Chapter 12: The Partial Equilibrium Competitive Models Prepared by Dr. M.Rafiquzzaman

For two goods, $x\ and\ y$, the Marshallian demands for $x\ and\ y$ are obtained by maximizing $U(x, y)$, subject to the budget constraint $p_x x + p_y y = I$. If we use i to reflect the ith individual in the market, then the ith individual's Marshallian demand function is

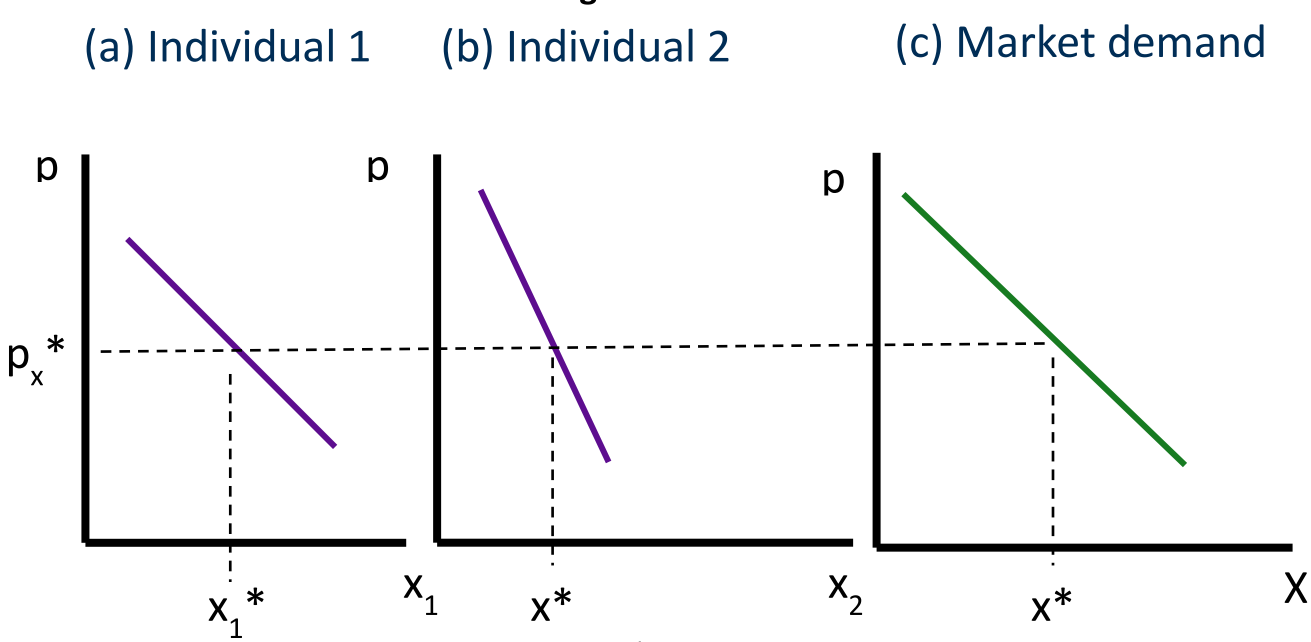
$x_i = x_i(p_x, p_y, I_i)$, where I_i is the income of the ith individual.

Suppose these are n consumers, then the market demand for good x is

$$X = \sum_{i=1}^{n} x_i(p_x, p_y, I_i)$$

- Characteristics of market demand curve for good X
  - p_x is allowed to vary
  - p_y and the income of each individual are held constant
  - If each individual's demand for *x* is downward sloping, the market demand curve will also be downward sloping

Figure 12.1



A market demand curve is the ‘‘horizontal sum’’ of each individual’s demand curve. At each price the quantity demanded in the market is the sum of the amounts each individual demands. For example, at p^*_x the demand in the market is $x^*_1 + x^*_2 = X^*$ **(Figure 12.1)**

Example

Suppose an individual 1’s demand for oranges is:

$x_1 = 10 - 2p_x + 0.1\, I_1 + 0.5p_y$,

and individual 2’s demand for oranges is:

$x_2 = 17 - p_x + 0.05\, I_2 + 0.5p_y$.

