Canadian Mortgages

By

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1 Mortgages

1.1 Mortgages

A mortgage is a loan which is secured with property. The borrower uses a mortgage to pledge property to the lender as security against the loan. In legal terms, the mortgage gives the legal title of the land to the lender and an equitable title (called "equity of redemption") to the borrower. The legal title only exists as a security for the debt and does not convey any title or powers to the lender. So, the borrower continues to live (or work) on the property and the lender does not have the right to use or occupy the property.

To protect the lender, a mortgage is recorded in the public records creating a lien. Before a mortgage is finalized, banks and other mortgage lenders run title searches of the real property to make certain that the lien of the mortgage is prior to anyone else's claim against the property.

Mortgages have been around for a long time. The Oxford English Dictionary shows the first usage of the word in 1393. Under English common law a mortgage was an actual transfer of title to the lender, with the borrower having the right to occupy the property while it was in effect, but non-payment ended the right of occupation.

Mortgage lending is a major category of the business of finance. In the U.S. and Canada mortgages are commercial paper and can be conveyed and assigned freely to other holders. In the U.S., the Federal Housing Administration administers the programs colloquially known as “Ginnie Mae,” “Fannie Mae,” and "Freddie Mac" (also known as the GSEs or government sponsored entities) to foster mortgage lending and thus to encourage home ownership and construction. In Canada, the Canada Mortgage and Housing Corporation (a Crown Corporation) performs the same role.

These programs work by buying a large number of mortgages from banks and issuing (at a slightly lower interest rate) "mortgage-backed securities" (MBS) to investors. This allows the banks to quickly relend the money to other borrowers and thereby to create more mortgages than the banks could with the money they have on deposit. This in turn allows the public to use these mortgages to purchase homes, which is something the government wishes to encourage.

1.2 Institutional Details

Mortgages have two relevant time periods: 1) the amortization period, and 2) the term. The amortization period is the length of time over which payments will be made. A typical residential amortization period is 25 years. Thus, it will take 25 years to fully repay the loan. The term refers to the length of time over which the interest rate is set. At the end of the term, the interest rate is renegotiated and loan payments are recalculated.

There are two basic types of mortgage loans: 1) Fixed Rate Mortgage (FRM), and 2) Adjustable Rate Mortgage (ARM). In a FRM, the interest rate, and hence monthly payment, remains fixed for the term of the loan. In the U.S., the term is usually for 10, 15, 20, or 30 years. However, the fixed term can be as short as five years. In Canada, most customers choose fixed loan terms between 1 and 5 years, but banks do offer longer terms such as 10 and even 20 years.

In an ARM, the interest rate is fixed for a period of time, after which it will periodically (quarterly or monthly) adjust up or down to some market index. Common indices include the prime rate, the London Inter-Bank Offer Rate (LIBOR), and the Treasury Index ("T-Bill").

1.3 Mortgage Payment Calculation

In Canada, mortgage lenders are restricted in the rate that they can charge on mortgages under the National Housing Act. Mortgage lenders are restricted to charging a rate that is “computed semi-annually and not in advance”. The phrase “computed semi-annually” means that the effective rate charged by the lender can be no greater than the quoted rate compounded semi-annually. The "not in advance" part means that the mortgage lender cannot charge discount interest (interest at the start of the month), rather the interest must be calculated at the end of the month or in “arrears” (which is the normal way that we calculate interest).

If a mortgage lender quoted a rate of 5%, then the highest effective rate that they can get is:

$$\text{EIR}(5\%, 2) = \left(1 + \frac{0.05}{2}\right)^2 - 1 = 0.0506.$$

Mortgage payments are paid at regular time intervals and are of equal size—they blend both interest and principal—so a mortgage is just an amortized loan.

To solve for mortgage payments we use the amortized loan equation of value, but there is one complicating feature. As we discussed above, lenders are restricted to semi-annual compounding, but borrowers can choose one of many different payment frequencies: weekly, bi-weekly (every two weeks), semi-monthly (every half month), or monthly. This creates a situation where the payment interval does not match the interest compounding interval.

We get around this problem by using the effective interest rate. If a bank quotes a rate of 5% on a home mortgage, then you know that with semi-annual compounding the EIR of the loan is 5.06%.

When calculating amortized loan payments we can choose any payment frequency, but we have to be careful that we use a corresponding interest rate which has an EIR of 5.06%. For example, if we choose monthly payments, m=12, then we must find a monthly rate which has an EIR of 5.06%. Let the Bank's quoted rate be denoted i, i = 5%. We need to find a periodic rate, denoted j (in this case monthly), which has the same effective interest rate as the EIR of i with semi-annual compounding.

Since the periodic rate compounded m times must have the same EIR, as the bank's quoted rate compounded twice, we find j using the following equality:

$EIR(i,2) = (1+ i/m)^m – 1$
$j= [1+ EIR(i,2)]^{1/m} – 1$

In this example, with i = 5% and m = 12:
EIR(5%,2) = 0.0506
So
$j = [1.0506]^{1/12} - 1 = 0.0041$

Keep in mind that j =0.0041 is a monthly rate in decimal form (not percentage).

