ELG3175
Introduction to Communication Systems

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Solutions

Assignment #5

#1 Consider "demodulator" operation in details.

$$x_1(t) = x^3(t) = A_c^3 m^3(t) \cos^3 \omega_c t.$$

Using $\cos^3 \alpha = \cos\alpha \cdot \frac{1}{2}(1+\cos 2\alpha) = \frac{3}{4}\cos\alpha + \frac{1}{4}\cos 3\alpha$

one obtains:

$$x_1(t) = \frac{3}{4} A_c^3 m^3(t) \cos\omega_c t + \frac{1}{4} A_c^3 m^3(t) \cos 3\omega_c t$$

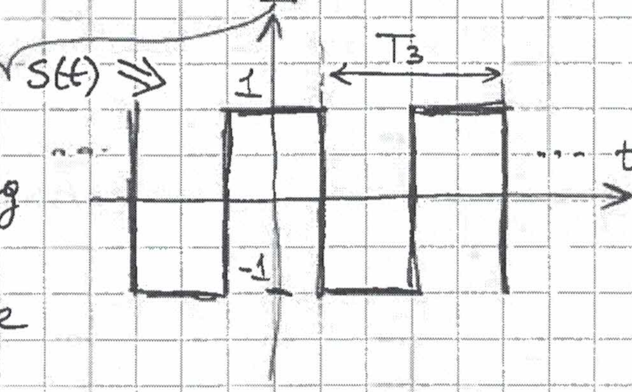
BPF at $f_0 = 3f_c$ rejects all terms except for $\cos 3\omega_c t$ term:

$$x_2(t) = \frac{1}{4} A_c^3 m^3(t) \cos 3\omega_c t$$

Ideal limiter produces the following:

$$x_3(t) = \text{sgn}[m(t)] \left[2 \sum_{n=-\infty}^{+\infty} \Pi\left(\frac{2}{T_3}t - 2n\right) - 1 \right], \quad T_3 = \frac{1}{3f_c} = \frac{2\pi}{3\omega_c}$$

Note that we must account for the sign of $m(t)$ (i.e., $\text{sgn}[m(t)] = \pm 1$, depending on $m(t) \gtrless 0$).



The pulse train $s(t)$ can be expanded in Fourier series:

$$s(t) = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \cos(2k+1)\omega_3 t = \frac{4}{\pi}\cos\omega_3 t + \text{high-frequency terms}$$

$\omega_3 = 3\omega_c$. BPF at $f_0 = 3f_c$ rejects all terms except for $\cos 3\omega_c t$:

