Answers to the second Midterm.

Q1

1) C
2) D
3) B
4) D
5) D
6) A
7) C
8) D
9) A

Q2

1) a. $\max_{Q} \pi = PQ - 2Q$ $\qquad P = \frac{480-Q}{2} = 240 - \frac{Q}{2}$

$= (240 - \frac{Q}{2}) \cdot Q - 2Q$

F.O.C. $\frac{d\pi}{dQ} = 0 \Rightarrow -\frac{Q}{2} + 240 - \frac{Q}{2} - 2 = 0$

$\therefore \quad -Q + 238 = 0$

$\therefore Q^* = 238 \qquad \therefore P^* = 121$

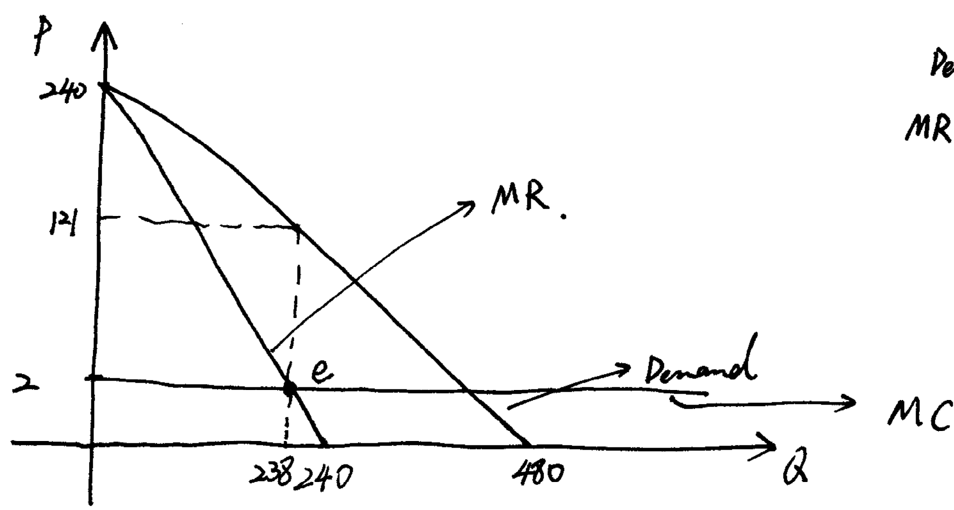
Or: $\max_{P} \pi = P(480-2P) - 2(480-2P)$

$\therefore$ F.O.C. $480 - 2P - 2P + 4 = 0$

$\therefore P^* = 121 \qquad Q^* = 238$

The above two methods are equivalent! Both are correct!

2)



Demand: $P = 240 - \frac{Q}{2}$

$MR = \cancel{240 - \frac{Q}{4}}\ 240 - Q$

Grading guidance: Demand curve 2 points

MR curve 2 ..

correctly labelling the ~~p~~ axis 2 points

MC curve 2 points

e point 2 points

2)

1) Firm A:

$$\max_{q_A} \Pi_A = P q_A - 4 - q_A$$

$$= (40 - 4q_A - 4q_B) q_A - 4 - q_A$$

F.O.C. $\frac{d\Pi_A}{dq_A} = 0 \Rightarrow \quad -4q_A + 40 - 4q_A - 4q_B - 1 = 0$

$\therefore \quad 8q_A = 39 - 4q_B$

$\therefore \quad q_A = \frac{39 - 4q_B}{8}$

$\therefore$ By symmetry,

$q_A^* = q_B^* = q^*$

$\therefore \quad q^* = \frac{39 - 4q^*}{8}$

$\therefore \quad \cancel{8}\,12 q^* = 39$

$\therefore \quad q_A^* = \cancel{\frac{39}{8}}\ \frac{39}{12}$

$\therefore \quad P^* = 40 - 4 \times \frac{39}{\cancel{8}\,6}$

$= \frac{240 - 156}{6} = 14$

$\therefore \quad \cancel{\Pi_A^* = 14 \times \frac{39}{8} - 4 - \frac{39}{8}}$

$\cancel{= 4.}$

$$\Pi_A^* = 14 \times \frac{39}{12} - 4 - \frac{39}{12}$$

$$= \cancel{5.75}\ 38.25$$

2) Firm A Leader:

$$\max_{q_A} \Pi_A = P q_A - 4 - q_A = \left[40 - 4q_A - 4\left(\frac{39 - 4q_A}{8}\right)\right] q_A - 4 - q_A$$

$\therefore$ F.O.C. $\frac{d\Pi_A}{dq_A} = (-4 + 2) q_A + \left(40 - 4q_A - \frac{39}{2} + 2q_A\right) - 1 = 0$

$-2q_A - 2q_A + 39 - \frac{39}{2} = 0$

$\therefore$

$\therefore \quad q_A^* = \frac{39}{8} = \frac{39}{8}$

$\therefore q_B^* = \dfrac{39-4q_A}{8} = \dfrac{39 - 4\times\frac{39}{8}}{8} = \dfrac{39}{16}$

