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MATH 1300-MIDTERM # 1-2009

NAME and I.D.# Kevin Tang 000558?638

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. **You may use the backs of pages.**

For long answer questions, YOU MUST SHOW YOUR WORK

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Answers:

#1	#2	#3	#4
E	A	D	A

Multiple Choice Section Questions (1-4)

Question 1 *Calculate:*

$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$$

A) 9 **B)** 18 **C)** 27 **D)** 36 **E)** The limit does not exist.

$$\cancel{\frac{x^3 - 3^3}{x-3}}$$

$$\cancel{(x-3)}(x^2+3x+9$$

Question 2 *Find the equation of the tangent line of the function* $f(x) = (2x+1)\sqrt{3x+1}$ *at* $x = 1$.

$f(1) = 6$

A) $y = \frac{25x}{4} - \frac{1}{4}$ **B)** $y = \frac{x}{9} + \frac{2}{9}$ **C)** $y = \frac{25x}{4} - \frac{7}{4}$ **D)** $y = \frac{x}{9} + \frac{2}{9}$ **E)** $y = \frac{25x}{2} - \frac{1}{2}$

$(2x+1)\frac{1}{2}(3x+1)^{-\frac{1}{2}}(3) + \sqrt{3x+1}\,(2)$

$(2+1)\frac{1}{2}(3+1)^{-\frac{1}{2}}(3) + \sqrt{3+1}\,(2)$

$9\left(\frac{1}{4}\right) + \frac{(16)}{4}$

$\frac{25}{4}$

$y = \frac{25}{4}x + b$

$6 = \frac{25}{4}(1) + b$

$\frac{24}{4} - \frac{25}{4} = b$

$b = -\frac{1}{4}$

Question 3 *Use implicit differentiation to find $\frac{dy}{dx}$ at the point (2,1) for the equation:*

$$x^2y + 2xy^2 = 8$$

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{4}{3}$ D) $\frac{-1}{2}$ E) $\frac{-2}{3}$

$x^2 yy' + 2xy + 2x\,2yy' + 2y^2 = 0$

$x^2yy' + 2x2yy' = -2xy - 2y^2$ $y' = \frac{-1}{2}$

$y'(x^2y + 2x2y) = -2xy - 2y^2$

$y'(2^2(1) + 2(2)2(1)) = -2(2)(1) - 2(1)^2$

8 -4 -2

$y'\frac{(12)}{12} = \frac{-6}{12}$ $-\frac{1}{2}$

Question 4 *Find the inverse of the function:*

$$f(x) = \frac{2x-1}{x+3}$$

A) $\frac{-3x-1}{x-2}$ B) $\frac{2x-3}{x-2}$ C) $\frac{3x-1}{x-3}$ D) $\frac{-3x-1}{2x+3}$ E) $\frac{2x-1}{3x+1}$

$x = \frac{2y-1}{y+3}$

$x(y+3) = 2y-1$

$xy + 3x - 2y + 1 = 0$

$(xy - 2y) = -3x - 1$

$y(x-2) = -3x - 1$

Long Answer Section Questions (5-7)

Question 5 (12 points) *Using only the definition of derivative as a limit, calculate $f'(x)$ where*

let $f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$

$$f(x) = (3 - 2x)^2$$

$$f'(x) = \lim_{h \to 0} \frac{(3-2x+h)^2 - (3-2x)^2}{h}$$

$(3-2x+h)(3-2x+h) - (3-2x)(3-2x)$

$\cancel{9} - \cancel{6x} + 3h - \cancel{6x} + \cancel{4x^2} - 2hx + 3h - 2hx + h^2 -$

$\cancel{9} + \cancel{6x} + \cancel{6x} - \cancel{4x^2}$

$3h - 2hx + 3h - 2hx + h^2$

$$= \lim_{h \to 0} \frac{3\cancel{h} - 2\cancel{h}x + 3\cancel{h} - 2\cancel{h}x + h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h \to 0} 3 - 2x + 3 - 2x + h$$

$$= \lim_{h \to 0} 6 - 4x + h$$

$$= 6 - 4x$$

$$\boxed{f'(x) = 6 - 4x}$$

4

Question 6 (14 points) *Suppose that a deposit of 3,000 \$ is made into a bank that gives 5% interest. Suppose that interest is compounded 6 times per year.*