Then market demand curve is $X = x_1 + x_2 = X(p_x, p_y, I_1, I_2) = (10 - 2p_x + 0.1\, I_1 + 0.5p_y) + (17 - p_x + 0.5\, I_2 + 0.5p_y) = 27 - 3p_x + 0.1\, I_1 + 0.05I_2 + p_y$

If $p_y = 4$, $I_1 = 40$, $I_2 = 20$, then the market demand is

$X = 27 - 3p_x + 4 + 1 + 4 = 36 - 3p_x$

If p_y increases to 6, and other parameters remain unchanged,

$X = 27 - 3p_x + 4 + 1 + 6 = 38 - 3p_x$

Note x and y are substitutes because as p_y increases demand for x increases.

Generalizations

Suppose there are n goods (denoted by $x_i, i = 1,2, \dots, n$) with prices $p_i, i = 1,2, \dots, n$. Assume that there are m individuals in society. Then the jth individual's demand for the ith good will depend on all prices and on I_j, the income of this person. This can be denoted by

$$x_{i,j} = x_{i,j}(p_1, p_2, \dots, p_n, I_j)$$

The market demand for a particular good (X_i) is the sum of each individual's demand for that good:

$$X_i(p_1, p_2, \dots, p_n, I_1, I_2, \dots, I_m) = \sum_{j=1}^{m} x_{i,j}(p_1, p_2, \dots, p_n, I_j)$$

Elasticity of market demand

The market demand function is represented by: $Q_D = D(P, P', I)$, then we define

Price Elasticity of Market Demand = $e_{D,P} = \frac{\partial D(P,P',I)}{\partial P} . \frac{P}{Q_D}$

Cross Price Elasticity of Market Demand = $e_{D,P'} = \frac{\partial D(P,P',I)}{\partial P'} . \frac{P'}{Q_D}$

Income Elasticity of Market Demand = $e_{D,I} = \frac{\partial D(P,P',I)}{\partial I} . \frac{I}{Q_D}$

If "own" price elasticity of demand, $e_{D,P} < -1$, the market demand is elastic, if $e_{D,P} > -1$, the market demand is inelastic, and if $e_{D,P} = 0$, the market demand is unitary elastic.

Short-Run Price Determination:

Detemination of supply depends on the strucure of the market in which firms operate. Let us now state explicitly the assumptions of a perfectly competitive model.

A perfectly competitive market is one that obeys the following assumptions.

1. There are a large number of firms, each producing the same homogeneous product.
2. Each firm attempts to maximize profits.
3. Each firm is a price-taker: It assumes that its action have no effect on market price.
4. Prices are assumed to be known by all market participants – information is perfect.
5. Transactions are costless: Buyers and sellers incur no costs in making exchanges.

You may recall that an individual firm's (firm i) short-run supply curve is given by

$$q_i = q_i(P, v, w)$$

If there are n firms in the industry, then the short-run market supply curve is

$$Q_s(P, v, w) = S(P, v, w) = \sum_{i=1}^{n} q_i(P, v, w)$$

The short-run market supply curve shows the two-dimensional relationship between Q and P, holding v, and w (and firm's underlying technology) constant.

Short-run elasticity of supply ($e_{S,P}$) is defined by

$$e_{S,P} = \frac{\partial Q_S}{\partial P} . \frac{P}{Q_S}$$

- Because price and quantity supplied are positively related, $e_{S,P} > 0$

Example:

A Short-Run Supply Fuction

Recall from Ch 11 (p. 49), with Cobb-Douglas production function $q = k^{\alpha} l^{\beta}$, where we fix k at k_1, a single firm's short-sun supply function was obtained as

$$q(P, v, w, k_1) = \left(\frac{w}{\beta}\right)^{\frac{\beta}{\beta-1}} p^{\frac{\beta}{1-\beta}}\ k_1^{\left(\frac{\alpha}{1-\beta}\right)} = \left(\frac{w}{\beta}\right)^{\frac{-\beta}{1-\beta}} p^{\frac{\beta}{1-\beta}}\ k_1^{\left(\frac{\alpha}{1-\beta}\right)},$$