$\therefore P^* = 40 - 4\left(\frac{39}{16} + \frac{39}{8}\right) = \frac{43}{4}.$

$\therefore \pi_A^* = \cancel{\frac{48}{4}}\ \frac{43}{4} \times \frac{39}{8} - 4 - \frac{39}{8} = \frac{1677}{32} - 4 - \frac{39}{8} = 43.531$

3) Cartel:

$\max_Q (PQ - 4 - Q) = \pi = (40-4Q)Q - 4 - Q$

$\therefore$ F.O.C. $\frac{d\pi}{dQ} = 0 \quad \therefore \quad -4Q + 40 - 4Q - 1 = 0$

$\therefore \quad 8Q = 39$

$\therefore Q^* = \frac{39}{8} \qquad P^* = \cancel{48}\ 40 - \frac{39}{2} = \frac{41}{2}$

$\therefore$ Firm A's profit: $\quad q_A = \frac{39}{16}$

$\frac{41}{2} \times \frac{39}{16} - 4 - \frac{39}{16}$

$= 43.531$

$\therefore \quad \pi_{A\,\text{cournot}}^* < \pi_{A\,\text{leader}}^*$

$\pi_{A\,\text{cournot}}^* < \pi_{A\,\text{cartel}}^*$

$\pi_{A\,\text{cartel}}^* = \pi_{A\,\text{leader}}^*$

3) $\underset{P_p}{Max}\, \Pi_p = P_p \cdot (50 - 2P_p + 1P_c) - m q_p = P_p(50 - 2P_p + P_c) - m(50 - 2P_p + 1P_c)$

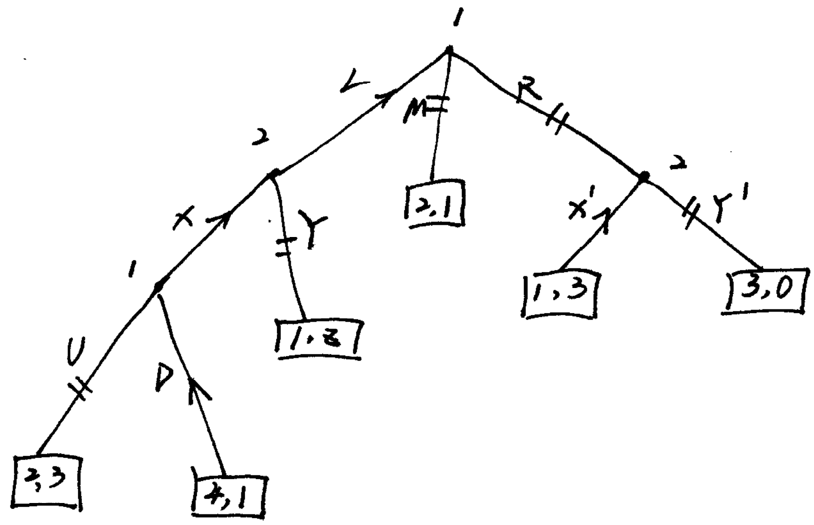
$\therefore$ F.O.C. $\frac{d\Pi_p}{dP_p} = 50 - 2P_p + P_c - 2P_p + 2m = 0$

$\therefore \quad P_p = \frac{50 + P_c + 2m}{4} \rightarrow$ B.R. Function

$\therefore \quad \frac{dP_p}{dm} = \frac{1}{2} = 0.5$

$\therefore$ $m \uparrow$ by $1\$ \Rightarrow P_p \uparrow$ by $0.5\$$.

4)



a) In the last subgame, player 1 plays D since $4 > 2$.
In the second last left game, player 2 plays X since $1 > 0$.
In the second last right game, player 2 plays X' since $3 > 0$
In the first subgame, player 1 plays L since $4 > 2 > 1$.
$\therefore$ The Subgame perfect N. E. is:
player 1 plays L at the beginning, then she plays D in the last subgame.
player 2 plays X if Player 1 plays L, and plays X' if player 1 plays R.

b) Player 1 plays M, the ~~pb~~ payoff is 2,
. . . . . R, ... 1, $\Big\} \Rightarrow 2 > 1$

In order to make player 1 plays M in the subgame perfect. N.E.
The payoff of playing L has to be smaller than 2. ☆

⇒ The value of z does not affect the behavior of player 1 in the last subgame. ⇒ player 1 will play D since $(4 > 2)$

In the last ~~b~~ second left subgame, player 2 plays X ⇒ payoff = 1
. . . . player 2 plays Y ⇒ payoff = z.

if $z > 1$ ⇒ player 2 plays Y ⇒ the payoff of player 1 plays L is 1 !
if $z < 1$ ⇒ . . . . X ⇒ . . . . 4 !

∵, In order to play M for player 1,
The ~~play~~ payoff of L has to be smaller than 2 !

∴ $z > 1$