(a) *(2 points) Write a formula for $A(t)$, the value of the investment, after t years in this case.*

(b) *(4 points) How much will the investment be worth after 3 years?*

(c) *(8 points) How long will it take for the investment to triple?*

Let $P =$ initial deposite, $n =$ # of time compounded per year, $I =$ interest, $t =$ time in years

a) $A(t) = P\left(1+\frac{I}{n}\right)^{nt}$

$A(t) = 3000\left(1+\frac{.05}{6}\right)^{6t}$

$\boxed{A(t) = 3000\left(1+\frac{.025}{3}\right)^{6t}}$

b) $\boxed{A(3) = 3000\left(1+\frac{.025}{3}\right)^{18}}$ $\frac{6.05}{6}$

c) $9000 = 3000\left(1+\frac{.025}{3}\right)^{6t}$

$3 = \left(1+\frac{.025}{3}\right)^{6t}$

$\ln(3) = \ln\left(1+\frac{.025}{3}\right)^{6t}$ $\frac{6.05}{6}$

$\ln(3) = 6t \ln\left(1+\frac{.025}{3}\right)$

$\boxed{t = \frac{\ln(3)}{6\ln\left(1+\frac{.025}{3}\right)}}$

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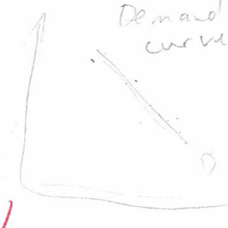
Question 7 (14 points)

- *(8 points) A business sells 5,000 radios per month at a price of 300 dollars each. It is estimated that monthly sales will increase by a level of 50 units for each 2 dollar decrease in price. Find the demand function, as well as the revenue function.*
- *(6 points) Suppose another business makes burglar alarms. Suppose that there is a initial fixed cost of 25,000 dollars. Suppose each alarm costs 90 dollars to make and that the manufacturer has set a price of 150 dollars per alarm. Write down a profit and cost function, and determine the breakeven point.*

RADIO BUSINESS 5000 Radios price 300 every $2 ↓ 50 unit ↑

let Revenue = R(x) & Demand = D(x)

Demand curve.



Demand function is negative

$R(x) = (5000 + 50(x))(300 - 2(x))$

$D(x) = -2(x)$

6

BURGLAR ALARM BUSINESS

let C(x) = Cost
R(x) = Revenue
x: # of unit

$C(x) = 25000 + 90(x)$

$R(x) = x(150)$

$P(x) = R(x) - C(x)$

$P(x) = x(150) - (25000 + 90x)$

BREAK EVEN POINT

$R(x) = C(x)$

$x(150) = 25000 + 90(x)$

$150x - 90x = 25000$

$\frac{60x}{60} = \frac{25000}{60}$

$x = \frac{25000}{60}$

$x \approx 416.6666$

$x \approx 417$

∴ - Profit function
$P(x) = 150x - (25000 + 90x)$
- Cost function
$C(x) = 25000 + 90x$,
- Break even point approximatly 417 units

Bonus Question (1 point): Name the two historical figures who can both claim to have invented calculus. **SPELLING COUNTS!**

Benjamin Franklin
Alfred Einstien

MATH 1300-MIDTERM # 2-2009

NAME and I.D.# Kevin TANG 5583638

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Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. Your answers for numerical questions should be ready to be plugged into a calculator (i.e. you may leave an answer such as $x = \frac{2}{3 \ln 4}$ without simplifying).

For long answer questions, YOU MUST SHOW YOUR WORK

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Answers:

F	F	F	C
#1	#2	#3	#4

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Multiple Choice Section Questions (1-4)

Question 1 *Find all asymptotes of the function* $f(x) = \frac{x^2-x-6}{x^2-4x+3}$.