$$x_4(t) = \text{sgn}[m(t)] \frac{4}{\pi} \cos 3\omega_c t$$

#1 Further,

$$x_5(t) = \text{sgn}[m(t)] \cdot \frac{4}{\pi} \cdot \cos \omega_c t$$

and

$$x_6(t) = \frac{4}{\pi} \text{sgn}[m(t)] \cos^2 \omega_c t \cdot A_c m(t) =$$

$$= \frac{4}{\pi} A_c |m(t)| \cdot \frac{1}{2} (1 + \cos 2\omega_c t).$$

Note that: $\text{sgn}[m(t)]\, m(t) = |m(t)|$

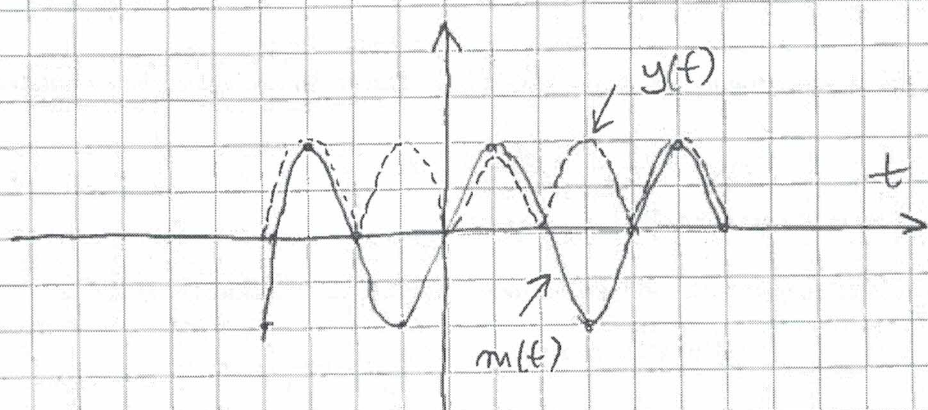
LPF rejects $\cos 2\omega_c t$ and

$$y(t) = \frac{2}{\pi} A_c \cdot |m(t)|$$

Hence, $y(t) \sim |m(t)|$ and we loose any information about polarity of $m(t)$!

It is not a demodulator → the loss of polarity cannot be recovered. For exampee, transmitting ± 1, we receive only $+1$!

(2b) $m(t) = \sin \Omega t$; $y(t) = |\sin(\Omega t)|$



#1

Problem [redacted]

1) When USSB is employed the bandwidth of the modulated signal is the same with the bandwidth of the message signal. Hence,

$$W_{\text{USSB}} = W = 10^4 \text{ Hz}$$

2) When DSB is used, then the bandwidth of the transmitted signal is twice the bandwidth of the message signal. Thus,

$$W_{\text{DSB}} = 2W = 2 \times 10^4 \text{ Hz}$$

3) If conventional AM is employed, then

$$W_{\text{AM}} = 2W = 2 \times 10^4 \text{ Hz}$$

4) Using Carson's rule, the effective bandwidth of the FM modulated signal is

$$B_c = (2\beta + 1)W = 2\left(\frac{k_f \max[|m(t)|]}{W} + 1\right) W = 2(k_f + W) = 140000 \text{ Hz}$$

Problem [redacted] 2

1) If SSB is employed, the transmitted signal is

$$u(t) = Am(t)\cos(2\pi f_0 t) \mp A\hat{m}(t)\sin(2\pi f_0 t)$$

Provided that the spectrum of $m(t)$ does not contain any impulses at the origin $P_M = P_{\hat{M}} = \frac{1}{2}$ and

$$P_{\text{SSB}} = \frac{A^2 P_M}{2} + \frac{A^2 P_{\hat{M}}}{2} = A^2 P_M = 400\frac{1}{2} = 200$$

The bandwidth of the modulated signal $u(t)$ is the same with that of the message signal. Hence,

$$W_{\text{SSB}} = 10000 \text{ Hz}$$

2) In the case of DSB-SC modulation $u(t) = Am(t)\cos(2\pi f_0 t)$. The power content of the modulated signal is

$$P_{\text{DSB}} = \frac{A^2 P_M}{2} = 200\frac{1}{2} = 100$$

and the bandwidth $W_{\text{DSB}} = 2W = 20000$ Hz.

3) If conventional AM is employed with modulation index $\alpha = 0.6$, the transmitted signal is

$$u(t) = A[1 + \alpha m(t)]\cos(2\pi f_0 t)$$

The power content is

$$P_{\mathrm{AM}} = \frac{A^2}{2} + \frac{A^2 \alpha^2 P_M}{2} = 200 + 200 \cdot 0.6^2 \cdot 0.5 = 236$$

The bandwidth of the signal is $W_{\mathrm{AM}} = 2W = 20000$ Hz.

