ELG3175
Introduction to Communication Systems

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Solutions

Assignment #6

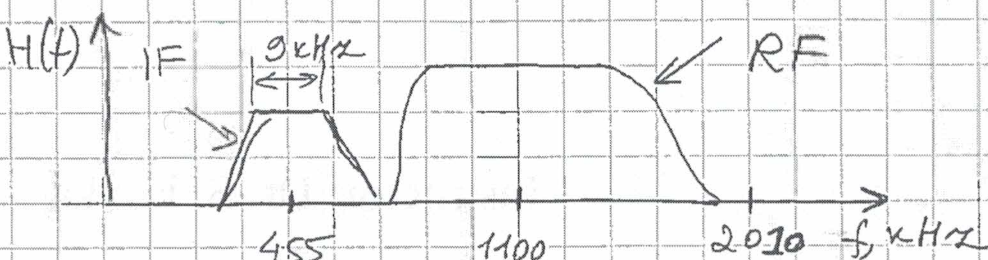
#1 Find f_{LO} first:

b+a) $f_{LO} = f_c + f_{IF} = 1.1\,MHz + 455\,kHz = 1555\,kHz$

where $f_c = 1.1\,MHz$ – AM signal carrier frequency.

Image frequency $f_{im} = f_c + 2f_{IF} = 1.1\,MHz + 910\,kHz =$
$= 2010\,kHz$.

IF filter bandwidth is $\Delta f_{IF} = \Delta f_s = 9\,kHz$.

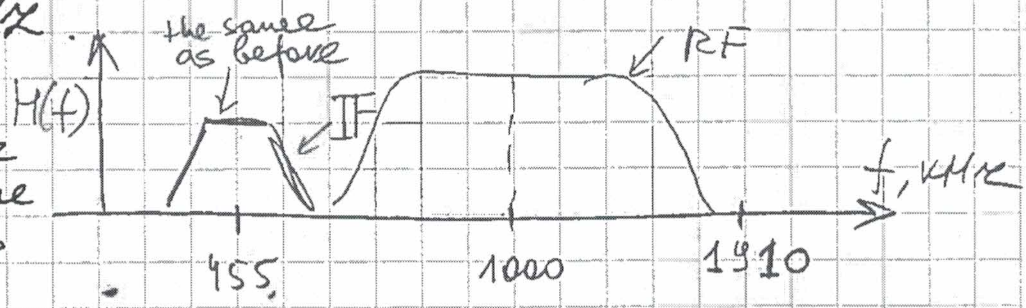


Normally, RF filter bandwidth is larger than IF one. However, RF filter must reject the image frequency that cannot be filtered out by the IF filter.

C) $f_c = 1\,MHz$; $f_{LO} = 1\,MHz + 455\,kHz = 1455\,kHz$

$f_{im} = f_c + 2f_{IF} = 1\,MHz + 910\,kHz = 1910\,kHz$

IF and RF filter bandwidth stay the same:
$\Delta f_{IF} = 9\,kHz$.



RF filter response may stay the same or may shift to around 1 MHz.

IF filter response always stay the same.

#2

(a) The center frequency of BPF

$$f_{BPF} = \frac{f_c}{8} = \frac{103.7}{8} = 12.96\ MHz$$

Peak deviation

$$\Delta f_{BPF} = \frac{\Delta f}{8} = \frac{75}{8} = 9.375\ kHz$$

Bandwith

$$BW_{BPF} = 2(\Delta f_{BPF} + F_m) = 2(9.375 + 15) =$$

$$= 48.75\ kHz$$

(b) $f_{BPF} = f_c + f_{os} \rightarrow f_{os} = f_{BPF} - f_c =$

$$= 12.96 - 5 = 7.96\ MHz$$

Another possibility:

$$f_{BPF} = f_{os} - f_c \rightarrow f_{os} = f_{BPF} + f_c = 17.96\ MHz.$$

(c) $\Delta f_{FM} = \frac{\Delta f}{8} = \Delta f_{BPF} = 9.375\ kHz$

where $\Delta f = 75\ kHz$ peak deviation.