If we substitute the formula for EIR(i,2) into the equation for j, then we can get an alternative formula for j that is a little easier to use:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

To solve for the mortgage payments, we use this periodic rate in the amortized loan equation of value as shown in the following example.

Example Solving for Mortgage Payments
What are the monthly payments on a mortgage for $300,000 with an amortization period of 25 years and a quoted rate of 5%?

Solution

Principal = $300,000
$i = 5\%$
$j = 0.0041$
$m = 12$
$n = 25$ years
$n*m = 12*25 = 300$ month

The amortized loan equation of value is:

$$\text{Principal} = \text{PMT} * \text{PVIFA}_{j,n*m}$$

$$\$300{,}000 = \text{PMT} * \text{PVIFA}_{0.41\%,300}$$
$$\$300{,}000 = \text{PMT} * 172.4321$$
$$\text{PMT} = \$1{,}739.82$$

While Canadian mortgages are simply amortized loans, you must remember to use the periodic rate, j, which incorporates the lending rate regulation, as we showed you above.

2 Practice Problems

1. **Periodic rate.** You have just purchased a $250000 home, and the bank has quoted you an interest rate of 6.5% on a 25-year mortgage, with payments occurring bi-weekly (at the end of the period) What is the bi-weekly interest rate?
2. **Mortgage payments.** You have just purchased a $250000 home, and the bank has quoted you an interest rate of 6.5% on a 25-year mortgage, with payments occurring bi-weekly (at the end of the period) What are your bi-weekly payments?
3. **Mortgage payments.** Your mortgage loan has a principal of $700,000, an amortization period of 20 years and a quoted rate of 5%. You elect to make 24 payments per year. What is the size of each mortgage loan payment?
4. **Periodic rate.** You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. What periodic (bi-weekly) rate will you use to solve for the mortgage payments?
5. **Mortgage payments.** You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. What are the bi-weekly mortgage payments?
6. **Principal Outstanding.** You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. After 3 years, how much principal is outstanding on this mortgage?
7. **Total Interest Paid.** You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. Over the life of the mortgage, what is the total amount of interest that you pay?
8. **Principal Outstanding.** Your mortgage loan has a principal of $800,000, a term of 40 years and a quoted rate of 8.75%. You elect to make semi-monthly payments. What is the amount of principal owing after five years of the 40 year term?

3 Solutions to Practice Problems

ID: 389a
TYPE: Topic
SECTION: Canadian Mortgages
DIFFICULTY: 2/4
DESCRIPTION: Calculate bi-weekly interest rate and payments on a mortgage.

QUESTION:

You have just purchased a $250000 home, and the bank has quoted you an interest rate of 6.5% on a 25-year mortgage, with payments occurring bi-weekly (at the end of the period) What is the bi-weekly interest rate? [express your answer in decimal form rounded to 6 places. i.e. 12.34567% = 0.123457]

SOLUTION:

First, calculate the EIR based on semi-annual compounding using the bank's quoted rate, i:

EIR = (1+ i/2)^2-1
EIR = (1+0.065/2)^2-1
EIR = 0.066056

Then, find the interest rate over the payment interval (bi-weekly) that has the same EIR. Let m be the payment (compounding) interval (26) and j is the bi-weekly rate.

EIR = (1+ j)^m - 1
0.066056 = (1 + j)^26 - 1
(0.066056+1)^(1/26) - 1 = j
j = 0.002463

This can be calculated in one step using the following formula:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

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ID: 389b
TYPE: EOT/EOC
SECTION: Canadian Mortgages
DIFFICULTY: 3/4

DESCRIPTION: Calculate payments on a mortgage.

QUESTION:

You have just purchased a $250000 home, and the bank has quoted you an interest rate of 6.5% on a 25-year mortgage, with payments occurring bi-weekly (at the end of the period) What are your bi-weekly payments?

SOLUTION:

Mortgage payments are just amortized loan payments and are solved using the amortized loan equation of value:

Principal = PMT * $PVIFA_{j,n*m}$

For Canadian mortgages the periodic rate is given by:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

j = (1 + 0.065/2)^(2/26) -1
j = 0.002463

n = 25
n * m = 25 * 26 = 650

PVIFA = (1-(1+0.002463)^(-26*25))/0.002463

PVIFA = 323.933015

PMT = Principal/ $PVIFA_{j,n*m}$
PMT = 250,000/323.933015
PMT = $771.72

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ID: 1270
TYPE: EOT/EOC
SECTION: Canadian Mortgages
DIFFICULTY: 3/4
DESCRIPTION: Calculate payments on a mortgage.

QUESTION:
Your mortgage loan has a principal of $700,000, an amortization period of 20 years and a quoted rate of 5%. You elect to make 24 payments per year. What is the size of each mortgage loan payment?

