ELG3175
Introduction to Communication Systems

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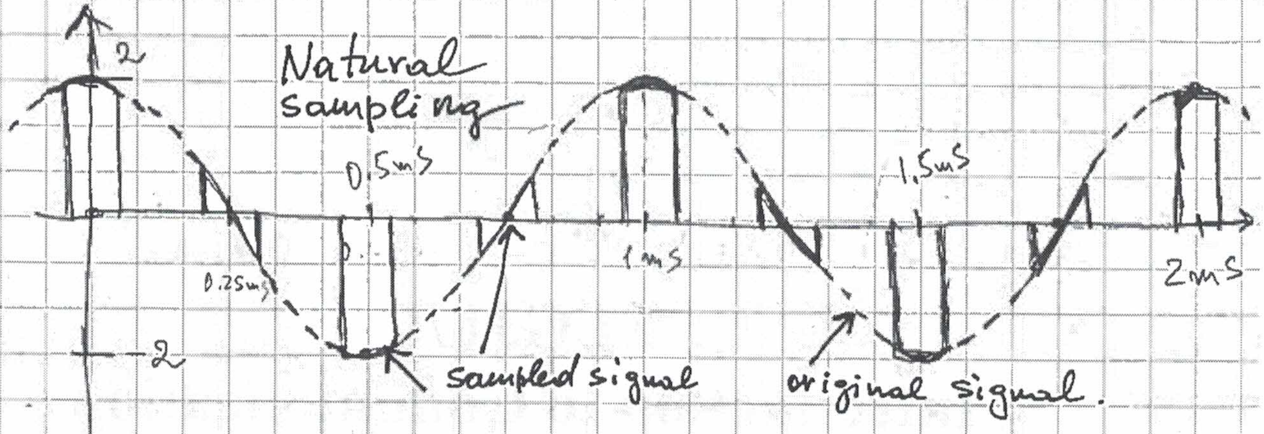
Solutions

Assignment #7

#3



Since the spectrum of the sampled signal is

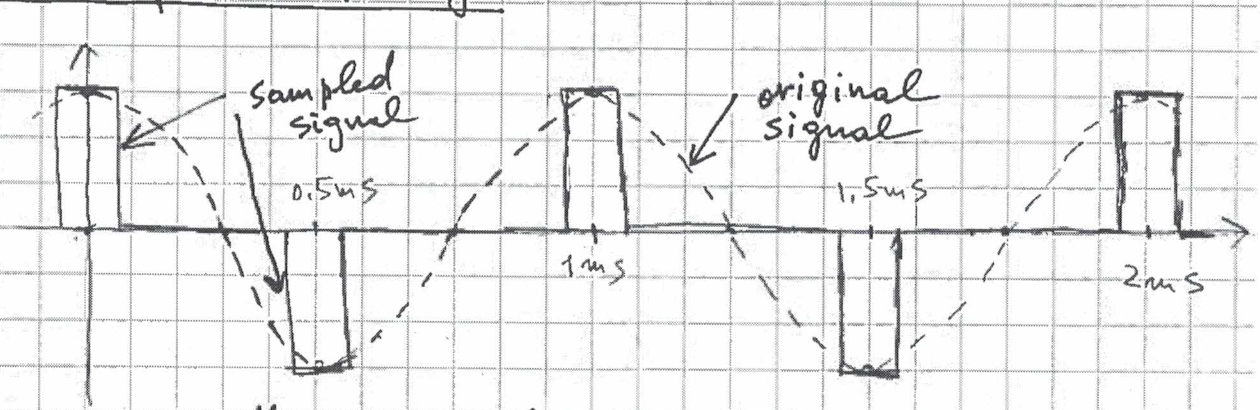
$$S_{x_S}(f) = d \sum_{k=-\infty}^{+\infty} \operatorname{sinc}(kd)\, S_x(f - kf_s)$$

where $S_x(f)$ is the spectrum of the original signal $x(t)$, the reconstruction filter will pass the copy with $k=0$:

$$S_{out}(f) = \frac{1}{d}\left(d \cdot S_x(f)\right) = S_x(f)$$

i.e., the same as the original signal $x(t)$! there is no distortions in this case.

