Solutions _ Version 2

(B)

Question 1. (4 points) Find and classify the critical points of the function $f(x, y) = 2x^3 + y^2 - 2xy$.

(2) for work

$f_x = 6x^2 - 2y$

$f_y = 2y - 2x = 2(y-x)$

$f_y = 0$ only if $x = y$

so then $f_x = 6x^2 - 2x = 2x(3x-1) = 0$ if $x = 0, 1/3$

thus there are 2 critical pts $(0,0)$, $(1/3, 1/3)$

$f_{xx} = 12x$, $f_{xy} = -2$, $f_{yy} = 2$

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2 = 24x - 4$

$D(0,0) < 0 \Rightarrow$ $\boxed{(0,0) \text{ saddle pt}}$ (1)

$D(1/3, 1/3) > 0$, $f_{xx}(1/3, 1/3) > 0$

$\Rightarrow$ $\boxed{(1/3, 1/3) \text{ local min}}$ (1)

B

Question 2. (4 points) Use Lagrange Multipliers to find the absolute maximum and minimum values of the function $f(x,y) = (x+y)^2$ subject to the constraint $x^2+y^2 = 8$.

3 for work

$$f_x = \lambda g_x \Rightarrow 2(x+y) = 2\lambda x \Rightarrow x+y = \lambda x$$

$$f_y = \lambda g_y \Rightarrow 2(x+y) = 2\lambda y \Rightarrow x+y = \lambda y$$

either $x=y$ or $\lambda = 0$ and $x=-y$

$$x^2+y^2 = 8 \Rightarrow 2x^2 = 8 \Rightarrow x = \pm 2$$

4 points $(2,2)$, $(2,-2)$, $(-2,2)$ and $(-2,-2)$

$f(2,2) = f(-2,-2) = 16$ max

$f(2,-2) = f(-2,2) = 0$ min

$\therefore$ $\boxed{\text{max is } 16 \quad \text{min is } 0}$

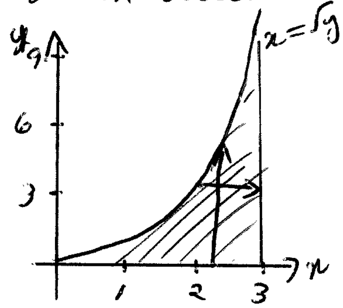
1

(B)

Question 3. (4 points) Evaluate the following integral

$$\int_0^9 \int_{\sqrt{y}}^3 \frac{1}{1+x^3}\, dx\, dy\,.$$

switch order



x goes $\sqrt{y}$ to 3

then y is 0 to 9

so y goes from 0 to x^2

and then x is 0 to 3

so $\displaystyle\int_0^9\int_{\sqrt{y}}^3 \frac{1}{1+x^3}\,dx\,dy = \int_0^3\int_0^{x^2}\frac{1}{1+x^3}\,dy\,dx$ 1

$\displaystyle = \int_0^3 \frac{x^2}{1+x^3}\,dx$ 2 for work

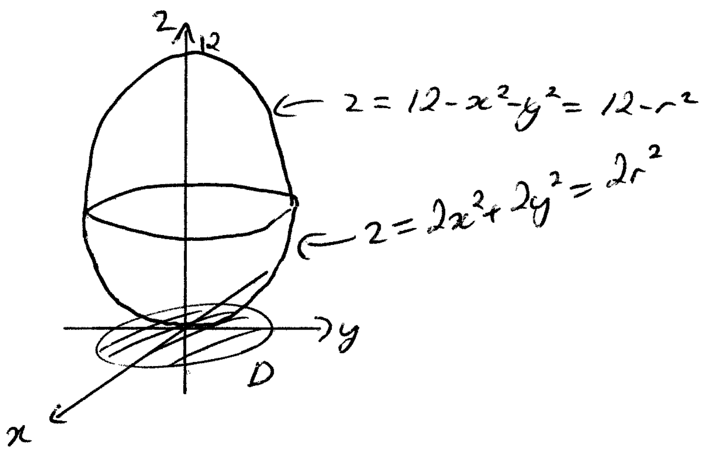
$\displaystyle = \frac{1}{3}\ln(1+x^3)\Big|_0^3$

$= \boxed{\frac{1}{3}\ln 28}$ 1

$\simeq \boxed{1.1107}$

(B)

Question 4. (4 points) Find the volume of the solid enclosed by the paraboloids $z = 12 - x^2 - y^2$ and $z = 2x^2 + 2y^2$.



intersection

$$12 - x^2 - y^2 = 2x^2 + 2y^2$$

$$x^2 + y^2 = 4$$

D is disk of radius 2

use polar coordinates

$$V = \int_0^{2\pi} \int_0^2 (12 - r^2 - 2r^2)\, r\, dr\, d\theta$$

$$= 2\pi \int_0^2 (12r - 3r^3)\, dr$$

$$= 2\pi \left(6r^2 - \frac{3}{4} r^4 \Big|_0^2\right)$$

$$= 2\pi (24 - 12)$$

$$= \boxed{24\pi}$$

$$\simeq \boxed{75.4}$$

3 for work

1

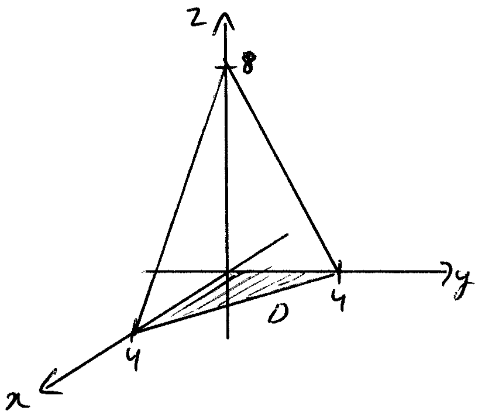
Also, view the volume as a triple integral;

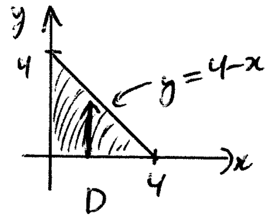
$$V = \int_0^{2\pi} \int_0^2 \int_{2r^2}^{12 - r^2} dz\, r\, dr\, d\theta = \int_0^{2\pi} \int_0^2 (12 - 3r^2)\, r\, dr\, d\theta$$

$$= \cdots$$

B

Question 5. (4 points) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $2x + 2y + z = 8$.





$$V = \int_0^4 \int_0^{4-x} \int_0^{8-2x-2y} dz\,dy\,dx$$

$$= \int_0^4 \int_0^{4-x} (8-2x-2y)\,dy\,dx$$

$$= \int_0^4 \left((8-2x)y - y^2 \Big|_0^{4-x} \right) dx$$

$$= \int_0^4 \left(2(4-x)^2 - (4-x)^2 \right) dx$$

$$= \int_0^4 (4-x)^2\,dx$$

$$= -\frac{1}{3}(4-x)^3 \Big|_0^4$$

$$= \boxed{\frac{64}{3}}$$

3 for work

1