(11.42) , (12.14)

If $\alpha = \beta = 0.5, v = 3, w = 12, and k_1 = 80$, this yields the single firm (Firm i) function supply function as

$$q_i(P, v, w = 12, k_1 = 80) = \left(\frac{w}{\beta}\right)^{\frac{-\beta}{1-\beta}} p^{\frac{\beta}{1-\beta}}\ k_1^{\left(\frac{\alpha}{1-\beta}\right)} =$$

$$\left(\frac{12}{0.5}\right)^{\frac{-0.5}{1-0.5}} 80\ ^{\left(\frac{0.5}{1-0.5}\right)} P = \frac{1}{24}(80)^1 P = \frac{10}{3} P$$

Now assume there are 100 such firms, and each firm faces the same market prices for both its output and its inputs. Then the shost-run market supply function is given by

$$S(P, v, w = 12, k_1 = 80) = \sum_{i=1}^{100} q_i = \sum_{i=1}^{100} \frac{10P}{3} = \frac{1.000P}{3} = Q_S$$

If, for example, market price P = 12, total market supply will be S=$\frac{1,000(12)}{3} = 4,000$. Each firm will supply $\frac{4,000}{100} = 40$ units.

Short-run elasticity of supply

$$e_{S,P} = \frac{\partial Q_S}{\partial P} . \frac{P}{Q_S} = \frac{1,000}{3}\left(\frac{12}{4000}\right) = 1$$

Effect of an increase in wage:

If all firms experience an wage increase of \$15, from (12.14)

$$q(P, v, w, k_1) = \left(\frac{w}{\beta}\right)^{\frac{-\beta}{1-\beta}} p^{\frac{\beta}{1-\beta}}\ k_1^{\left(\frac{\alpha}{1-\beta}\right)}, \qquad (12.14)$$

If $\alpha = \beta = 0.5, v = 3, w = 15, and k_1 = 80$, this yields the single firm (Firm i) function supply function as

$$q_i(P, v, w = 15, k_1 = 80) = \left(\frac{w}{\beta}\right)^{\frac{-\beta}{1-\beta}} p^{\frac{\beta}{1-\beta}}\ k_1^{\left(\frac{\alpha}{1-\beta}\right)} =$$

$$\left(\frac{15}{0.5}\right)^{\frac{-0.5}{1-0.5}} 80^{\ \left(\frac{0.5}{1-0.5}\right)} P = \frac{1}{30}(80)^1 P = \frac{8}{3}P$$

Market supply function is $S(P, v, w = 15, k_1 = 80) = \sum_{i=1}^{100} q_i = \sum_{i=1}^{100} \frac{8P}{3} = \frac{800P}{3} = Q_S$

Now at P=12, industry supply is S=$\frac{800(12)}{3} = 3{,}200$. Each firm will supply $\frac{3{,}200}{100} = 32$ units.

In other words, the supply curve have shifted upward because of the increase in wage. However the price elasticity of supply is still equal to 1. **(Prove this).**

Equilibrium price determination

An equilibrium price is one at which quantity demanded is equal to quantity supplied. At such a price, neither demanders nor suppliers have an incentive to alter their economic decisions. Mathematically, an equilibrium price P* solves the equation

$$D(P^*, P', I) = S(P^*, v, w)$$

More compactly,

$D(P^*) = S(P^*)$

A Comparative Statics Model of Market Equilibrium:

Assume that market demand function is given by $Q_D = D(P, \alpha)$, where α is an exogenous variable that shifts the demand function (such as income or price of another good). Similarly, the short-run supply function is given by $Q_S = S(P, \beta)$, where β is an exogenous variable that shifts supply function (such as input prices or technological progress).

Market equilibrium values of price (P*) and quantity (Q*) are determined by

$$Q_D = Q_S = Q^* = D(P^*, \alpha) = S(P^*, \beta) \qquad \textbf{(12.22)}$$

To show how these equilibrium values change when one of the exogenous variables changes, from (12.22), we write the equilibrium conditions as

$$D(P^*, \alpha) - Q^* = 0$$

$$S(P^*, \beta) - Q^* = 0 \qquad \textbf{(12.23)}$$

These two equations are solved simultaneously to get the equilibrium values of P* and Q*.