c) <u>flat-top sampling:</u>



The spectrum of the sampled signal is

$$S_{x_S}(f) = d \cdot \operatorname{sinc}(\tau f) \sum_{k=-\infty}^{+\infty} S_x(f - kf_s)$$

Ideal LPF will pass $k=0$ component only: ⇓

#3 $S_{out}(f) = \frac{1}{d}\cdot\left[d\ \mathrm{sinc}(\tau f)\cdot S_x(f)\right] =$

$= \mathrm{sinc}(\tau f)\cdot S_x(f)$, where $\tau = T_s \cdot d =$

$= 0.125\ mS.$

The spectrum of $x(t)$ is

$$S_x(f) = \delta(f-F) + \delta(f+F)$$

Hence, $S_{out}(f) = \mathrm{sinc}(\tau \cdot F)\,\delta(f-F) + \mathrm{sinc}(-\tau F)\cdot$

$\cdot\,\delta(f+F) \Rightarrow X_{out} = \mathrm{sinc}(\tau\cdot F)\cdot \cos(2\pi F t) =$

$= \mathrm{sinc}(\tau\cdot F)\cdot X(t) = \mathrm{sinc}(0.125\cdot 10^{-3}\cdot 10^{3})\, x(t) =$

$= 0.97\ x(t)$

i.e. the amplitude is reduced by 3%.

It is different from (b) because flat-top sampling requires a reconstruction filter other than an ideal low-pass filter (see lecture notes). However, when pulse width τ is small, the distortion is small as well. In this case, flat-top sampling is close to ideal sampling using delta-functions.

Problem [redacted] **5**

The sampling rate is $f_s = 44100$ meaning that we take 44100 samples per second. Each sample is quantized using 16 bits so the total number of bits per second is 44100×16. For a music piece of duration 50 min $= 3000$ sec the resulting number of bits per channel (left and right) is

$$44100 \times 16 \times 3000 = 2.1168 \times 10^9$$

and the overall number of bits is

$$2.1168 \times 10^9 \times 2 = 4.2336 \times 10^9 \approx 505 \text{ MB}$$

For a sinusoid, $\beta = 2 = 3\text{dB}$ and

$$SQNR = -\beta + 6\nu + 4.8 = -3 + 6 \cdot 16 + 4.8 \approx 98\text{dB}$$

If $SQNR = 60\text{dB}$, then

$$\nu = \frac{1}{6}(SQNR + \beta - 4.8) = \overbrace{\frac{1}{6}(60 + 3 - 4.8)}^{=a} \Rightarrow 10$$

(the closest integer which is $\geq a$ is used).

if sinusoid level is $\frac{1}{2}$ of the maximum allowed level, then $\Rightarrow \beta = \dfrac{X_{max}^2}{A^2/2} = 2\left(\dfrac{X_{max}}{A}\right)^2 = 8 = 9\text{dB}$

where $A \to$ sinusoid amplitude, $X_{max} \to$ max. allowed level (if signal $>$ max. allowed level $\to$ saturation distortion will appear). Hence,

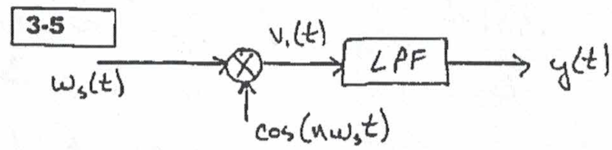
$$SQNR = -\beta + 6\nu + 4.8 = -9 + 6 \cdot 16 + 4.8 \approx 92\text{dB}$$

if $SQNR = 60\text{dB}$, then

$$\nu = \frac{1}{6}(SQNR + \beta - 4.8) = \frac{1}{6}(60 + 9 - 4.8) \Rightarrow 11$$

$\to$ one more bit is required. This is a manifestation of <u>6-dB rule</u>.

Problem 3.