4) If the modulation is FM with $k_f = 50000$, then

$$P_{\mathrm{FM}} = \frac{A^2}{2} = 200$$

and the effective bandwidth is approximated by Carson's rule as

$$B_c = 2(\beta + 1)W = 2\left(\frac{50000}{W} + 1\right) W = 120000 \text{ Hz}$$

Problem [redacted] 7

1) Since $\mathcal{F}[\mathrm{sinc}(400t)] = \frac{1}{400}\Pi(\frac{f}{400})$, the bandwidth of the message signal is $W = 200$ and the resulting modulation index

$$\beta_f = \frac{k_f \max[|m(t)|]}{W} = \frac{k_f 10}{W} = 6 \Longrightarrow k_f = 120$$

Hence, the modulated signal is

$$\begin{aligned} u(t) &= A\cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau) \\ &= 100\cos(2\pi f_c t + +2\pi 1200 \int_{-\infty}^{t} \mathrm{sinc}(400\tau)d\tau) \end{aligned}$$

2) The maximum frequency deviation of the modulated signal is

$$\Delta f_{\max} = \beta_f W = 6 \times 200 = 1200$$

3) Since the modulated signal is essentially a sinusoidal signal with amplitude $A = 100$, we have

$$P = \frac{A^2}{2} = 5000$$

4) Using Carson's rule, the effective bandwidth of the modulated signal can be approximated by

$$B_c = 2(\beta_f + 1)W = 2(6 + 1)200 = 2800 \text{ Hz}$$

Problem 3[redacted]

1) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have

$$P = \frac{A_c^2}{2} \Longrightarrow P = \frac{100^2}{2} = 5000$$

The same result is obtained if we use the expansion

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t)$$

along with the identity

$$J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1$$

2) The maximum phase deviation is

$$\Delta\phi_{\max} = \max|4\sin(2000\pi t)| = 4$$

3) The instantaneous frequency is

$$\begin{aligned} f_i &= f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) \\ &= f_c + \frac{4}{2\pi}\cos(2000\pi t)2000\pi = f_c + 4000\cos(2000\pi t) \end{aligned}$$

Hence, the maximum frequency deviation is

$$\Delta f_{\max} = \max|f_i - f_c| = 4000$$

4) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with phase deviation constant $k_p = 4$ and message signal $m(t) = \sin(2000\pi t)$ and it is an FM signal with frequency deviation constant $k_f = 4000$ and message signal $m(t) = \cos(2000\pi t)$.

Problem [redacted] 4

1)

$$\begin{aligned} \beta_p &= k_p \max[|m(t)|] = 1.5 \times 2 = 3 \\ \beta_f &= \frac{k_f \max[|m(t)|]}{f_m} = \frac{3000 \times 2}{1000} = 6 \end{aligned}$$

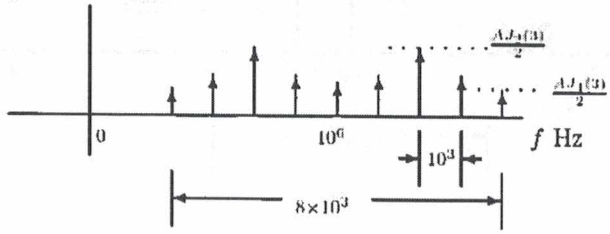
2) Using Carson's rule we obtain

$$\begin{aligned} B_{\text{PM}} &= 2(\beta_p + 1)f_m = 8 \times 1000 = 8000 \\ B_{\text{FM}} &= 2(\beta_f + 1)f_m = 14 \times 1000 = 14000 \end{aligned}$$

3) The PM modulated signal can be written as

$$u(t) = \sum_{n=-\infty}^{\infty} AJ_n(\beta_p)\cos(2\pi(10^6 + n10^3)t)$$

The next figure shows the amplitude of the spectrum for positive frequencies and for these components whose frequencies lie in the interval $[10^6 - 4 \times 10^3, 10^6 + 4 \times 10^3]$. Note that $J_0(3) = -.2601$, $J_1(3) = 0.3391$, $J_2(3) = 0.4861$, $J_3(3) = 0.3091$ and $J_4(3) = 0.1320$.