#3

DSB-SC modulated signal:

$$x(t) = A_c \cdot m(t) \cdot \cos \omega_c t$$

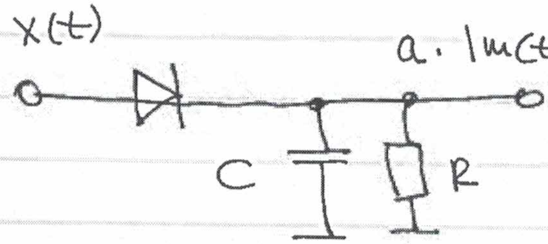
Its envelope $E(t)$:

$$E(t) = |A_c\, m(t)| = A_c |m(t)|$$

If $\underline{m(t) \geq 0}$: $|m(t)| = m(t)$

$$E(t) = A_c\, m(t) \sim m(t).$$

i.e. envelope detector can be used:



$a \cdot |m(t)| = a \cdot m(t)$

if $m(t) \geq 0$.

↑ simpliest AM detector.

FDM Broadcast of FM stations

#4

FM mod. signal bandwidth (large $\beta \gg 1$):

$$\Delta f \approx 2(F_{max} + \Delta f) = 2(20+75)\,\text{kHz} = 190\,\text{kHz}$$

where $F_{max} = 20\,\text{kHz}$ = max. message freq.

$\Delta f = 75\,\text{kHz}$ = frequency deviation.

Total bandwidth per channel (incl. saftey margin $\delta f = 10\,\text{kHz}$):

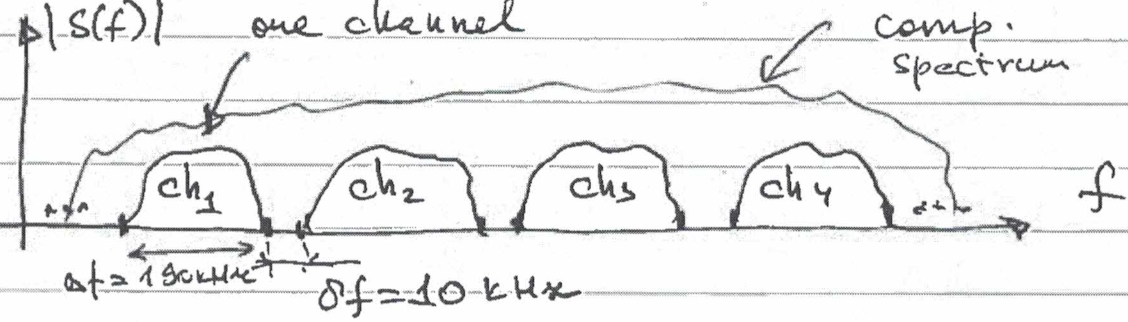
$$\Delta f' = \Delta f + \delta f = 200\,\text{kHz}$$

Number N of stations:

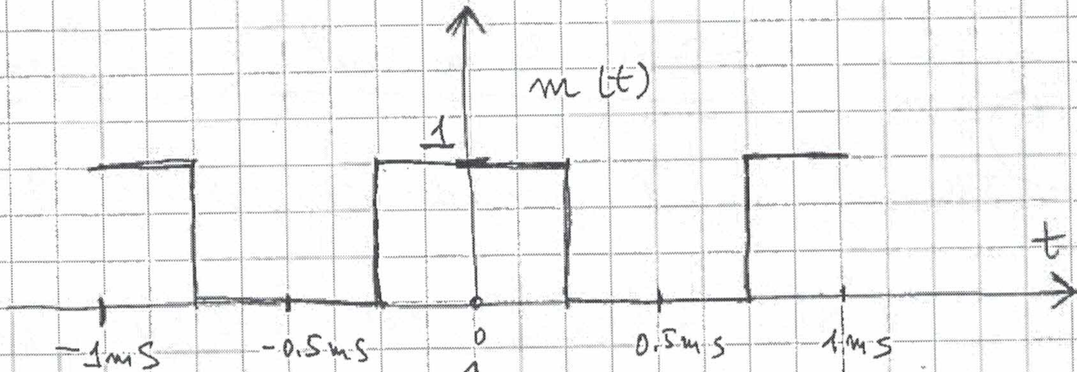
$$N = \frac{F_2 - F_1}{\Delta f'} = \frac{(110-80)\,\text{MHz}}{200\,\text{kHz}} = \frac{30\,\text{MHz}}{200\,\text{kHz}} = 150$$