3- 5 Cont'd (a.)

$$w_s(t) = w(t)\left[d + 2d\sum_{k=1}^{\infty}\frac{\sin(k\pi d)}{k\pi d}\cos(k\omega_s t)\right]$$

$v_1(t) = w_s(t)\cos(n\omega_s t)$ Deleted by LPF Deleted by LPF

$$= w(t)\Big\{d\,\cancel{\cos(n\omega_s t)} + 2d\sum_{\substack{k=1\\k\neq n}}^{\infty}\frac{\sin(k\pi d)}{k\pi d}\,\cancel{\cos(k\omega_s t)\cos(n\omega_s t)}$$

$$+ 2d\,\frac{\sin(n\pi d)}{n\pi d}\underbrace{\cos^2(n\omega_s t)}_{\frac{1}{2}+\frac{1}{2}\cancel{\cos(2n\omega_s t)}}$$ Deleted by LPF

$$\Rightarrow y(t) = \frac{d\sin(n\pi d)}{n\pi d}\,w(t) = C\,w(t)$$

(b) $$C = \frac{d\sin(n\pi d)}{n\pi d}$$

#3 for a sinusoid $\beta = 2$ and

$$SQNR = \frac{3N^2}{2} \Rightarrow N \geq \sqrt{\frac{2\,SQNR}{3}} = 25.8$$

Hence, $N = 26$. The number of bits per sample:

$$\nu = \log_2 N = \lceil 4.7 \rceil = 5 \text{ bits/sample.}$$

The minimum bandwidth is

$$\Delta f = \frac{\nu}{2} f_s = 20 \text{ kHz}.$$

For μ-law companding ($\mu = 255$),

$$N \geq \ln(1+\mu)\sqrt{SQNR/3} = 101.2$$

Hence, $N = 102$; $\nu = \log_2 N = \lceil 6.7 \rceil = 7$.

and $\Delta f = \frac{\nu}{2} f_s = 28 \text{ kHz}$.

Companding provides advantage when probability of small signal levels is high, i.e. when β is large. In our case, $\beta = 2$ (sinusoid) $\rightarrow$ hence, no advantage due companding.

#1 The required sampling frequency is

$$f_s = \left(\frac{8\pi^2}{3} f_m^2 f_{LPF} \cdot SQNR\right)^{1/3} = 3.5\ MHz$$

where $f_m = 20\ kHz$ (max. message frequency), $f_{LPF} = 40\ kHz$, $SQNR = 10^5$.

The step size is (minimum)

$$\Delta = 2\pi A\, f_m / f_s = 0.036\ V$$

(i) when $f_m = 200\ kHz$ and $f_{LPF} = 400\ kHz$, f_s changes to 35 MHz, but Δ stays the same.

(ii) when $A = 10\ V$, f_s stays the same, but Δ changes to 0.36 V.

The effect of max. message frequency is to increase f_s. The effect of message amplitude is to increase Δ.

(#2) The required transmission rate is

$$R = 10^4 \text{ samples/s} \quad (\text{since binary})$$

Hence,
$$\Delta f = \frac{1+\alpha}{2} R = 5.5 \text{ kHz} \text{ for } \alpha = 0.1$$

and $\Delta f = 9.5 \text{ kHz}$ for $\alpha = 0.9$.

If 16-level PAM is used, than the required transmission rate is

$$R = \frac{10^4}{4} \text{ samples/s} \quad (4 \text{ bits/sample!})$$

and $\Delta f = \frac{1+\alpha}{2} R = 1.4 \text{ kHz}$ for $\alpha = 0.1$ and

$\Delta f = 2.4 \text{ kHz}$ for $\alpha = 0.9 \rightarrow$ substantially less than for binary!

(#4) The number of quantizing levels is

$$\frac{\Delta}{2} = \frac{A}{N} \leq 0.01 \cdot A \implies N \geq 100$$

where A - signal amplitude.

Choose $N = 2^7 = 128$, then $\nu = \log_2 N = 7$ bit.

Nyquist rate is $f_N = 2 \cdot f_{max} = 2 \cdot 2\,kHz = 4\,kHz$ (for each signal!), and sampling rate:

$$f_s = 1.25\, f_N = 5000 \text{ samples/s}$$

The total sampling rate (for all 8 signals) is

$$f_{s,t} = 8 \cdot f_s = 40 \cdot 10^3 \text{ samples/s}.$$

The total transmission rate is

$$R = 7 \cdot f_{s,t} = 280\, kb/s$$

The min. bandwidth is

$$\Delta f = \frac{1+\alpha}{2} R = \frac{1+0.2}{2} 280\, kb/s =$$

$$= 168\, kHz.$$