Consider a shift in the demand function shown by a change in α. Differentiating (12.23) with respect to α, we get

$$\frac{\partial D(P^*, \alpha)}{\partial P^*}\left(\frac{dD(P^*\alpha)}{d\alpha}\right) + \frac{\partial D(P^*, \alpha)}{\partial \alpha} - \frac{dQ^*}{d\alpha} = 0$$

$$\Rightarrow D_P \frac{dP^*}{d\alpha} + D_\alpha - \frac{dQ^*}{d\alpha} = 0 \Rightarrow D_P \frac{dP^*}{d\alpha} - \frac{dQ^*}{d\alpha} = -D_\alpha$$

$$S_P \frac{dP^*}{d\alpha} - \frac{dQ^*}{d\alpha} = 0$$

The above two equations are written in matrix form:

$$\begin{bmatrix} D_P & -1 \\ S_P & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{dP^*}{d\alpha} \\ \frac{dQ^*}{d\alpha} \end{bmatrix} = \begin{bmatrix} -D_\alpha \\ 0 \end{bmatrix}$$

Applying Cramer's rule, changes in equilibrium price and quantiy are obtained as

$$\frac{dP^*}{d\alpha} = \frac{\begin{vmatrix} -D_\alpha & -1 \\ 0 & -1 \end{vmatrix}}{\begin{vmatrix} D_P & -1 \\ S_P & -1 \end{vmatrix}} = \frac{D_\alpha}{S_P - D_P}$$

$$\frac{dQ^*}{d\alpha} = \frac{\begin{vmatrix} D_P & -D_\alpha \\ S_P & 0 \end{vmatrix}}{\begin{vmatrix} D_P & -1 \\ S_P & -1 \end{vmatrix}} = \frac{D_\alpha S_P}{S_P - D_P}$$

Because $S_P > 0, D_P < 0$, the denominator of these expressions is positive. Now if

- α = income, or α = price of a substitute good, $D_\alpha > 0 \Rightarrow \frac{dP^*}{d\alpha} > 0 \ and \ \frac{dQ^*}{d\alpha} > 0$. Thus, income or price a substitute good increase, both equilibrium price and quantity increase.
- If α = income, or α = price of a substitute good, and value of $\alpha \ increases,$ and $D_\alpha > 0 \Rightarrow \frac{dP^*}{d\alpha} > 0 \ and \ \frac{dQ^*}{d\alpha} > 0$. Thus, increases in income or price of a substitute good increase equilibrium price and quantity.

- If α = income, or α = price of a complementary good, and value of α *increases*, *and* $D_\alpha < 0 \Rightarrow \frac{dP^*}{d\alpha} < 0$ *and* $\frac{dQ^*}{d\alpha} < 0$. Thus, increases in income or price a complementary good reduce equilibrium price and quantity.

Equilibrium with constant elasticity functions

Demand and supply for automobiles are given by:

$$D(P,I) = 0.1P^{-1.2}I^3; \quad S(P,w) = 6{,}400Pw^{-0.5}$$

Here P and I are measured in dollars, and w is hourly wage of automobile workers. If the value of exogenous variables I and w are \$20,000 and \$25, respectively, then demand – supply requires

$$D(P,I) = 0.1P^{-1.2}I^3 = 0.1P^{-1.2}(20{,}000)^3 = 2{,}000(20{,}000)^2P^{-1.2}$$

$= S(P,w) = 6{,}400P(25)^{-0.5} = \frac{6{,}400P}{5} = 1{,}280P \Rightarrow \frac{P}{P^{-1.2}} = \frac{2{,}000(20{,}000)^2}{1{,}280} \Rightarrow P^{2.2} = 1.5625(20{,}000)^2 \Rightarrow P^* = (1.5625)^{\frac{1}{2.2}}(20{,}000)^{2/2.2} = (1.5625)^{0.4545}(20{,}000)^{0.9091} = 1.22487(8129.5474) = 9957.6 \approx 9{,}957$

Either use supply or demand function to find Q^*. From the supply equation,

$$Q^* = 1{,}280P^* = 1{,}280(9{,}957) = 12{,}744{,}960$$

Hence, the equilibrium in the automobile market has a price of nearly \$10,000 with approximately 12,745,000$\approx$ 13 million cars being sold.

