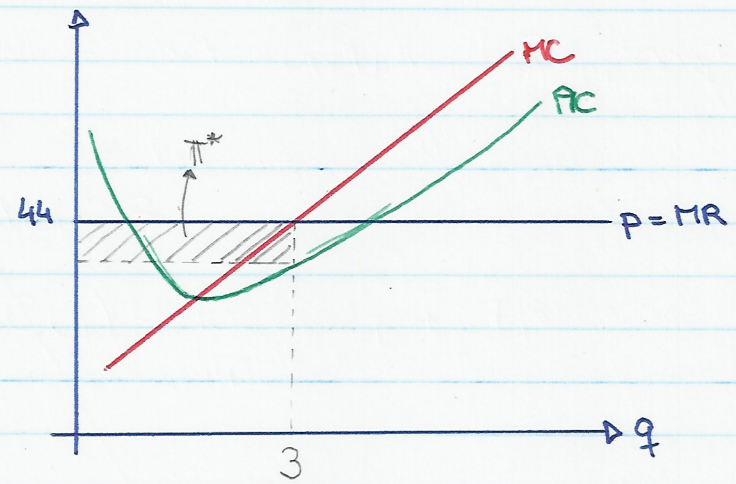
b)



c) $$\max_q \pi = pq - 10 - 2q - 7q^2 - tq$$

$$\frac{d\pi}{dq} = p - 2 - 14q - t = 0$$

$$\Leftrightarrow \quad 42 - 14q - t = 0 \quad \text{(substitute } p\text{)}$$

$$\Leftrightarrow \quad q^* = \frac{42 - t}{14}$$

d) The change in output resulting from a change in the tax is expressed as:

$$\frac{dq}{dt} = -\frac{1}{14} < 0$$

As the tax increases, output decreases.

2) $\min_{K,L} \quad C = wL + rK \quad \text{s.t.} \quad \bar{Q} = 4LK$

$$\mathcal{L} = wL + rK + \lambda(\bar{Q} - 4LK)$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda 4K = 0 \qquad \text{Eq (1)} \implies w = \lambda 4K$$

$$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda 4L = 0 \qquad \text{Eq (2)} \implies r = \lambda 4L$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{Q} - 4LK = 0 \qquad \text{Eq (3)} \implies \bar{Q} = 4LK$$

Divide Eq(1) by Eq (2):

$$\frac{w}{r} = \frac{K}{L} \iff \frac{wL}{r} = K$$

Substitute for K in Eq (3)

$$\bar{Q} = 4L \cdot \frac{wL}{r} \iff \bar{Q} = \frac{4wL^2}{r}$$

$$\iff \frac{\bar{Q}r}{4w} = L^2$$

$$\iff \boxed{L^* = \sqrt{\frac{\bar{Q}r}{4w}}}$$

Replace L and solve for K

$$K = \frac{wL}{r} \iff K = \frac{w}{r}\sqrt{\frac{\bar{Q}r}{4w}}$$

$$\iff \boxed{K^* = \sqrt{\frac{\bar{Q}w}{4r}}}$$

The long-run cost function is:

$$C^* = wL^* + rK^* \iff C^* = w\sqrt{\frac{Qr}{4w}} + r\sqrt{\frac{Qw}{4r}}$$

$$\iff C^* = \sqrt{\frac{Qrw}{4}} + \sqrt{\frac{Qrw}{4}}$$

$$\iff C^* = \cancel{2}\frac{\sqrt{Qrw}}{\cancel{2}} \qquad \left[\text{note } \frac{1}{\sqrt{4}} = \frac{1}{2}\right]$$

$$\iff \boxed{C^* = \sqrt{Qrw}}$$

3)

Agent 2

Agent 1	Contribute	Free Ride
Contribute	$4-c_1$, $4-c_2$	$4-c_1^*$, 4^*
Free Ride	4^* , $4-c_2^*$	0 , 0

b) Agent 1 :
- If Agent 2 contributes ⇒ best response for Agent 1 is to not contribute
- If Agent 2 does not contribute ⇒ best response for Agent 1 is to contribute

Agent 2 :
- If Agent 1 contributes ⇒ best response for Agent 2 is to not contribute
- If Agent 1 does not contribute ⇒ best response for Agent 2 is to contribute.

Pure Nash Equilibria

i) A1 : NOT A2 : CONTRIBUTE

ii) A1 : CONTRIBUTE A2 : NOT.