A) vertical asymptote at $x = 1$, no horizontal asymptotes
B) vertical asymptotes at $x = 1, x = 3$, no horizontal asymptotes
C) vertical asymptote at $x = 1$, horizontal asymptote at $y = 0$
D) vertical asymptote at $x = 1$, horizontal asymptote at $y = 1$
E) vertical asymptotes at $x = 1, x = 3$, horizontal asymptote at $y = 0$
F) vertical asymptotes at $x = 1, x = 3$, horizontal asymptote at $y = 1$

Question 2 *Suppose that the demand function for a product is given by* $p = 8 + \frac{100}{x+5}$. *What is the elasticity of demand when* $x = 45$? *Is demand elastic or inelastic?*

A) $\eta = -\frac{10}{45}$, elastic **B)** $\eta = -\frac{10}{45}$, inelastic **C)** $\eta = -\frac{1}{25}$, elastic

D) $\eta = -\frac{1}{25}$, inelastic **E)** $\eta = -\frac{50}{9}$, elastic **F)** $\eta = -\frac{50}{9}$, inelastic

Question 3 *Given the function $f(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - 3x + 2$, which of the following statements is correct?*

A) $f(x)$ has a local minimum at $x = 0$
B) $f(x)$ has a local maximum at $x = 0$
C) $f(x)$ has an inflection point at $x = 0$
D) $f(x)$ has a local maximum at $x = -\frac{1}{4}$
E) $f(x)$ has a local minimum at $x = -\frac{1}{4}$
F) $f(x)$ has an inflection point at $x = -\frac{1}{4}$

Question 4 *Consider the function $g(x) = x^3 - \frac{3}{2}x^2 - 6x$. Where does the absolute maximum of $g(x)$ on the interval $[-2, 5]$ occur?*

A) At $x = -2$ **B)** At $x = -1$ **C)** At $x = 0$ **D)** At $x = 2$ **E)** At $x = 5$

$g'(x) = 3x^2 - 2x - 6$

CP there are no C.P

x	y
-2	+10
-1	-2
0	-4
2	-2
5	

Long Answer Section Questions (5-7)

Question 5 (12 points) *A frog population on a tropical island is growing exponentially. In 1999, there were 200 animals, and in 2004 there were 220 animals.*

(a) *(6 points) Find a formula which describes the size of the frog population as a function of time (measured in years).*

(b) *(2 points) What will the population size be in 2024?*

(c) *(4 points) How many years will it take before there are 1000 frogs on the island?*

a) let pop. formula $= A(t) = P(b)^t$

where t = time in yr, b = Growth rate or frog

P = inital amount of frogs, $A(t)$ = Population of frogs

let 1999 be inital time $t=0$

$$A(t) = P(b)^t$$

$$220 = 200(b)^5$$

$$\frac{220}{200} = b^5 \rightarrow \frac{11}{10} = b^5 \qquad b = \left(\frac{11}{10}\right)^{\frac{1}{5}}$$

$$\boxed{A(t) = 200\left(\frac{11}{10}\right)^{\frac{t}{5}}}$$

b)

$$\begin{array}{r} 2024 \\ 1999 \\ \hline 25 \end{array}$$

$$A(25) = 200\left(\frac{11}{10}\right)^{\frac{25}{5}}$$

$$= 200\left(\frac{11}{10}\right)^5$$

12

c)

$$1000 = 200\left(\frac{11}{10}\right)^{t/5}$$

$$\frac{1000}{200} = \frac{11}{10}^{\frac{t}{5}}$$

$$5 = \frac{11}{10}^{t5} \rightarrow \ln(5) = \ln\left(\frac{11}{10}\right)^{t/5} \rightarrow \ln(5) = \frac{t}{5}\ln\left(\frac{11}{10}\right)$$

$$\boxed{t = \frac{5\ln(5)}{\ln\left(\frac{11}{10}\right)}}$$

5

Question 6 (14 points) *Consider the function* $f(x) = 3x^{\frac{2}{3}} - 2x + 1$.

(a) *(2 points) Find the derivative of* f.