Using the above procedure, please complete the parts: (1) **A shift in demand** and (2) **A shifi in supply** in the text book p. 417, or text book slides 35 (ch 12).

Long-Run Analysis

In the long run

- A firm may adapt all of its inputs to fit market conditions
- Profit-maximization for a price-taking firm:
  - Price is equal to long-run *MC*
- Firms can also enter and exit an industry
- Perfect competition: there are no special costs of entering or exiting an industry

In the long run new firms will be lured into any market where economic profits are > 0

- The short-run industry supply curve will shift outward
- Market price and profits will fall
- The process will continue until economic profits are zero
- Existing firms will leave any industry where economic profits are negative
  - The short-run industry supply curve will shift inward
  - Market price will rise and losses will fall
  - The process will continue until economic profits are zero

Long-Run Competitive Equilibrium

Assumptions

- All firms in an industry have identical cost curves
  - No firm controls any special resources or technology

- The equilibrium long-run position requires that each firm earn zero economic profit
  - $P = MC$ (profit maximization condition)
  - $P = AC$ (zero profit condition)

- A perfectly competitive industry is in long-run equilibrium
  - If there are no incentives for profit-maximizing firms to enter or to leave the industry
  - When the number of firms is such that
    - *(1)* $P = MC = AC$
    - (2) And each firm operates at minimum *AC*

Classifications of Long-Run Supply Curves

- Constant Cost
  - Entry does not affect input costs
  - **Horizontal long-run supply curve** at the long-run equilibrium price
- Increasing Cost
  - Entry increases inputs costs
  - **Positively sloped long-run supply** curve
- Decreasing Cost
  - Entry reduces input costs
  - **Negatively sloped long-run supply** curve

For graphical representation of these supply curves, please go through Figures 12.7 12.8, and 12.9 in the textbook. You may read the textbook slides on these supply curves in Ch 12.

<u>Example:</u>

Handmade bicycle frames are produced by a number of identical firms. Total long-run monthly costs for a typical firm is given by

$C(q) = q^3 - 20q^2 + 100q + 8{,}000$
where q is the number of frames produced per month. Demand for bicycle frames is
$Q_D = D(P) = 2{,}500 - 3P$

where Q_D is the quantity demanded per month and P is the price per frame.

Find the long-run equilibrium in this market.

Step 1. To obtain a firm's long-run supply function. A firm's supply curve is the LRMC curve starting from the minimum of the LRAC curve. Note that the minimum of AC occurs when AC = MC.

From the cost function, AC(q) = $\frac{C(q)}{q} = \frac{q^3-20q^2+100q+8{,}000}{q} = q^2 - 20q \quad + 100 + \frac{8{,}000}{q}$

MC = $\frac{\partial C(q)}{\partial q} = 3q^2 - 40q + 100$

AC = MC $\Rightarrow q^2 - 20q \quad + 100 + \frac{8{,}000}{q} = 3q^2 - 40q + 100 \Rightarrow 2q^2 - 20q \quad - \frac{8{,}000}{q} = 0 \Rightarrow 2q^2 - 20q \quad = \frac{8{,}000}{q} \Rightarrow 2q^3 - 20q^2 \quad = 8{,}000 \Rightarrow$
$2q^2(q-10) = 8000$

Suppose q = 20, then left hand side of the above expression = 2(20)(20)(20-10) = 8,000 = Right hand side.

At q = 20, AC = MC = 500.

This suggests that Long-run price = $500, because at this price every firm makes zero profit in the long-run.

Substituting P= 500, in the demand function, $Q_D = 2{,}500 - 3(500) = 2{,}500 - 1{,}500 = 1{,}000.$

So, the number of firms in the long-run equilibrium = $\frac{1{,}000}{20} = 50$

If the demand increases to $Q_D = D(P) = 3{,}000 - 3P$ *, other things remain unchanged, solve the new long-run equilibrium price, output and number of firms.*