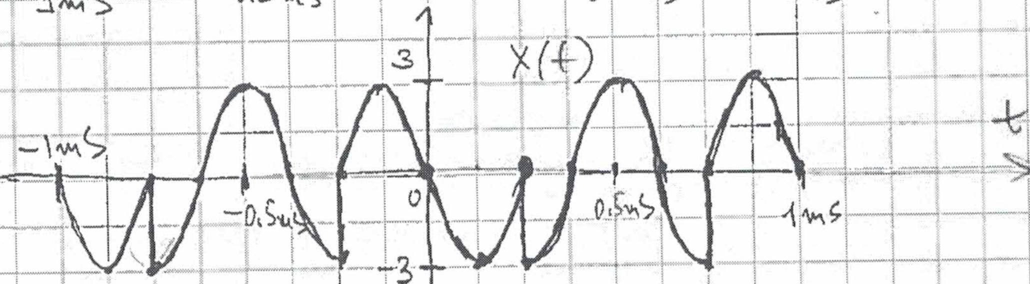
where $F_1 = 80\,\text{MHz}$, $F_2 = 110\,\text{MHz}$.

Typical spectrum of comp. signal:



#4





Since $\beta_D = 1 \Rightarrow \Delta\varphi = \frac{\pi}{2}\cdot\beta_D = \pi/2$

when $m(t)=1$, $x(t) = 3\cos(\omega_c t + \frac{\pi}{2}) = -3\sin\omega_c t$

when $m(t)=0$, $x(t) = 3\cos(\omega_c t)$, $f_c = 2kHz \rightarrow T_c = 0.5mS$.

To find the spectrum of $x(t)$, we present it as

$$x(t) = \frac{1}{2}\left(e^{j\omega_c t}e^{j\Delta\varphi m(t)} + e^{-j\omega_c t}e^{-j\Delta\varphi m(t)}\right)$$

and find the spectrum of $z(t) = e^{j\Delta\varphi m(t)}$ first:

$$z(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\Omega t}, \text{ where } \Omega = \frac{2\pi}{T_m},\ T_m = 1mS.$$

$$C_n = \frac{1}{T_m}\int_{-T_m/4}^{3T_m/4} z(t)e^{-jn\Omega t}dt = \frac{1}{T_m}\int_{-T_m/4}^{T_m/4} e^{j\Delta\varphi}e^{-jn\Omega t}dt +$$

$$+\frac{1}{T_m}\int_{T_m/4}^{3T_m/4} e^{-jn\Omega t}dt = \frac{1}{T_m}e^{j\Delta\varphi}\frac{e^{-jn\Omega t}}{-jn\Omega}\Bigg|_{-T_m/4}^{T_m/4} + \frac{1}{T_m}\frac{e^{-jn\Omega t}}{-jn\Omega}\Bigg|_{T_m/4}^{3T_m/4} =$$

$$= \frac{\sin(\frac{\pi}{2}n)}{\pi n}\left[e^{j\pi/2} - 1\right] = \frac{1}{2}\mathrm{sinc}\left(\frac{n}{2}\right)(j-1);$$

#4 Using this result, we present $x(t)$ as

$$x(t) = \frac{1}{2}\sum_{n=-\infty}^{+\infty} C_n e^{j(\omega_c + n\Omega)t} + \frac{1}{2}\sum_{n=-\infty}^{+\infty} C_n^* e^{-j(\omega_c + n\Omega)t}$$

Hence, FT of $x(t)$ is

$$S_x(f) = \frac{1}{2}\sum_{n=-\infty}^{+\infty} C_n \delta(f - f_c - nF) + \frac{1}{2}\sum_{n=-\infty}^{+\infty} C_n^* \delta(f + f_c + nF).$$

where $f_c = \omega_c/2\pi$, $F = \Omega/2\pi$.

When $f_c = 2\,MHz \gg F = 1\,kHz$, positive and negative frequency parts of the spectrum (1st and 2nd terms correspondingly) do not overlap and the null-to-null bandwidth is determined by C_n:

$$C_n = 0 \Rightarrow \text{sinc}\left(\frac{n}{2}\right) = 0 \Rightarrow n = 2 \text{ (1st null)}.$$

Hence, $\Delta f = 2 \cdot n \cdot F = 2 \cdot 2 \cdot 1\,kHz = 4\,kHz$.

To compare with the random message case, find R first: $R = 2F = 2\,kHz$, hence

$\Delta f_{random} = 2R = 4\,kHz \rightarrow$ the same as for determenistic square-wave of the same period.

Hence, determenistic square-wave provides good approximation for a random message (sometimes!).