(b) *(3 points) Find all critical points of this function.*

(c) *(4 points) Classify the critical points. Indicate which test(s) you use and how.*

(d) *(5 points) Find all inflection points and intervals where the function is concave up and all intervals where it is concave down.*

a) $f(x) = 3x^{2/3} - 2x + 1$

$\boxed{f'(x) = 2x^{-1/3} - 2}$

b) CP

$0 = 2x^{-\frac{1}{3}} - 2$

$\left(\frac{2}{2}\right)^{-\frac{1}{3}} = x^{-\frac{1}{3}} \quad (1)^{-\frac{1}{3}} = x \quad x = 1$

, $x = 0$

There is one CP at $x = 1$

c) Use the first Derivative test to find the classification of the CP
1st Derivative is already calculated & CP

$-$ 0 $-$

1

The CP is neither a local max or min & neither a global max or min

d) Using second Derivative test

$f''(x) = -\frac{2}{3}x^{-\frac{4}{3}} \cancel{-2}$

$0 = -\frac{2}{3}x^{-4/3} - 2$

$\cancel{-2}(3) \; x^{-4/3}$

$\cancel{-2}$

$3^{-3/4} = x$

$+ \quad | \quad -$

$\cap \cancel{\cup} \quad 3^{-\frac{3}{4}} \quad \cap$

Concave up $(-\infty, -\cancel{9}^{-\frac{3}{4}})$

Concave Down $(-\cancel{9}^{-\frac{3}{4}}, \infty)$

Question 7 (14 points) *A company produces household appliances, and has found that the demand function for refrigerators is given by* $p = 200 + \frac{80{,}000}{\sqrt{x}}$. *The company has an initial cost of 100,000 dollars and each unit costs 600 dollars to make.*

How many refrigerators should the company produce in order to maximize profit? Be sure to explain why your answer is an absolute maximum.

$R(x)$ = revenue, $C(x)$ = cost, $P(x)$ = Profit

let $R(x) = x \cdot P$, $C(x) = 100000 + 600x$, $P(x) = R(x) - C(x)$

8

$R(x) = x \cdot 200 + \frac{80000}{\sqrt{x}}x$ +3

$= 200x + \left(\frac{80000}{\sqrt{x}}\right)x$ ok

+3

$P(x) = 200x + \left(\frac{80000}{\sqrt{x}}\right)x - (100000 + 600x)$

$= 800x + \left(\frac{80000}{\sqrt{x}}\right)x, -100000$

$P'(x) = 800 + \frac{80000}{\sqrt{x}}$ +2

$= 800 + 80000(x)^{-\frac{1}{2}}$

$\frac{-800}{80000} =$

$\left(-\frac{1}{100}\right)^{-2} = x \qquad x = \frac{-100}{(1)^{-2}} \qquad x = 100$

Since there is only 1 CP & there is a local max, By therm not in the book it is also a global max at $x = 100$

$+ \quad 100 \quad -$ local max

Bonus Question (1 point): Name either Professor Blute's favorite musician or his favorite band, according to his website.

+6 Beatles

Space for additional work

$x = 45 \quad P = 8 + \frac{100}{45+5}$

$= 8 + 2$

$= 10$

$P' = \cancel{8+} \frac{100}{x+5}$

$-100(x+5)^{-2}(1)$

$= \frac{-100}{(x+5)^2}$

$\frac{-100}{(x+5)^2}$

$= \frac{-100}{(45+5)^2}$

$\frac{-100}{2500}$

$-\frac{1}{25}$

$2x^2 + x - 3 = 0$

$(2x-3)(x-1)$

$4x + 1 \quad x = -\frac{1}{4}$ inflection

$x = 1, \frac{3}{2}$

$2x^{-\frac{1}{3}} - 2$

$2(8)^{-\frac{1}{3}} - 2$

$2 \frac{1}{\sqrt[3]{8}}$

$2\left(\frac{1}{2}\right) - 2 \quad 1 - 2$

$\frac{2}{2} - 2 \quad = -1$

$2(1)^{-\frac{1}{3}} - 2$

$\frac{1}{\sqrt[3]{1}}$

$2(1) - 2 = 0$

$\frac{1}{2(9)}$

$\frac{1}{18} \quad \frac{1}{4}$