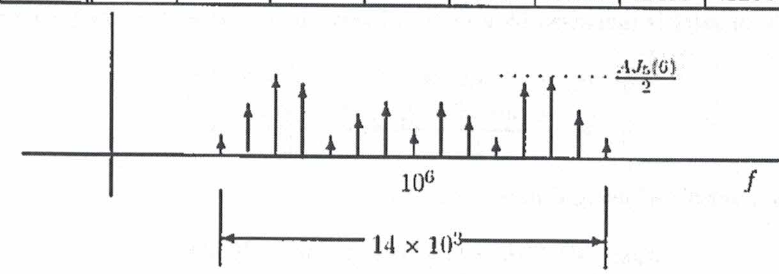


In the case of the FM modulated signal

$$\begin{aligned} u(t) &= A\cos(2\pi f_c t + \beta_f \sin(2000\pi t)) \\ &= \sum_{n=-\infty}^{\infty} AJ_n(6)\cos(2\pi(10^6 + n10^3)t + \phi_n) \end{aligned}$$

The next figure shows the amplitude of the spectrum for positive frequencies and for these components whose frequencies lie in the interval $[10^6 - 7 \times 10^3, 10^6 - 7 \times 10^3]$. The values of $J_n(6)$ for $n = 0, \ldots, 7$ are given in the following table.

n	0	1	2	3	4	5	6	7
$J_n(6)$	.1506	-.2767	-.2429	.1148	.3578	.3621	.2458	.1296



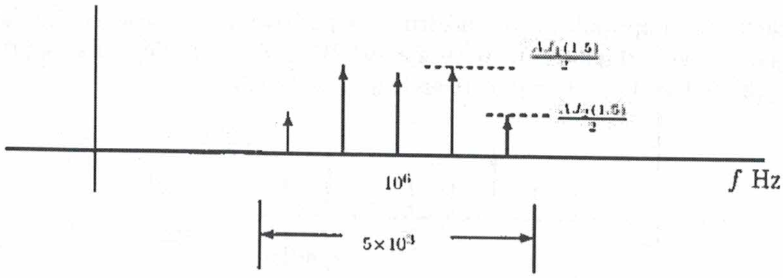
4) If the amplitude of $m(t)$ is decreased by a factor of two, then $m(t) = \cos(2\pi 10^3 t)$ and

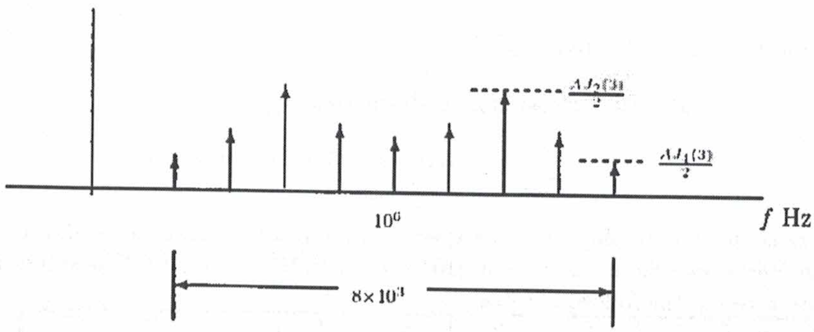
$$\begin{aligned} \beta_p &= k_p \max[|m(t)|] = 1.5 \\ \beta_f &= \frac{k_f \max[|m(t)|]}{f_m} = \frac{3000}{1000} = 3 \end{aligned}$$

The bandwidth is determined using Carson's rule as

$$\begin{aligned} B_{PM} &= 2(\beta_p + 1)f_m = 5 \times 1000 = 5000 \\ B_{FM} &= 2(\beta_f + 1)f_m = 8 \times 1000 = 8000 \end{aligned}$$

The amplitude spectrum of the PM and FM modulated signals is plotted in the next figure for positive frequencies. Only those frequency components lying in the previous derived bandwidth are plotted. Note that $J_0(1.5) = .5118$, $J_1(1.5) = .5579$ and $J_2(1.5) = .2321$.