SOLUTION:

Mortgage payments are just amortized loan payments and are solved using the amortized loan equation of value:

Principal = PMT * $PVIFA_{j,n*m}$

For Canadian mortgages the periodic rate is given by:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

j = (1 + 0.05/2)^(2/24) -1
j = 0.00206

n = 20
n * m = 20 * 24 = 480

PVIFA = (1-(1+0.00206)^(-20*24))/ 0.002060

PVIFA = 304.6693

PMT = Principal/ $PVIFA_{j,n*m}$
PMT = 700,000/304.6693
PMT = $2297.57

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ID: 685a
TYPE: EOT/EOC
SECTION: Canadian Mortgages
DIFFICULTY: 2/4
DESCRIPTION: Calculate periodic rate for mortgage calculation.

QUESTION:

You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. What periodic (bi-weekly) rate will you use to solve for the mortgage payments?

SOLUTION:
For Canadian mortgages the periodic rate is given by:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

Where

i = the quoted rate
m the number of payments

j = (1 + 0.06/2)^(2/26) -1
j = 0.002276

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ID: 685b
TYPE: EOT/EOC
SECTION: Canadian Mortgages
DIFFICULTY: 3/4
DESCRIPTION: Calculate mortgage payments.

QUESTION:

You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. What are the bi-weekly mortgage payments?

SOLUTION:
Mortgage payments are just amortized loan payments and are solved using the amortized loan equation of value:

Principal = PMT * $PVIFA_{j,n*m}$

For Canadian mortgages the periodic rate is given by:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

j = (1 + 0.06/2)^(2/26) -1
j = 0.002276

n = 30
n * m = 30 * 26 = 780

PVIFA = (1 - (1.002276)^-780)/0.002276

PVIFA = 364.7375

PMT = Principal/ $PVIFA_{j,n*m}$

PMT = 270,000/364.7375
PMT = $740.1876

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ID: 685c
TYPE: EOT/EOC
SECTION: Canadian Mortgages
DIFFICULTY: 4/4
DESCRIPTION: Calculate principal outstanding

QUESTION:

You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. After 3 years, how much principal is outstanding on this mortgage?

SOLUTION:
For Canadian mortgages the periodic rate is given by:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

j = (1 + 0.06/2)^(2/26) -1
j = 0.002276

PMT = Principal/ $PVIFA_{j,n*m}$

n = 30
n * m = 30 * 26 = 780
PVIFA = (1 - (1.002276)^-780)/0.002276
PVIFA = 364.7375

PMT = Principal/ $PVIFA_{j,n*m}$
PMT = 270,000/364.7375
PMT = $740.1876

After three years the principal outstanding is the present value of the remaining payments. After three years there are 780 – 3*26 = 702 payments remaining.

Principal = PMT * $PVIFA_{j,n*m}$

Where

n*m = 702

Principal Outstanding = $740.1876 * [(1-(1.002276)^(-702)) / 0.002276]
Principal Outstanding = $259,287.29

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ID: 685d

DESCRIPTION: Calculate interest paid over life of mortgage

QUESTION:

You bought a ski chalet at Whistler for $270000. You borrow the full amount over a 30-year term. The bank quotes you a rate of 6%. You select bi-weekly payments. Over the life of the mortgage, what is the total amount of interest that you pay?

SOLUTION:

Interest paid = total payments - principal repaid

For Canadian mortgages the periodic rate is given by:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

j = (1 + 0.06/2)^(2/26) -1
j = 0.002276

PMT = Principal/ $PVIFA_{j,n*m}$

n = 30
n * m = 30 * 26 = 780
PVIFA = (1 - (1.002276)^-780)/0.002276
PVIFA = 364.7375

PMT = Principal/ $PVIFA_{j,n*m}$
PMT = 270,000/364.7375
PMT = $740.1876

Interest paid = (780 x $740.1876) - $270,000

Interest paid = $307,346

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ID: 887
TYPE: EOT/EOC
SECTION: Canadian Mortgages
DIFFICULTY: 4/4
DESCRIPTION: Calculate principal outstanding

QUESTION:
Your mortgage loan has a principal of $800,000, a term of 40 years and a quoted rate of 8.75%. You elect to make semi-monthly payments. What is the amount of principal owing after five years of the 40 year term?

SOLUTION:

For Canadian mortgages the periodic rate is given by:

$$j = \left(1 + \frac{i}{2}\right)^{2/m} - 1$$

j = (1 + 0.0875/2)^(2/24) -1
j = 0.00357471

PMT = Principal/ $PVIFA_{j,n*m}$

n = 40
n * m = 40 * 24 = 960
PVIFA = (1 - (1. 00357471)^-960)/0. 00357471
PVIFA = 3270.6432

PMT = Principal/ $PVIFA_{j,n*m}$
PMT = 800,000/270.6432
PMT = $2955.923

After five years the principal outstanding is the present value of the remaining payments. After five years there are 960 – 5*24 = 840 payments remaining.

Principal = PMT * $PVIFA_{j,n*m}$

Where
n*m = 840

Principal Outstanding = $2955.923 * [(1-(1.00357471)^(-840)) / 0.00357471]
Principal Outstanding = $785,622.80