9 \end{bmatrix}$ (c) $\begin{bmatrix} 5 \\ -7 \end{bmatrix}$ (d) $\begin{bmatrix} -5 \\ 7 \end{bmatrix}$ (e) $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

2. Suppose that a system of equations has the augmented matrix $\left[\begin{array}{cc|c} 1 & h & 3 \\ 5 & -10 & k \end{array}\right]$. If the system is inconsistent, then the values of h and k must be:

(a) $h = -2, k = 15$
(b) $h \neq -2, k \neq 15$
(c) $h \neq -2, k = 15$
(d) $h = -2, k \neq 15$
(e) $h = 2, k = 15$

3. Find all the values of h such that the vectors $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \\ h \end{bmatrix}$ are linearly dependent:

(a) $h = 2$ (b) $h \neq 2$ (c) $h = -2$ (d) $h \neq -2$ (e) $h \neq \pm 2$

4. Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ x+y-w \\ y+z \end{bmatrix}.$$ Which of the following is true?

(a) T is onto and one-to-one

(b) T is onto, but not one-to-one

(c) T is one-to-one, but not onto

(d) T is neither onto nor one-to-one

(e) none of the above statements is true

5. Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \text{ Find } T\left(\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}\right).$$

(a) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$ **(b)** $\begin{bmatrix} 9 \\ -6 \end{bmatrix}$ **(c)** $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$ **(d)** $\begin{bmatrix} -11 \\ 2 \end{bmatrix}$ **(e)** $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$

6. Let $A = [v_1\ v_2\ v_3\ v_4] = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Which of the following can not be a basis of Col$A$?

(a) $\{v_1, v_3\}$ **(b)** $\{v_2, v_4\}$ **(c)** $\{v_3, v_4\}$ **(d)** $\{v_1, v_2\}$ **(e)** $\{v_2, v_3\}$

7. Let A, B and C be $n \times n$ invertible matrices. What is $\left(A^{-1}B^T(C^{-1})^T\right)^{-1}$?

(a) AB^TC^T (b) CBA (c) $A(B^T)^{-1}C^T$ (d) $C^T(B^{-1})^TA$ (e) $C^T(B^T)^{-1}A^{-1}$

8. Let A be a 5×7 matrix such that $\dim \mathrm{Col}A = 4$. What is $\dim \mathrm{Nul}A$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

9. Let $\mathcal{B} = \{v_1, v_2\}$ be a basis for a subspace H, and let $x = 2v_2 - v_1$. Find $[x]_{\mathcal{B}}$, i.e, the coordinate vector of x relative to $\mathcal{B}$.

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

10. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 4 & 4 \\ 5 & 5 & 5 & 5 & 6 \end{bmatrix}$. What is $\det A$?

(a) 0 (b) 6 (c) 12 (d) 24 (e) 48

11. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} g & h & i \\ d-a & e-b & f-c \\ -2a & -2b & -2c \end{bmatrix}$

$A = B$ change row or column by any number

If $\det(A) = 2$, what is $\det(B)$?

(a) 2 (b) 4 (c) -4 (d) -2 (e) 0

2

12. Let A be an 2×2 matrix and $\det A = -3$. What is $\det(2A^2A^TA^{-1})$?

(a) -36 (b) -18 (c) 18 (d) 36 (e) 9

13. Consider the matrix equation $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

What is the value of x_1? (Hint: Use Cramer's Rule)

(a) $\frac{2}{17}$ (b) $\frac{4}{17}$ (c) 17 (d) 2 (e) 0

14. Let $z = \cos(\pi/16) + i\sin(\pi/16)$ and $w = \cos(\pi/8) + i\sin(\pi/8)$. What is z^2w ?

(a) $\frac{1}{\sqrt{2}}(1+i)$ (b) $\frac{1}{\sqrt{2}}(1-i)$ (c) $1+i$ (d) $1-i$ (e) 1

15. What is the standard form of the complex number $\frac{2+4i}{1+i}$?

(a) $1-3i$ (b) $1-i$ (c) $1+2i$ (d) $3+i$ (e) $3-i$

16. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix}$. What are the eigenvalues of A?

(a) 1, -2, 2 (b) 1, 2, -4 (c) 2, 2, 4 (d) 0, 1, 2 (e) -2, 1, 6

17. Let $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$. You are given that $\lambda = 1$ is an eigenvalue of A.

What is the dimension of the corresponding eigenspace?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

18. Which of the following sets are subspaces of R^3.

$$U=\left\{\begin{bmatrix}a\\b\\0\end{bmatrix} \mid a+b=1\right\}, \quad V=\left\{\begin{bmatrix}a\\b\\c\end{bmatrix} \mid a-b=c\right\},$$

$$W=\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}, \quad H=\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$$

(a) U and W (b) V and W (c) V and H (d) U and W (e) V only

19. Let $A=\begin{bmatrix}1 & -3\\3 & 1\end{bmatrix}$. What are the eigenvalues of A?

(a) $1\pm 3i$ (b) $-1\pm 3i$ (c) $\pm 3i$ (d) ± 1 (e) 1 and 3i

20. Let A be a 2×2 matrix with eigenvectors $v_1=\begin{bmatrix}4\\-1\end{bmatrix}$ and $v_2=\begin{bmatrix}-3\\1\end{bmatrix}$ corresponding to eigenvalues $\lambda_1=1$ and $\lambda_2=-1$, respectively. What is the matrix A?

(a) $\begin{bmatrix}7 & -2\\-7 & 25\end{bmatrix}$ (b) $\begin{bmatrix}4 & -1\\-3 & 1\end{bmatrix}$ (c) $\begin{bmatrix}7 & -2\\24 & -7\end{bmatrix}$ (d) $\begin{bmatrix}-3 & 1\\4 & -1\end{bmatrix}$ (e) $\begin{bmatrix}7 & 24\\-2 & -7\end{bmatrix}$

21. Let $x = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $W = \text{Span}\{u, v\}$.

What is the orthogonal projection of the vector x onto the subspace W?

(a) $\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ -4 \\ -7 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

22. Let $u_1 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$, and $u_3 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.

You are given that $S = \{u_1, u_2, u_3\}$ is an orthogonal basis for R^3.

Let $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. What is the value of c_3 in the equation $x = c_1u_1 + c_2u_2 + c_3u_3$?

(a) -1 (b) 1 (c) -2 (d) 2 (e) 4

23. Let $x = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

What is the distance from x to the line through u and the origin?

(a) $\sqrt{20}$ (b) $\sqrt{45}$ (c) $\sqrt{50}$ (d) $\sqrt{90}$ (e) $\sqrt{125}$

24. Let $u = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. What is the angle between the vectors u and v?

(a) $\pi/3$ (b) $-\pi/3$ (c) $2\pi/3$ (d) $-2\pi/3$ (e) $\pi/2$

25. Let $A = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$. Which of the following vectors is in $(\mathrm{Col}A)^{\perp}$?

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ (e) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$