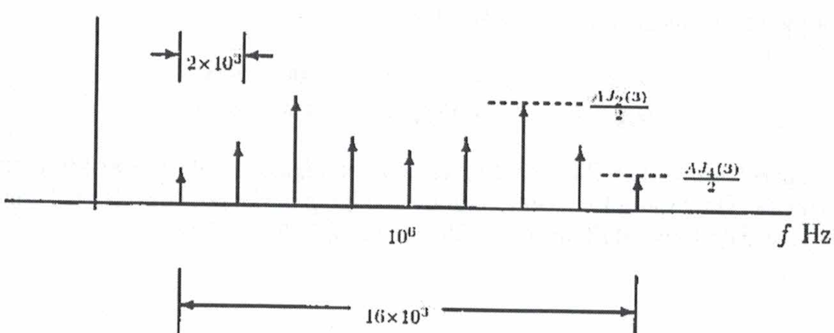
5) If the frequency of $m(t)$ is increased by a factor of two, then $m(t) = 2\cos(2\pi 2 \times 10^3 t)$ and

$$\begin{aligned} \beta_p &= k_p \max[|m(t)|] = 1.5 \times 2 = 3 \\ \beta_f &= \frac{k_f \max[|m(t)|]}{f_m} = \frac{3000 \times 2}{2000} = 3 \end{aligned}$$

The bandwidth is determined using Carson's rule as

$$\begin{aligned} B_{\text{PM}} &= 2(\beta_p + 1) f_m = 8 \times 2000 = 16000 \\ B_{\text{FM}} &= 2(\beta_f + 1) f_m = 8 \times 2000 = 16000 \end{aligned}$$

The amplitude spectrum of the PM and FM modulated signals is plotted in the next figure for positive frequencies. Only those frequency components lying in the previous derived bandwidth are plotted. Note that doubling the frequency has no effect on the number of harmonics in the bandwidth of the PM signal, whereas it decreases the number of harmonics in the bandwidth of the FM signal from 14 to 8.



#5 Frequency separation Δf_s in the original signal spectrum is $\Delta f_s = F_m = 5\,kHz$,

wher $F_m = 5\,kHz$ → message frequency.

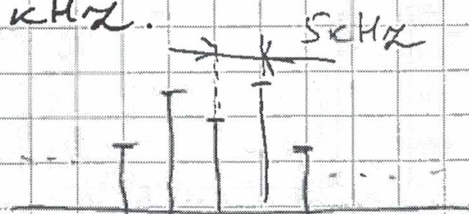
Frequecy multiplication does not change it because it does no affect message frequency, i.e.

$x(t) = a\cos\left(\omega_c t + \beta \int_0^t m(t)\,dt\right)$ after freq. multiplication becomes:

$$x_1(t) = a_1 \cos\left(n\cdot\omega_c t + n\beta \int_0^t m(t)\,dt\right)$$

Clearly, message frequency is not affected, but modulation index β is increased n times.

Hence, $\Delta f_{s,1} = \Delta f_{s,2} = \Delta f_s = 5\,kHz$.



Peak frequency deviation increases 2 times for 1st multiplier and 2×5 times for the 2nd:

$f_{d1} = ~~\ldots~~ = 2\cdot\Delta f = 4\,kHz$; $f_{d2} = 2\cdot 5\cdot \Delta f = 20\,kHz$.

Output signal bandwidth is

$$\Delta f_{out} = 2(\beta_{out}+1)F_m = 2(f_{d2}+F_m) = 2(20+5) =$$

$$= 50\,kHz.$$

Output modulation index is $\beta_{out} = \frac{f_{d2}}{F_m} = 4$, while the input modulation index is $\beta_{in} = \frac{\Delta f}{F_m} = 0.4$.

Problem # 5: Spectra (assuming f_c=10 kHz)